Postulates and Theorems of Boolean Algebra

$x+0=x$. $\qquad x \cdot 1 = x$.

$x+x'=1 \qquad x \cdot x' = 0$.

$x+x=x \qquad x \cdot x = \not{x}\, x$

$x+1=1 \qquad x \cdot 0 = 0$.

Involution. $\overline{(\overline{x})} = x$.

Commutative $\quad x+y=y+x \qquad xy=yx$

Associative $\quad x+(y+z)=(x+y)+z \qquad x(yz)=(xy)z$

Distributive $\quad x(y+z)=xy+xz \qquad x+yz=(x+y)(x+z)$

De Morgan $\quad (x+y)'=x'y' \qquad (xy)'=x'+y'$.

$\overline{(x+y)} = \bar{x}\,\bar{y} \qquad \overline{(xy)} = \bar{x}+\bar{y}$

Absorption $\quad x+x.y=x \qquad x(x+y)=x$.

Duality Theorem:

$x+0=x \qquad x \cdot 1 = x$

$x+1=1 \qquad x \cdot 0 = 0$

$x+x=x \qquad x \cdot x = x$

$x+x'=1 \qquad x \cdot x' = 0$

$x+y=y+x \qquad x \cdot y = y \cdot x$

$x+(y+z)=$.

$(x+y)+z$.

De-Morgan's Theorems -

De-Morgan's theorem states that

The complement of a product of variables is equal to the sum of the complement of the variables.

(or)

The complement of two or more variables ANDed is equal to the ~~product of the~~ OR. ~~Complements of the~~ of the complements of the individual variables.

$\overline{XY} = \overline{X} + \overline{Y}$

The complement of sum of variables is equal to the product of the complement of the variables $\overline{X+Y} = \overline{X} \cdot \overline{Y}$'

$\overline{X+Y} = \overline{X} \cdot \overline{Y}$'

X	Y.	X+Y.	$\overline{X+Y}$ ①	$\overline{X}$	$\overline{Y}$	$\overline{X} \cdot \overline{Y}$ ②
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

L.H.S = R.H.S

$\overline{XY} = \overline{X} + \overline{Y}$'

X	Y	XY	$\overline{XY}$	$\overline{X}$	$\overline{Y}$	$\overline{X} + \overline{Y}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0.

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L.H.S = R.H.S

1) Apply De Morgan's theorems to the expressions $\overline{XYZ} = \overline{X+Y+Z}$

$$\overline{XYZ} = \bar{X} + \bar{Y} + \bar{Z}$$

$$\overline{X+Y+Z} = \bar{X} \cdot \bar{Y} \cdot \bar{Z}$$

2) Apply De Morgan's theorems of the following expression:

(a) $\overline{(A+B+C)D} = \overline{(A+B+C)} + \bar{D}$

$$\bar{A} \cdot \bar{B} \cdot \bar{C} + \bar{D} \quad //$$

(b) $\overline{ABC + DEF} = \overline{ABC} + \overline{DEF}$

$$= (\bar{A} + \bar{B} + \bar{C}) + (\bar{D} + \bar{E} + \bar{F})$$

(c) $\overline{A\bar{B} + \bar{C}D + EF} = (\overline{A\bar{B}})(\overline{\bar{C}D})(\overline{EF})$

$$= \overline{A\bar{B}} \quad \bar{A} + \bar{\bar{B}} + \bar{\bar{C}} + \bar{D} + \bar{E} + \bar{F}$$

$$= \bar{A} + B + C + \bar{D} + \bar{E} + \bar{F}$$

$$(\overline{A\bar{B}}) \cdot (\overline{\bar{C}D}) \cdot (\overline{EF}) = (\bar{A} + \bar{\bar{B}}) \cdot (\bar{\bar{C}} + \bar{D}) \cdot (\bar{E} + \bar{F}).$$

$$= (\bar{A} + B) \cdot (C + \bar{D}) \cdot (\bar{E} + \bar{F})$$

Apply De Morgan's theorem to each of the following:

(a) $\overline{\overline{(A+B)} + \bar{C}}$

(b) $\overline{(\bar{A} + B) + CD}$

(c) $\overline{(\bar{A} + B)\bar{C}\bar{D} + E + \bar{F}}$

Simplify the following Boolean functions to a min number of literals.

1. $x(x'+y) = \cancel{xx'+xy} \cdot xx'+xy.$

$= 0+xy$

$= xy$

2. $x+x'y. = (x+x')(x+y)$

$= 1\,(x+y) = x+y.$

3. $(x+y)(x+y') = x\cdot x + xy + xy' + yy'.$

$= x + xy + xy' + 0.$

$= x(1+y+y').$

4) $xy + x'z + yz. = xy+x'z+yz(x+x').$

$= xy+x'z+xyz+x'yz.$

$= xy(1+z)+x'z(1+y)$

$= xy+x'z$

5) $(x+y)(x'+z)(y+z)$

$(xx+xy+yz+xz)(y+z)$

$(x+xy+yz+xz)(y+z)$

$xy+xz+xy\cdot y+xyz+xyz+x\cdot z\cdot z.$

$xy+xz+xy+xyz+xz.$

$xy+xz+xyz \cancel{+xz}.$

$xy(1+z)+xz$

$xy+xz.$

Rule 1 : $A + 0 = A$

Rule 2 : $A + 1 = 1$

Rule 3 : $A \cdot 0 = 0$

Rule 4 : $A \cdot 1 = A$

Rule 5 : $A + A = A$.

