Topic: 4.5 – TAYLOR SERIES FOR FUNCTIONS OF TWO VARIABLES

(3) Expand $\sin(xy)$ in powers of $x-1$ and $y-\frac{\pi}{2}$ upto second degree terms.

Function	Value at $(1, \pi/2)$
$f(x,y) = \sin(xy)$	$f = 1$
$f_x = y\cos(xy)$ $f_y = x\cos(xy)$	$f_x = 0$ $f_y = 0$
$f_{xx} = -y^2\sin(xy)$ $f_{xy} = -xy\sin(xy) + \cos xy$ $f_{yy} = -x^2\sin(xy)$	$f_{xx} = -\pi^2/4$ $f_{xy} = -\pi/2$ $f_{yy} = -1$

Here $a = 1$, $b = \pi/2$, $h = x-1$ and $K = y-\pi/2$

$$f(x,y) = 1 + \left[(x-1)\,0 + (y-\pi/2)\,0\right] + \frac{1}{2!}\left[(x-1)^2\left(-\frac{\pi^2}{4}\right) + 2(x-1)(y-\pi/2)(-\pi/2) + (y-\pi/2)^2(-1)\right] + \cdots$$

$$= 1 + \frac{1}{2}\left[-\frac{\pi^2}{4}(x-1)^2 - \pi(x-1)(y-\pi/2) - (y-\pi/2)^2\right] + \cdots$$

(4) Find the Taylor's series expansion of $x^2y^2+2x^2y+3xy^2$ in powers of $(x+2)$ and $(y-1)$ upto third degree terms.

Function	Value at $(-2,1)$
$f(x,y) = x^2y^2+2x^2y+3xy^2$	$f = 6$
$f_x = 2xy^2+4xy+3y^2$ $f_y = 2x^2y+2x^2+6xy$	$f_x = -9$ $f_y = 4$
$f_{xx} = 2y^2+4y$ $f_{xy} = 4xy+4x+6y$ $f_{yy} = 2x^2+6x$	$f_{xx} = 6$ $f_{xy} = -10$ $f_{yy} = -4$

$f_{xx} = 2y^2+4y$ $f_{xy} = 4xy+4x+6y$ $f_{yy} = 2x^2+6x$	$f_{xx} = 6$ $f_{xy} = -10$ $f_{yy} = -4$
$f_{xxx} = 0$ $f_{xxy} = 4y+4$ $f_{xyy} = 4x+6$ $f_{yyy} = 0$	$f_{xxx} = 0$ $f_{xxy} = 8$ $f_{xyy} = -2$ $f_{yyy} = 0$

$$f(x,y) = 6 + \frac{1}{1!}\left[-9(x+2)+4(y-1)\right] + \frac{1}{2!}\left[6(x+2)^2-20(x+2)(y-1)-4(y-1)^2\right] + \frac{1}{3!}\left[24(x+2)^2(y-1)-6(x+2)(y-1)^2\right]+\cdots$$