



TOPIC: 4.4- TAYLOR SERIES FOR FUNCTIONS OF TWO VARIABLES.

Taylor's series for functions of two variables

$$\begin{aligned} f(x, y) = & f(a, b) + \frac{1}{1!} [h f_x(a, b) + k f_y(a, b)] \\ & + \frac{1}{2!} [h^2 f_{xx}(a, b) + 2hk f_{xy}(a, b) + k^2 f_{yy}(a, b)] \\ & + \frac{1}{3!} [h^3 f_{xxx}(a, b) + 3h^2k f_{xxy}(a, b) \\ & + 3hk^2 f_{xyy}(a, b) + k^3 f_{yyy}(a, b)] + \dots \end{aligned}$$

where $h = x - a$ and $k = y - b$



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② Expand $e^x \log(1+y)$ in powers of x and y upto terms of third degree.

Function	Value at (0,0)
$f(x,y) = e^x \log(1+y)$	$f = 0$
$f_x = e^x \log(1+y)$ $f_y = e^x \frac{1}{1+y}$	$f_x = 0$ $f_y = 1$
$f_{xx} = e^x \log(1+y)$ $f_{xy} = e^x \cdot \frac{1}{1+y}$ $f_{yy} = -\frac{e^x}{(1+y)^2}$	$f_{xx} = 0$ $f_{xy} = 1$ $f_{yy} = -1$



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$$f_{xxx} = e^x \log(1+y)$$

$$f_{xxy} = \frac{e^x}{1+y}$$

$$f_{xyy} = -\frac{e^x}{(1+y)^2}$$

$$f_{yyy} = +\frac{2e^x}{(1+y)^3}$$

$$f_{xxx} = 0$$

$$f_{xxy} = 1$$

$$f_{xyy} = -1$$

$$f_{yyy} = 2$$

Here $a=0$, $b=0$, $h=x$ and $k=y$.

$$\begin{aligned} f(x,y) &= 0 + \frac{1}{1!} [x(0) + y(1)] + \frac{1}{2!} [x^2(0) + 2xy(1) + y^2(-1)] \\ &\quad + \frac{1}{3!} [x^3(0) + 3x^2y(1) + 3xy^2(-1) + y^3(2)] \\ &= y + \frac{1}{2!} [2xy - y^2] + \frac{1}{3!} [3x^2y - 3xy^2 + 2y^3] \end{aligned}$$