Rule 6 : $A + \bar{A} = 1$

Rule 7 : $A \cdot A = A$.

Rule 8 : $A + \bar{A} = 0$.

Rule 9 : $\bar{\bar{A}} = A$.

Rule 10 : $A + AB = A + B$

Rule 11 : $\bar{A} + \bar{A}B = A + B$.

12 : $(A+B)(A+C) = A + BC$.

~~Property of~~.

Duality property :-

Find the complement of the function F1 & F2 by taking their duals and complementing each literal.

1. $F_1 = x'yz' + x'y'z$

dual of func. $F_1 = (x' + y + z) \cdot (x' + y' + z)$

Complement of $F_1 = (x + y' + z)(x + y + z')$.

by complementing each literal

2. $F_2 = x(y'z' + yz)$

$x + (y' + z') \cdot (y + z)$.

Complement of $F_2 = x' + (y+z) \cdot (y' + z')$.

3 $F_2{}' = x'yz' + x'y'z$

$= (x' + y + z') \cdot (x' + y' + z)$

$= (x + y' + z) \cdot (x + y + z')$.

F_2 - $F_2 = x(y'z' + yz)$.

Co duality $= x + (y'+z')\cdot(y+z)$

$= x' + (y+z)(y'+z')$.

Find the complement of functions for following. By applying De Morgan's theorems as many times as necessary, the complements are obtained as follows.

1) $F_1 = x'yz' + x'y'z$.

$F_1' = \overline{\bar{x}y\bar{z} + \bar{x}\bar{y}z}$

$= \overline{\bar{x}y\bar{z}} \cdot \overline{\bar{x}\bar{y}z} = \bar{\bar{x}} + \bar{y} + \bar{\bar{z}} \cdot \bar{\bar{x}} + \bar{\bar{y}} + \bar{z}$

$= x + \bar{y} + \bar{z} \cdot \bar{x} + \bar{y} + \bar{z}$,

$= (x + \bar{y} + z)(x + y + \bar{z})$

2) $F_2 = x(y'z' + yz)$

$F_2' = \overline{x(\bar{y}\bar{z} + yz)}$

$= \bar{x} + \overline{(\bar{y}\bar{z} + yz)}$

$= \bar{x} + (\overline{\bar{y}\bar{z}})\cdot(\overline{yz})$

$= \bar{x} + (\bar{\bar{y}} + \bar{\bar{z}})\cdot(\bar{y} + \bar{z})$

$= \bar{x} + (y+z)\cdot(\bar{y} + \bar{z})$.

Canonical Forms :-

To place a SOP equation into canonical form using Boolean algebra, we do the following:

1. Identify the missing variables in each AND term.

2. AND the missing term and its complement with the original AND term.

$xy(z+\bar{z})$. $(z+\bar{z}) = 1$.

3. Expand the term by application of the property of distribution

$$xyz + xy\bar{z}$$

To place a pos eqn into canonical form using Boolean algebra, we do this

1. Identify the missing in each OR term.
2. OR the missing terms (s) and its complement with the original OR term, $x+\bar{y}+(z\bar{z})$ $[z\bar{z}=0]$

$(x+\bar{y}+z)\cdot(x+\bar{y}+\bar{z})$.

1) Place the following equations into proper Canonical form.

ii) $P = ab' + ac' + bc$ → sop.

$= ab'(c+c') + ac'(b+b') + bc(a+a')$

$= ab'c + ab'c' + abc' + ab'c' + abc + a'bc$.

2) a Minterms and Maxterms:

Consider 2 binary numbers in AND operations.

Consider xy.

Possible Combinations ⇒ $xy + x'y + xy' + x'y'$.

These four AND terms is called minterm.

In a similar ~~functions~~ fashion, n variables forming an OR term, with each variable provide 2^n possible Combinations, max terms or Standard Sums.

Minterms & max terms for 3 Binary Variables

			Minterms		Maxterms	
x	y	z	Term	Designation	Term	Designation
0	0	0	$x'y'z'$	m_0	$x+y+z$	M_0
0	0	1	$x'y'z$	m_1	$x+y+z'$	M_1
0	1	0	$x'yz'$	m_2	$x+y'+z$	M_2
0	1	1	$x'yz$	m_3	$x+y'+z'$	M_3
1	0	0	$xy'z'$	m_4	$x'+y+z$	M_4
1	0	1	$xy'z$	m_5	$x'+y+z'$	M_5
1	1	0	xyz'	m_6	$x'+y'+z$	M_6
1	1	1	xyz	m_7	$x+y+z$	M_7

Convert the following Boolean expression in standard POS form:

$$(A+\bar{B}+C)(\bar{B}+C+\bar{D})(A+\bar{B}+\bar{C}+D)$$

$$A+\bar{B}+C+(D\bar{D})$$

$$=(A+\bar{B}+C+D).(A+\bar{B}+C+\bar{D}).$$

$$\bar{B}+C+\bar{D} = \bar{B}+C+\bar{D}+(A.\bar{A})$$

$$=(A+\bar{B}+C+\bar{D}).(\bar{A}+\bar{B}+C+\bar{D})$$

$$\Rightarrow (A+\bar{B}+C+D).(A+\bar{B}+C+\bar{D})(A+\bar{B}+C+\bar{D}).$$

$$(\bar{A}+\bar{B}+C+\bar{D}).$$

$$(A+\bar{B}+\bar{C}+D)$$