



**TOPIC: 4.7 MAXIMA AND MINIMA OF FUNCTIONS OF TWO VARIABLES**

② Find the dimensions of the rectangular box, open at the top, of maximum capacity whose surface area is 432 square meter.

Let  $x, y, z$  be the rectangular box dimension

$$\text{Volume } f(x, y, z) = xyz$$

$$\text{Surface area } g(x, y, z) = xy + 2yz + 2xz = 432$$

$$g(x, y, z) = xy + 2yz + 2xz - 432$$

$$\text{Hence } F(x, y, z) = f(x, y, z) + \lambda g(x, y, z)$$

$$= xyz + \lambda (xy + 2yz + 2xz - 432) \rightarrow \textcircled{1}$$

Diff.  $\textcircled{1}$  partially w.r.t 'x', 'y' and 'z'



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$$\frac{\partial F}{\partial x} = yz + \lambda (y + 2z) = 0$$

$$\lambda (y + 2z) = -yz$$

$$\lambda = \frac{-yz}{y+2z} \rightarrow \textcircled{2}$$

$$\frac{\partial F}{\partial y} = xz + \lambda (x + 2z) = 0$$

$$\lambda (x + 2z) = -xz$$

$$\lambda = \frac{-xz}{x+2z} \rightarrow \textcircled{3}$$

$$\frac{\partial F}{\partial z} = xy + \lambda (2y + 2x) = 0$$

$$\lambda (2y + 2x) = -xy$$

$$\lambda = -\frac{xy}{2y+2x} \rightarrow \textcircled{4}$$

From  $\textcircled{2}$  &  $\textcircled{3}$ ,

$$\frac{-yz}{y+2z} = \frac{-xz}{x+2z} \Rightarrow \frac{y}{y+2z} = \frac{x}{x+2z}$$

$$\Rightarrow xy + 2yz = xy + 2xz$$

$$\Rightarrow 2yz = 2xz \Rightarrow \boxed{x = y}$$



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From (3) & (4),  
$$\leftarrow -\frac{xz}{x+2z} = -\frac{xy}{2y+2x} \Rightarrow \frac{z}{x+2z} = \frac{y}{2y+x}$$

$$\Rightarrow 2yz + 2xz = xy + 2yz$$

$$\Rightarrow 2xz = xy \Rightarrow \boxed{y = 2z}$$

$$\therefore \boxed{x = y = 2z}$$

sub. in  $g(x, y, z)$

$$xy + 2yz + 2xz = 432$$

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$$xy + 2yz + 2xz = 432$$

$$(2z)(2z) + 2(2z)z + 2(2z)z = 432$$

$$\Rightarrow 4z^2 + 4z^2 + 4z^2 = 432$$

$$\Rightarrow 12z^2 = 432 \Rightarrow z^2 = 36$$

$$\Rightarrow \boxed{z = 6}$$

$$\therefore \boxed{x = 12}$$

$$\boxed{y = 12}$$