Double integration method

Deflection of beam

The deflection of any point in the beam is the deformation measured at the point to the deflected shape from its original position.

$$0 = \frac{-10 \times 6^3}{6} - \frac{6^4}{12} + (C_1 \times 6) + C_2$$

Let us discuss the methods of slope and deflection of beams

1.Double Integration method

2.Macaulay's method

3.Area Moment method

4.Conjugate beam method

5.Energy method

6.Matrix method

Double integration method:

From the basic differential equation for the deflection curve,

integrating once, we get slope and integrating twice, we get deflection.

i.e., From the equation

$$EI \frac{d^2y}{dx^2} = M \qquad \text{....(1)}$$

where M is the Bending moment Integrating the equation (1), we get slope

$$\text{i.e.,} \qquad EI \frac{dy}{dx} = \int M$$

$$EI\,\theta = \int M \qquad \text{....(2)}$$

Again integrating the equation (2), we get deflection

$$\text{i.e.,} \qquad EI\,y = \int\int M \qquad \text{....(3)}$$

The assumptions made By Double Integration Method:

1. The whole deflection is due to BM only and that the deflection caused by SF is negligible.

2. Deflection is small compared to cross sectional dimension of the beam.

3. The beam is uniform cross section and straight before application of load.

4. Modulus of elasticity in tension are equal.

The relation of slope,deflection and radius of curvature are:

$$\frac{1}{R} = \frac{d^2y}{dx^2} \qquad \text{....(i)}$$

$$M = EI \frac{d^2y}{dx^2} \qquad \text{....(ii)}$$

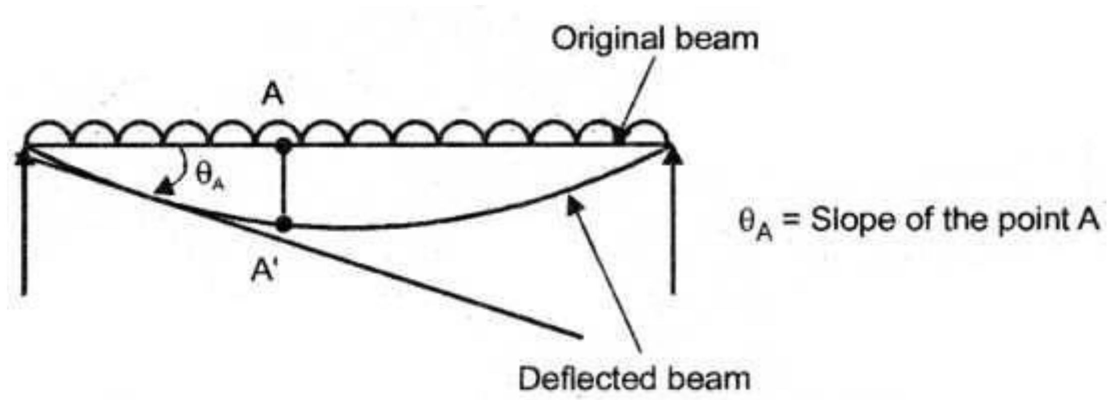
Where R = Radius of curvatureM = Bending moment

y = Deflection $\frac{dy}{dx}$ = Slope

I = Moment of InertiaE = Young's modulus

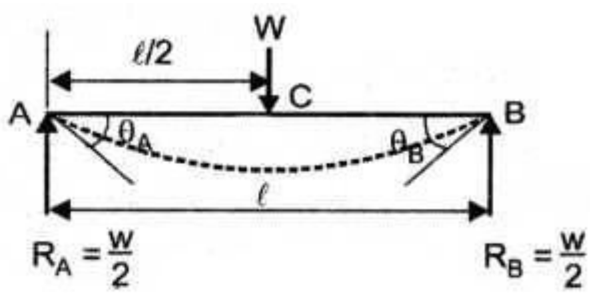
Slope of beam

The slope at any point of beam is the angle made by the tangent drawn through the point, but located on the deflected shape with the axis of beam.



