Deformation in spherical shells

Thin spherical shell expression:

Change in dia, δ_d = $\frac{pd^2}{4tE}\left(1-\frac{1}{m}\right)$

Problems

Let us solve A thick spherical shell of 200 mm internal diameter is subjected to an internal fluid pressure of 7 N/mm^2. If the permissible tensile stress in the shell material is 8 N/mm^2, find the thickness of the shell.

For thick spherical shell, Lame's equations are:

$$f_r = \frac{2b}{r^3} - a \quad \text{and} \quad f_c = \frac{b}{r^3} + a$$

Given:

$$r = \frac{200}{2} = 100 \text{ mm}$$

f_r (at r = 100 mm) = 7 N/mm^2

f_c (at r = 100 mm) = 8 N/mm^2

Solution:

Substituting these values in Lame's equations,

$$7 = \frac{2b}{100^3} - a \quad \text{.... (i)}$$

$$8 = \frac{b}{(100)^3} + a \quad \text{.... (ii)}$$

Adding equations (i) and (ii), we get

$$15 = \frac{2b}{100^3} + \frac{b}{100^3}$$

or 15 = $\frac{3b}{100^3}$

∴ b = $\frac{100^3 \times 15}{3}$ = 5×10^6

Substituting this value in equation (i),

$$7 = \frac{2 \times 5 \times 10^6}{100^3} - a$$

Solving, a = 3

∴ Lame's equations are

$$f_r = \frac{2 \times 5 \times 10^6}{r^3} - 3$$ and

$$f_c = \frac{5 \times 10^6}{r^3} + 3$$

To Find thickness

At outer radius, radial stress, f_r is zero

Let r_2 = outer radius

∴at $r = r_2$; $f_r = 0$

Substituting these values, in f_r equation,

$$f_r = \frac{2b}{r_2^3} - a$$

i.e., $$0 = \frac{2 \times 5 \times 10^6}{r_2^3} - 3$$

or $$r_2^3 = \frac{2 \times 5 \times 10^6}{3}$$

or $$r_2 = \sqrt[3]{\frac{2 \times 5 \times 10^6}{3}} = 149.3 \text{ mm}$$

∴ Thickness of shell = Outer radius – Inner radius

= 149.3 – 100 = 49.3 mm, say 50 mm