SNS COLLEGE OF ENGINEERING

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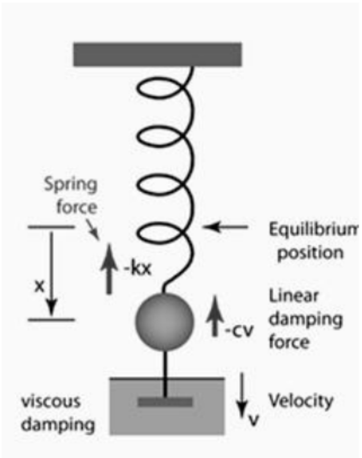
AN AUTONOMOUS INSTITUTION

Approved by AICTE, New Delhi and Affiliated to Anna University, Chennai.

UNIT -II WAVES & OPTICS

TOPIC – II DAMPED OSCILLATION

Let us consider a spring in which a body of mars 'm' is suspended as shown in figure.



Here two types of forces act on it.

1. Restoring Force 2. A frictional Force

Restoring Force that act on opposite direction to the displacement in order to restore its original position.

$F1 \alpha - y$

$F1 = -Ky$ ------------- (1)

Where, K → Force Constant

y → Displacement

-ve sign indicates restoring force is acting in the Opposite direction

A Frictional force (or) damping force due to air resistance, which also acts opposite to the direction of motion

$F2 \alpha - \frac{dy}{dt}$

$$F2 = -r\frac{dy}{dt} \text{ ----------------- (2)}$$

Where, r → Frictional Force

$\frac{dy}{dt}$ → Velocity

-ve sign indicates damping force act along the opposite direction

Therefore Total Force (F) = F1 + F2

$$F = -ky - r\frac{dy}{dt} \text{ -------------- (3)}$$

From Newton's II law

F = ma

$$F = m\frac{d^2y}{dt^2} \text{ ---------------------- (4)}$$

Equating (3) and (4),

$$m\frac{d^2y}{dt^2} = -ky - r\frac{dy}{dt}$$

$$\frac{d^2y}{dt^2} = -\frac{ky}{m} - \frac{r}{m}\frac{dy}{dt}$$

$$\frac{d^2y}{dt^2} + \frac{ky}{m} + \frac{r}{m}\frac{dy}{dt} = 0 \text{ ---------------- (5)}$$

Let $\frac{r}{m} = 2b$ & $\frac{k}{m} = \omega^2$

Therefore equation can be written as,

$$\frac{d^2y}{dt^2} + 2b\,\frac{dy}{dt} + \omega^2 y = 0 \quad \text{----------------} \ (6)$$

Equation 6 is II order differential Equation, for which the solution is,

$$y = A\,e^{\alpha t} \quad \text{----------------} \ (7)$$

Where, A and α are arbitrary constant

Differentiate equation (7) twice with respect to "t",

$$\frac{dy}{dt} = A\,e^{\alpha t}\,\alpha \quad \text{------------} \ (8)$$

$$\frac{d^2y}{dt^2} = = A\,e^{\alpha t}\,\alpha^2 \quad \text{------------} \ (9)$$

Substitute (7), (8) and (9) in (6) we get,

$$A\alpha^2 e^{\alpha t} + 2b\,A\,\alpha\,e^{\alpha t} + \omega^2 A\,e^{\alpha t} = 0$$

$$A\,\alpha\,e^{\alpha t}\,[\alpha^2 + 2b\alpha + \omega^2] = 0$$

Here,

$$Ae^{\alpha t} \neq 0$$

Therefore

$$\alpha^2 + 2b\alpha + \omega^2 = 0$$

Solving this we get,

$$\alpha = -b \pm \sqrt{(b^2 - \omega^2)}$$

Substitute "α" in equation (7),

Y= A exp [-b $\pm\sqrt{(b^2 - \omega^2)}$] t

Y= A1 exp [-b + $\sqrt{b^2 - \omega^2} t + A2$exp[-b - $\sqrt{(b^2 - \omega^2)}$] t ------------- (11)

Where A1 and A2 are arbitrary constant.

Special Cases :

Case 1 : When $b^2 > \omega^2$ → Real Type on Motion is called Over damped Oscillation (or) Dead Beat

Eg : Pendulum moving in a thick coil media

Case 2 : When $b^2 = \omega^2$ not exactly zero but has a small value say "β" type of motion is called critical dmaped motion or critical oscillation

Eg : Sensitive Galvanometer

Case 3 : When $b^2 < \omega^2$ Imaginary. This type of motion is called under damped motion

Eg : Motion of Simple pendulum.