Let us derive deflection of simply supported beam

Solution:



Boundary Conditions

at A, $y_A = 0$

at B, $y_B = 0$

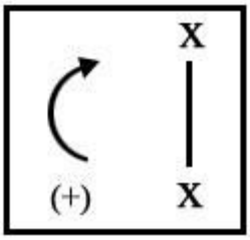
at C, $\theta_C = 0$

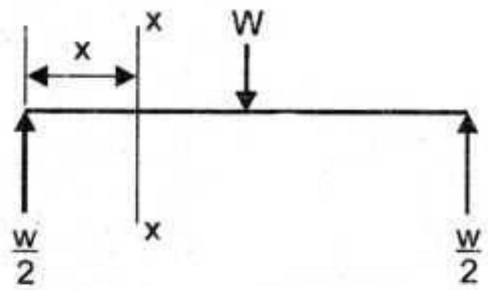
Consider a simply supported beam AB supported at A and B, subjected to a central point load W at center (at C) as shown in figure.

Reactions $R_A = R_B = \dfrac{\text{Total load}}{2} = \dfrac{W}{2}$

Consider a section XX at a distance x from A.

$$(BM)_x = \frac{W}{2}x$$





$$\boxed{M = \frac{W}{2}x}$$

Using the relation $EI\dfrac{d^2y}{dx^2} = M$

or $$EI\frac{d^2y}{dx^2} = \frac{W}{2}x \quad \text{.... (1)}$$

Integrating the equation (1), we get

$$\int EI\frac{d^2y}{dx^2} = \int\frac{W}{2}x$$

or $$EI\frac{dy}{dx} = \frac{W}{2}\frac{x^2}{2} + C_1$$

To Find constant C_1

Apply the Boundary condition that at center (where deflection is maximum) slope is zero, i.e., at $x = \frac{l}{2},\ \frac{dy}{dx} = 0$

Substituting these values in the above equation,

$$EI \times 0 = \frac{W}{2} \times \frac{1}{2} \times \left(\frac{\ell}{2}\right)^2 + C_1$$

or $$0 = \left\{\frac{W}{4} \times \left(\frac{\ell^2}{4}\right)\right\}$$

$$0 = \frac{Wl^2}{16} + C_1$$

$$\therefore \qquad C_1 = \frac{-W\ell^2}{16}$$

Substituting C_1 in the above equation.

$$\boxed{EI\frac{dy}{dx} = \frac{Wx^2}{4} - \frac{Wl^2}{16}} \qquad \text{....} \quad (2)$$

known as slope equation.

Integrating the above equation, we get

$$EI\ y = \frac{W}{4}\left(\frac{x^3}{3}\right) - \frac{Wl^2}{16}x + C_2$$

To find C_2

Apply the boundary condition that, at the support A, deflection is zero.

i. e.,at x = 0, y = 0

Substituting these value in the above equation,

$E\ I \times 0 = 0 - 0 + C_2$

$C_2 = 0$

Substituting C_2 in the above equation,

$$\boxed{EI\ y = \frac{Wx^3}{12} - \frac{Wl^2}{16}x} \qquad \text{.... (3)}$$

known as Deflection equation.

To find slope at A

Substitutex = 0 in slope equation (i.e., equation (2))

$$EI\frac{dy}{dx} = \frac{Wx^2}{4} - \frac{Wl^2}{16}$$

or $EI\,\theta_A = \dfrac{0 - Wl^2}{16}$ $(\because x = 0)$

or $\theta_A = \dfrac{-Wl^2}{16EI}$ (– indicates clockwise slope)

$$\boxed{\theta_A = \frac{Wl^2}{16EI}}$$

Due to symmetry $\theta_B = \dfrac{Wl^2}{16EI}$ (But anticlockwise slope)

To Find deflection under point load

Substitute x = $\dfrac{l}{2}$ in deflection equation (i.e, equation (3))

$$E\,I\,y = \frac{Wx^2}{12} - \frac{W\ell^2}{16}x$$

$$E\,I\,y_C = \frac{W}{12}\left(\frac{l}{2}\right)^3 - \frac{Wl^2}{16}\left(\frac{l}{2}\right)$$

$$= \frac{Wl^3}{96} - \frac{Wl^3}{32} \qquad \left(\because x = \frac{1}{2}\right)$$

$$= \frac{W\ell^3 - 3W\ell^3}{96}$$

$$= \frac{-2W\ell^3}{96} = \frac{-W\ell^3}{48}$$ (– indicates downward deflection)

$$\boxed{y_C = \frac{W\ell^3}{48EI}}$$,downward