Lame’s theorem

Lame’s theory assumption

1.The material of the shell is homogeneous and isotropic.

2.The plane section of cylinder perpendicular to the longitudinal axis remain plane under the effect of internal pressure.

The second assumption implies that the longitudinal strain is same at all points, i.e., the strain is independent of the radius.

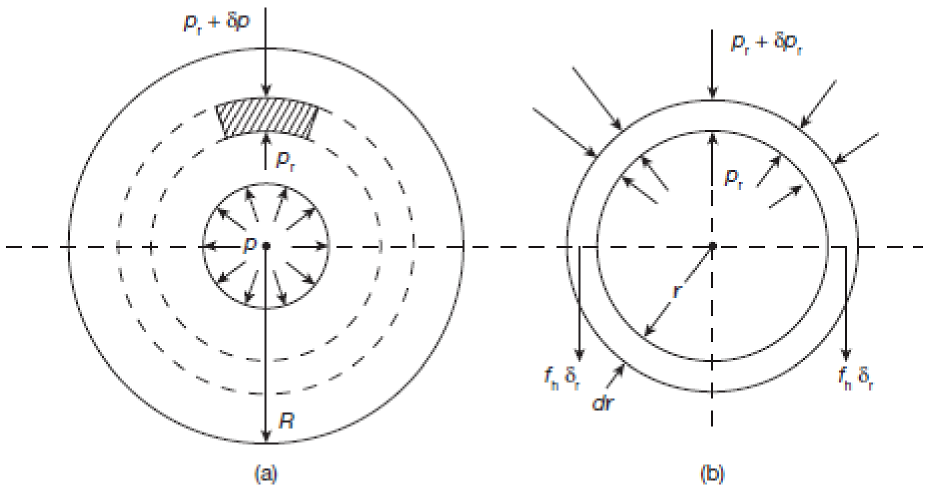


Fig.: Stress condition of a thick cylinder

Problems

A thick cylinder of 150 mm outside radius and 100 mm inner radius is subjected of an internal pressure 60 MN/m^2, external pressure 30 MN/m^2. Calculate the maximum shear stress in the material of the cylinder at the inner radius.

Given :

Outside radius,r_2 = 150 mm

Inside radius,r_1 = 100 mm

Internal pressure P = 60 MN/m^2 (i.e. at r = 100 mm)

External pressure p = 30 MN/m^2 (i.e. at r = 150 mm)

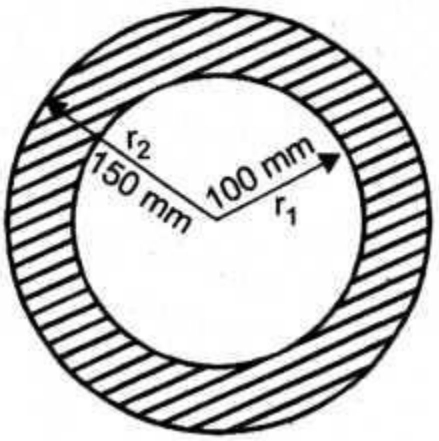
Find: Maximum shear stress

Solution:

Lame's equations are:

$$f_r = \frac{b}{r^2} - a \qquad \text{and}$$

$$f_c = \frac{b}{r^2} + a$$



To Find Lame's constants 'a' and 'b'

It is given,

f_r = 60 MN/m^2 at r = 100 mm

$$\text{i.e., } \frac{60 \times 10^6}{(1000)^2} = 60 \text{ N/mm}^2 \text{ at r = 100 mm}$$

f_r = 30 MN/m^2 at r = 150 mm

$$\text{i.e., } \frac{30 \times 10^6}{(1000)^2} = 30 \text{ N/mm}^2 \text{ at } r = 150 \text{ mm}$$

Substituting these boundary conditions in f_r equation, we get,

$$60 = \frac{b}{100^2} - a \qquad \text{....(i)}$$

$$30 = \frac{b}{150^2} + a \qquad \text{....(ii)}$$

Equations (1) + Equation (2), gives,

$$90 = \frac{b}{100^2} + \frac{b}{150^2} = \frac{b}{10000} + \frac{b}{22500} = \frac{2.25b + b}{22500} = \frac{3.25b}{22500}$$

$$\text{or } b = \frac{22500 \times 90}{3.25} = 623076$$

Substituting the value of 'b' in equation (i), we get,

$$a = \frac{b}{100^2} - 60 = \frac{623076}{100^2} - 60 = 2.30$$

∴Lame's equations are

$$f_r = \frac{623076}{r^2} - 2.30 \text{ and } fc = \frac{623076}{r^2} + 2.30$$

To Find Hoop stress:

i)At Inner radius (i.e., at r = 100 mm)

$$f_c = \frac{b}{r^2} + a = \frac{623076}{100^2} + 2.30 = 64.6 \text{ N/mm}^2.$$

ii)At Outer radius (i.e., at r = 150 mm)

$$f_c = \frac{b}{r^2} + a = \frac{623076}{150^2} + 2.30 = 30 \text{ N/mm}^2.$$

To Find Maximum Shear stress at Inner radius:

Maximum shear stress, τ_{max} is given by the equation,

$$\tau_{max} = \frac{1}{2}(f_r + f_c) = \frac{1}{2}(60 + 64.6) = 62.3 \text{ N/mm}^2.$$