

# Quine - Mc Cluskey Minimization Technique.

The Quine - Mc Cluskey minimization technique is an algorithm that uses the same Boolean algebra postulates that were used with K-maps, but in a form suitable for a computer solution.

Larger the K-map, the more difficult the pattern recognition becomes. The Quine - McCluskey - approach eliminates the need for pattern recognition.

This method is used to minimize the boolean eqn.

1) Simplify the following using Quine - McCluskey minimization technique.

$$D = f(a, b, c, d) = \sum (0, 1, 2, 3, 6, 7, 8, 9, 14, 15)$$

	a	b	c	d
0 -	0	0	0	0
1 -	0	0	0	1
2 -	0	0	1	0
3 -	0	0	1	1
6 -	0	1	1	0
7 -	0	1	1	1
8 -	1	0	0	0
9 -	1	0	0	1
14 -	1	1	1	0
15 -	1	1	1	1

Grouping minterms according to the number of 1s

Group	Minterms	Variables				
		a	b	c	d	
0	0	0	0	0	0	✓
1	1	0	0	0	1	✓
	2	0	0	1	0	✓
	8	1	0	0	0	✓
	9	1	0	0	1	✓
2	6	0	1	1	0	✓
	7	0	1	1	1	✓
	14	1	1	1	0	✓
3	15	1	1	1	1	✓
	15	1	1	1	1	✓

## Creation of minterm groups of two

Group	Minterm	Variables				
		a	b	c	d	
0	0,1	0	0	0	-	✓
0	0,2	0	0	-	0	✓
0	0,8	-	0	0	0	✓
1	1,3	0	0	-	1	✓
1	1,9	-	0	0	1	✓
1	2,3	0	0	1	-	✓
1	2,6	0	-	1	0	✓
1	8,9	1	0	0	-	✓
2	3,7	0	-	1	1	✓
2	6,7	0	1	1	-	✓
2	6,14	-	1	1	0	✓
3	7,15	-	1	1	1	✓
3	14,15	1	1	1	-	✓

## Creation of minterm groups of four

Group	Minterm	Variable:			
		a	b	c	d
0	0,1,2,3	0	0	-	-
0	0,1,8,9	-	0	0	-
1	2,6,3,7	0	-	1	-
2	6,7,14,15	-	1	1	-

$$\bar{a}\bar{b} + \bar{b}\bar{c} + \bar{a}c + bc.$$

Minimized func. will have only essential prime implicant.

Prime Implicant	Min term Individual	0									
		0	1	2	3	6	7	8	9	14	15
$\bar{b}\bar{a}\bar{b}$	0, 1, 2, 3	x	x	x	x						
$\bar{b}\bar{c}$	0, 1, 8, 9	x	x					(x)	(x)		
$\bar{a}\bar{c}$	2, 6, 3, 7			x	x	x	x				
$bc$	6, 7, 14, 15					x	x			(x)	(x)

Circle the x where there only one x.

$$D = \bar{b}\bar{c} + bc$$

From this (0, 1, 8, 9) & (6, 7, 14, 15).

Implicants. (2, 3) minterms are missing.

So write either  $\bar{a}\bar{b}$  or  $\bar{a}\bar{c}$ .

$$D = \bar{b}\bar{c} + bc + \bar{a}\bar{b}$$

(or)

$$D = \bar{b}\bar{c} + bc + \bar{a}\bar{c}$$

2)  $Y(A, B, C, D) = \sum m (0, 1, 3, 7, 8, 9, 11, 15)$

Group	Minterms	Variable					A	B	C	D
		A	B	C	D		0	0	0	0
0	0	0	0	0	0	✓	0	0	0	0
		1	0	0	0	1	✓	0	0	0
1	(1, 8)	3	0	0	1	✓	0	0	1	1
		8	1	0	0	0	✓	1	0	0
2	(3, 9)	7	0	0	1	✓	0	1	0	0
		9	1	0	0	1	✓	1	0	1
3	(7, 11)	15	1	1	1	✓	1	1	1	1
		11	0	1	1	1	✓	0	1	1
4	15	1	1	1	1	✓	1	1	1	1

