Duality :-

Boolean algebra is an algebraic structure defined by a set of elements, together with two binary operators '+' &

This important ~~property~~ of Boolean algebra is called

Duality principle states that every algebric expression deducible from the postulates of Boolean algebra remains valid if the operators and identify elements are interchanged.

The duality principle has many applicat expression. If the dual of an algebric expression is desired, we simply interchange OR & AND operators and replace 1's by 0's and 0's by 1's

1) Using Boolean algebra techniques, simplify this expression

$$AB + A(B+C) + B(B+C)$$

$$AB + AB + AC + B \cdot B + B \cdot C$$

$$AB + AC + B + BC$$

$$AB + AC + B(1+C)$$

$$AB + AC + B$$

$$AC + B(1+A) = AC + B$$

2) $[A\bar{B}(C + BD) + \bar{A}\bar{B}]C$

$$[A\bar{B}C + A\bar{B}BD + \bar{A}\bar{B}]C$$

$$[A\bar{B}C + \bar{A}\bar{B}]C = A\bar{B}C \cdot C + \bar{A}\bar{B}C$$

$$= A\bar{B}C + \bar{A}\bar{B}C = \bar{B}C(A+\bar{A})$$

$$= \bar{B}C$$

Rule 1 : $A+0=A$

Rule 2 : $A+1=1$

Rule 3 : $A \cdot 0 = 0$

Rule 4 : $A \cdot 1 = A$

Rule 5 : $A+A=A$.

Rule 6 : $A+\bar{A}=1$

Rule 7 : $A \cdot A = A$.

Rule 8 : $A+\bar{A}=0$.

Rule 9 : $\bar{\bar{A}}=A$.

Rule 10 : $A+AB = A+B$

Rule 11 : $\bar{A}+AB = A+B$.

12 : $(A+B)(A+C) = A+BC$.

~~Property of~~.

Duality property :-

Find the complement of the function F_1 & F_2 by taking their duals and complementing each literal.

1. $F_1 = x'yz' + x'y'z$

dual of func. $F_1 = (x'+y+z) \cdot (x'+y'+z)$

Complement of F_1 = $(x+y'+z)(x+y+z')$.

by complementing each literal

2. $F_2 = x(y'z'+yz)$

$x+(y'+z') \cdot (y+z)$.

Complement of $F_2 = x' + (y+z) \cdot (y'+z')$.

3 F_2, $= x'yz' + x'y'z$

$= (x'+y+z') \cdot (x'+y'+z)$

$= (x+y'+z) \cdot (x+y+z')$.

F_2 = $x(y'z' + yz)$.

Co duality = $x + (y'+z')\cdot(y+z)$

= $x' + (y+z)(y'+z')$.

Find the complement of functions for following. By applying De Morgan's theorems as many times as necessary, the complements are obtained as follows:

1) $F_1 = x'yz' + x'y'z$.

$F_1' = \overline{\bar{x}y\bar{z} + \bar{x}\bar{y}z}$

$= \overline{\bar{x}y\bar{z}} \cdot \overline{\bar{x}\bar{y}z} = \bar{\bar{x}} + \bar{y} + \bar{\bar{z}} \cdot \bar{\bar{x}} + \bar{\bar{y}} + \bar{z}$

$= x + \bar{y} + \bar{\bar{z}} \cdot \bar{\bar{x}} + \bar{\bar{y}} + \bar{z}$,

$= (x + \bar{y} + z)(x + y + \bar{z})$

2) $F_2 = x(y'z' + yz)$

$F_2' = \overline{x(\bar{y}\bar{z} + \bar{\bar{y}}\bar{\bar{z}})}$

$= \bar{x} + \overline{(\bar{y}\bar{z} + yz)}$

$= \bar{x} + (\overline{\bar{y}\bar{z}})\cdot(\overline{yz})$

$= \bar{x} + (\bar{\bar{y}} + \bar{\bar{z}})\cdot(\bar{y} + \bar{z})$

$= \bar{x} + (y+z)\cdot(\bar{y} + \bar{z})$.

Canonical Forms :-

To place a SOP equation into canonical form using Boolean algebra, we do the following:

1. Identify the missing variables in each AND term.

2. AND the missing term and its complement with the original AND term.

$xy(z + \bar{z})$. $(z + \bar{z}) = 1$.

3. Expand the term by application of the property of distribution

$$xyz + xy\bar{z}$$

To place a pos eau into canonical form using Boolean algebra, we do this

1. Identify the missing in each OR term.
2. OR the missing terms (s) and its complement with the original OR term, $x+\bar{y}+(z\bar{z})$ $[z\bar{z}=0]$
$(x+\bar{y}+z)\cdot(x+\bar{y}+\bar{z})$.

1) Place the following equations into proper Canonical form.

ii) $P = ab' + ac' + bc$ → sop.

$= ab'(c+c') + ac'(b+b') + bc(a+a')$

$= ab'c + ab'c' + abc' + ab'c' + abc + a'bc$.

2) a Minterms and Maxterms:

Consider 2 binary numbers in AND operations.

Consider xy.

Possible Combinations ⇒ $xy + x'y + xy' + x'y'$.

These four AND terms is called minterm.

In a similar ~~functions~~ fashion, n variables forming an OR term, with each variable provide 2^n possible combinations, max terms or standard sums.

Minterms & max terms for 3 Binary Variables

x	y	z	Minterms Term	Minterms Designation	Maxterms Term	Maxterms Designation
0	0	0	$x'y'z'$	m_0	$x+y+z$	M_0
0	0	1	$x'y'z$	m_1	$x+y+z'$	M_1
0	1	0	$x'yz'$	m_2	$x+y'+z$	M_2
0	1	1	$x'yz$	m_3	$x+y'+z'$	M_3
1	0	0	$xy'z'$	m_4	$x'+y+z$	M_4
1	0	1	$xy'z$	m_5	$x'+y+x'$	M_5
1	1	0	xyz'	m_6	$x'+y'+x$	M_6
1	1	1	xyz	m_7	$x+y+z$	M_7

Convert the following Boolean expression in standard POS form:

$$(A+\overline{B}+C)(\overline{B}+C+\overline{D})(A+\overline{B}+\overline{C}+D)$$

$$A+\overline{B}+C+(D\overline{D})$$

$$=(A+\overline{B}+C+D).(A+\overline{B}+C+\overline{D}).$$

$$\overline{B}+C+\overline{D} = \overline{B}+C+\overline{D}+(A.\overline{A})$$

$$=(A+\overline{B}+C+\overline{D}).(\overline{A}+\overline{B}+C+\overline{D})$$

$$\Rightarrow (A+\overline{B}+C+D).(A+\overline{B}+C+\overline{D})(A+\overline{B}+C+\overline{D}).$$
$$(\overline{A}+\overline{B}+C+\overline{D}).$$
$$(A+\overline{B}+\overline{C}+D)$$