Spherical shells subjected to internal pressure

Spherical shell expression

1)Circumferential stress, $\sigma c = \frac{pd}{4t}$

2)Longitudinal stress, $\sigma_l = \frac{pd}{4t}$

Let us derive an expression for change in diameter and volume when a thin spherical shell is subjected to internal pressure.

Consider a thin spherical shell as shown in fig, subjected to an internal pressure.

Let p = Internal pressure

d = diameter of shell

t = thickness of shell

The force (P) which has a tendency to split the shell = p × $\frac{\pi}{4}d^2$.

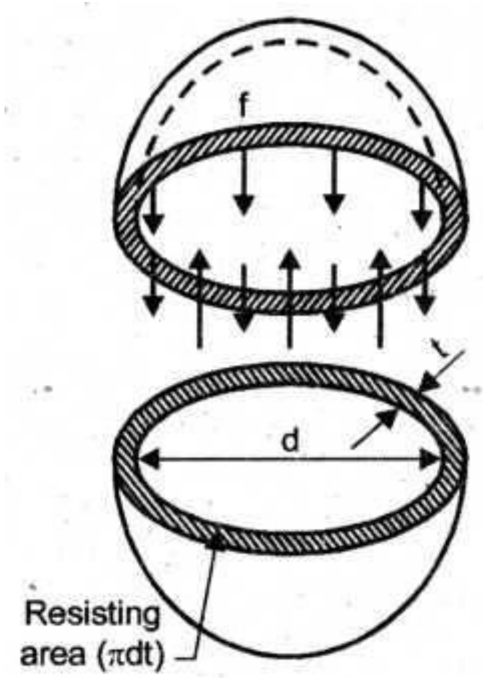

Fig.Thin spherical shell

But the area which is resisting this force = πdt

∴ Hoop (or) circumferential stress,

$$\sigma_c = \frac{\text{Force, P}}{\text{Area resisting the force P}}$$

$$= \frac{p \times \frac{\pi}{4} d^2}{\pi . d . t} = \frac{pd}{4t}$$

Similarly longitudinal stress (i.e., along yy axis)

$$\sigma_\ell = \frac{Pd}{4t}$$

We know, maximum shear stress = $\frac{\sigma_c - \sigma_l}{2}$

$$= \frac{\left(\frac{Pd}{4t}\right) - \left(\frac{Pd}{4t}\right)}{2} = 0$$

i.e., No shear stress. Hence σ_c and σ_1 are acting at right angles to each other.

∴ Strain in any direction is given by,

$$e = \frac{\sigma_c}{E} - \frac{\sigma_l}{mE}$$

$$= \frac{\sigma_c}{E}\left(1 - \frac{1}{m}\right) \quad (\because \sigma_c = \sigma_l)$$

$$= \frac{pd}{4tE}\left(1 - \frac{1}{m}\right) \qquad \left(\because \sigma_c = \frac{pd}{4t}\right)$$

We know that, strain in any direction = $\frac{\delta d}{d}$

$$\therefore \qquad \frac{\delta d}{d} = \frac{pd}{4tE}\left(1 - \frac{1}{m}\right)$$

∴ Change in diameter,

$$\boxed{\delta d = \frac{pd^2}{4tE}\left(1 - \frac{1}{m}\right)} \qquad \text{....(i)}$$

Let, V = original volume of shell = $\frac{4}{3}\pi r^3 = \frac{\pi}{6}d^3$

V1 = Final volume of shell

$$= \frac{\pi}{6}\{(d + \delta d)^3\}$$

$$= \frac{\pi}{6}\{d^3 + 3d^2\,\delta d\}$$

(second and higher power of **δ**d are neglected).

∴ Change in volume, $\delta v = V_1 - V$

$$= \frac{\pi}{6}\{d^3 + 3d^2\,\delta d\} - \frac{\pi}{6}d^3$$

$$= \frac{\pi}{6}\{d^3 + 3d^2\,\delta d - d^3\}$$

$$= \frac{\pi}{6}\{3d^2\,\delta d\}$$

To write δv in terms of p, t and E:

We know, Volumetric strain, ev = $\frac{\delta v}{V}$

$$\therefore \quad \frac{\delta v}{V} = \frac{\frac{\pi}{6}\left(3d^2\,\delta d\right)}{\frac{\pi}{6}d^3}$$

$$= \frac{3\,\delta d}{d}$$

Now substituting eqn. (i), we get,

$$\frac{\delta v}{V} = \frac{pd^2}{4tE}\left(1-\frac{1}{m}\right)$$

$$\frac{\delta v}{V} = \frac{3}{d}\left[\frac{pd^2}{4tE}\left(1-\frac{1}{m}\right)\right]$$

$$= \frac{3pd}{4tE}\left(1-\frac{1}{m}\right)$$

orchange in volume, **δ**v = $\left\{\frac{3pd}{4tE}\left(1-\frac{1}{m}\right)\right\} \times V$

Now substitutingV = $\frac{\pi}{6}d^3$, we get

$$\delta v = \frac{3pd}{4tE}\left(1 - \frac{1}{m}\right) \times \frac{\pi}{6} d^3$$

$$\boxed{\delta v = \frac{\pi p d^4}{8tE}\left(1 - \frac{1}{m}\right)}$$