Creation of min term groups of two

Group	Minterms	A	B	C	D
0	(0, 1)	0	0	0	- ✓
0	(0, 8)	-	0	0	0 ✓
1	(1, 3)	0	0	-	1 ✓
	(1, 9)	-	0	0	1 ✓
	(8, 9)	1	0	0	- ✓
2	(3, 7)	0	-	1	1 ✓
	(3, 11)	-	0	1	1 ✓
	(9, 11)	1	0	-	1 ✓
3	(7, 15)	-	1	1	1 ✓
	(11, 15)	1	-	1	1 ✓

Creation of minterm groups of four

Group	Minterm	A	B	C	D	
0	0, 1, <del>8</del> , 9	-	0	0	-	BC
	0, 8, 1, 9	-	0	0	-	
1	1, 3, 9, 11	-	0	-	1	$\overline{B}D$
	1, 9, 3, 11	-	0	-	1	
2	3, 7, 11, 15	-	-	1	1	$\overline{C}D$
	2, 11, 7, 15	-	-	1	1	

PI	Minterm	0	1	3	7	8	9	11	15
$\overline{B}C$	0, 1, <del>8</del> , 9	(X)	X			(X)	X		
$\overline{B}D$	1, 3, 9, 11		X	X			X	X	
$\overline{C}D$	3, 7, 11, 15			X	(X)			X	(X)

$\Rightarrow \overline{B}C + \overline{C}D$

0, X, 8, 9, 3, X, 11, 15

Quine Mc Clusky Using Don't Care Terms.

$$S = f(w, x, y, z) = \sum (1, 3, 13, 15) + \sum d(8, 9, 10, 11)$$

Group	Minterms	Variables				e
		a	b	c	d	
1	1	0	0	0	0	1
1	4	0	0	1	0	0
2	3	0	0	0	1	1
2	5					
2	6					
2	10					
2	12					
2	18					
2	20					

$(1, 9, 13, 11) \Rightarrow -00-1$   
 $(1, 3, 9, 11) =$

Quine - Mc Cluskey Using Don't care terms

$$S = f(w, x, y, z) = \sum (1, 3, 13, 15) + 2d (8, 9, 10, 11)$$

Group	Minterms	Variables			
		w	x	y	z
1	1	0	0	0	1 ✓
1	8*	1	0	0	0 ✓
2	9*3	0	0	1	1 ✓
2	9*	1	0	0	1 ✓
2	10*	1	0	1	0 ✓
3	11*	1	0	1	1 ✓
3	13	1	1	0	1 ✓
4	15	1	1	1	1 ✓

Creation of min term/\* groups of two

Group	Minterms	w	x	y	z
1	1, 3	0	0	-	1 ✓
1	1, 9*	-	0	0	1 ✓
1	8*, 9*	1	0	0	- ✓
1	8*, 10*	1	0	-	0 ✓
2	3, 11*	-	0	1	1 ✓
2	9*, 11*	1	0	-	1 ✓
2	9*, 13	1	-	0	1 ✓
2	10*, 11*	1	0	1	- ✓
3	11*, 15	1	-	1	1 ✓
3	13, 15	1	1	-	1 ✓

# Creation of minterms /<sup>x</sup> group of 4

Group Minterms Variables

		w	x	y	z
1	1, 3, 9*, 11*	-	0	-	1
1	8*, 9*, 10*, 11*	1	0	-	-
2	9*, 13, 11*, 15	1	-	-	1

→ All are don't care. So neglect

## Prime Implicant table:-

PI terms	Minterms	1	3	13	15
$\bar{x}z$	(1, 3, 9*, 11*)	(x)	(x)		
wz	(9*, 13, 11*, 15)			(x)	(x)

S =  $\bar{x}z + wz$

Reduce  $Q = f(a, b, c, d, e)$

$= \Sigma (1, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 18, 19, 20, 21, 22, 23, 26, 27.)$

Sol Grouping minterms according to no of 1's.

Group	Minterms	<del>16</del> a	<del>8</del> b	84 c	2 d	1 e
1	1	0	0	0	0	1
1	4	0	0	1	0	0
2	3	0	0	0	1	1
2	5	0	0	1	0	1
2	6					1
2	10					1



Obtain the minimal expression for  $f = \sum m(1, 2, 3, 5, 6, 7, 8, 9, 12, 13, 15)$  using tabular method.

