



TOPIC:4.6- MAXIMA AND MINIMA OF FUNCTIONS OF TWO VARIABLES

Maxima and Minima

State the condition for $f(x, y)$ be a maximum or minimum.

<u>Maximum</u>	<u>Minimum</u>
(i) $f_x = 0$	(i) $f_x = 0$
(ii) $f_y = 0$	(ii) $f_y = 0$
(iii) $f_{xx} f_{yy} - (f_{xy})^2 > 0$	(iii) $f_{xx} f_{yy} - (f_{xy})^2 > 0$
(iv) $f_{xx} < 0$ (or) $f_{yy} < 0$	(iv) $f_{xx} > 0$ (or) $f_{yy} > 0$



① A rectangular box open at the top, is to have a volume of 32 cc. Find the dimensions of the box, that requires least materials for its construction.

Let x, y, z be the dimensions of the rectangular box.

Surface area $f(x, y, z) = xy + 2xz + 2yz$

Volume $g(x, y, z) = xyz = 32$

$$g(x, y, z) = xyz - 32$$

$$\text{Hence } F(x, y, z) = f(x, y, z) + \lambda g(x, y, z)$$

$$= xy + 2xz + 2yz + \lambda(xyz - 32)$$

→ (1)



Diff. ① partially w.r.t. 'x', 'y', 'z'.

$$\frac{\partial F}{\partial x} = y + 2z + yz\lambda = 0$$

$$yz\lambda = -y - 2z$$

$$\lambda = - \left[\frac{y + 2z}{yz} \right]$$

$$\lambda = - \left[\frac{1}{z} + \frac{2}{y} \right] \rightarrow \textcircled{2}$$

$$\frac{\partial F}{\partial y} = x + 2z + xz\lambda = 0$$

$$xz\lambda = -x - 2z$$

$$\lambda = - \left[\frac{x + 2z}{xz} \right]$$

$$\lambda = - \left[\frac{1}{z} + \frac{2}{x} \right] \rightarrow \textcircled{3}$$



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$$\frac{\partial F}{\partial z} = 2x + 2y + xy\lambda = 0$$

$$xy\lambda = -2x - 2y$$

$$\lambda = - \left[\frac{2x+2y}{xy} \right]$$

$$\lambda = - \left[\frac{2}{y} + \frac{2}{x} \right] \rightarrow \textcircled{4}$$

From $\textcircled{2}$ & $\textcircled{3}$

$$- \left[\frac{1}{z} + \frac{2}{y} \right] = - \left[\frac{1}{z} + \frac{2}{x} \right]$$

$$\frac{2}{y} = \frac{2}{x} \Rightarrow \boxed{x = y}$$

From $\textcircled{3}$ & $\textcircled{4}$,

$$- \left[\frac{1}{z} + \frac{2}{x} \right] = - \left[\frac{2}{y} + \frac{2}{x} \right]$$

$$\frac{1}{z} = \frac{2}{y} \Rightarrow \boxed{y = 2z}$$

$$\therefore x = y = 2z$$

Sub. in $g(x, y, z)$

$$xyz = 32 \Rightarrow 2z \cdot 2z \cdot z = 32$$

$$\Rightarrow 4z^3 = 32 \Rightarrow z^3 = 8 \Rightarrow \boxed{z = 2}$$

$$\boxed{x = 4} \text{ and } \boxed{y = 4}$$