Grouping minterms according to no. of 1s

Group	Minterms	Variables				
		A	B	C	D	
1	1	0	0	0	1	✓
1	2	0	0	1	0	✓
1	4, 8	0	1	0	0	✓
2	3	0	0	1	1	✓
	5	0	1	0	1	✓
	6	0	1	1	0	✓
	9	1	0	0	1	✓
	12	1	1	0	0	✓
3	7	0	1	1	1	✓
	13	1	1	0	1	✓
4	15	1	1	1	1	✓

Creation of minterm group of two

Group	Minterms	8	4	2	1	
		A	B	C	D	
1	(1, 3)	0	0	-	1	✓
1	(1, 5)	0	-	0	1	✓
1	(1, 9)	-	0	0	1	✓
1	(2, 3)	0	0	1	-	✓
1	(2, 6)	0	-	1	0	✓
1	(8, 9)	1	0	0	-	✓
1	(8, 12)	1	-	0	0	✓
2	(3, 7)	0	-	1	1	✓

2	(5,7)	0	1	-	1	✓
2	(5,13)	-	1	0	1	✓
2	(6,7)	0	1	1	-	✓
2	(9,13)	1	-	0	1	✓
2	(12,13)	1	1	0	-	✓
3	(7,15)	-	1	1	1	✓
3	(13,15)	1	1	-	1	✓

Creation of minterms group of four:

Group	Minterms	A	B	C	D	
1	1,3,5,7	0	-	-	1	} → $\bar{A}D$
	1,5,3,7	0	-	-	1	
	1,5,9,13	-	-	0	1	} → $\bar{C}D$
	1,9,5,13	-	-	0	1	
2	2,3,6,7	0	-	1	-	} → $B\bar{A}C$
	2,6,3,7	0	-	1	-	
	8,9,12,13	1	-	0	-	} → $A\bar{C}$
	8,12,9,13	1	-	0	-	
2	5,7,13,15	-	1	-	1	} → $BD$
	5,13,7,15	-	1	-	1	

Prime Implicant chart

PI	Minterms	1	2	3	5	6	7	8	9	12	13	15
$\bar{A}D$	(1,3,5,7)	x		x	x		x					
$\bar{C}D$	(1,5,9,13)	x			x			(x)			x	
$\bar{A}C$	(2,3,6,7)		(x)	x		(x)	x					
$A\bar{C}$	(8,9,12,13)							x		(x)	x	
$BD$	(5,13,7,15)				x		x				x	(x)

$\bar{C}D + \bar{A}C + A\bar{C} + BD$

POS -

Find the minimal expressions for  
 $f = \prod M(2, 3, 8, 12, 13) + d(10, 14)$

Group	Minterms	A	B	C	D
1	2	0	0	0	1 ✓
	8	1	0	0	0 ✓
2	3	0	0	1	1 ✓
	10*	1	0	0	1 ✓
	12	1	1	0	0 ✓
3	13	1	1	0	1 ✓
	14*	1	1	1	0 ✓

Creation of minterm group of 2.

Group	Minterms	A	B	C	D	
1	(2, 3)	0	0	1	-	⊗
	(2, 10*)	-	0	1	0	⊗
	(8, 10*)	1	0	-	0	✓
	(8, 12)	1	-	0	0	✓
2	(10*, 14*)	1	-	1	0	✓
	(12, 13)	1	1	0	0	⊗
	(12, 14*)	1	1	-	0	✓

Creation of minterm group of 4:

Group	Minterm	A	B	C	D	
1	(8, 10*, 12, 14*)	1	-	-	0	} A $\bar{D}$
	(8, 12, 10*, 14*)	1	-	-	0	

PPI	Minterms	2	3	8	12	13
$A \bar{D}$	$(8, 10^*, 12, 14^*)$			(x)	x.	
$A \bar{B} C$	$(2, 3)$	x	(x)			
$\bar{B} C \bar{D}$	$(2, 10^*)$	x.				
$AB \bar{C}$	$(12, 13)$				x	(x)

$$f = \cancel{A \bar{D}} + \cancel{A \bar{B} C} + \cancel{AB \bar{C}}$$

$$(\bar{A} + D)(A + B + \bar{C})(\bar{A} + \bar{B} + C) //$$