Problems

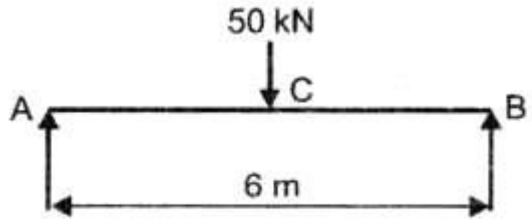
A beam 6m long simply supported at its ends carrying a point load of 50 KN at center. The moment of inertia of the beam is given as equal to $78 \times 10^6 mm^4$. If E for material of the beam = $2.1 \times 10^5 N/mm^2$, calculate,i)deflection at the center of the beam and ii)slope at the supports.

Given:

$I = 78 \times 10^6\ mm^4$ $E = 2.1 \times 10^5\ N/mm^2$

$y_c = ?$ $\theta_A = ?$ $\theta_B = ?$

Solution:



To find deflection at center of beam

$W = 50\ KN = 50 \times 10^3 N$

1 = 6m = 6000 mm

Using the relation, $$y_C = \frac{Wl^3}{48EI}$$

$$= \frac{50 \times 10^3 \times (6000)^3}{48 \times 2.1 \times 10^5 \times 78 \times 10^6}$$

$$= .13.736\ mm$$

To find slope at supports

Using the relation, $$\theta_A = \frac{wl^2}{16EI}$$

$$= \frac{50 \times 10^3 \times (6000)^2}{16 \times 2.1 \times 10^5 \times 78 \times 10^6}$$

$$= 6.868 \times 10^{-3} \text{ radians}$$

$$= 6.868 \times 10^{-3} \frac{180}{\pi} \text{ degrees}$$

= 0.394°

Due to symmetry, θ_B = 0.394°

Let us consider a beam of 5 m long simply supported at its ends, carryinga point load W at its center. If the slope at the ends of the beam is not to exceed 1°, find the deflection at the center of the beam.

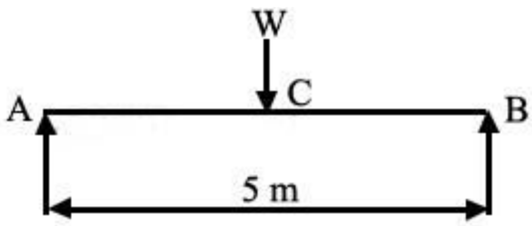
Given:

1 = 5m = 5000 mm.

$\theta_A = \theta_B$ $$= \int^{o} = 1 \times \frac{\pi}{180}$$

= 0.0174 radians

y_c = ?



Solution:

Using the relation,

$$\theta_A = \frac{Wl^2}{16EI} \quad \text{or} \quad 0.0174 = \frac{Wl^2}{16EI}$$

To find deflection at center

Using the relation,

$$y_c = \frac{Wl^3}{48EI}$$

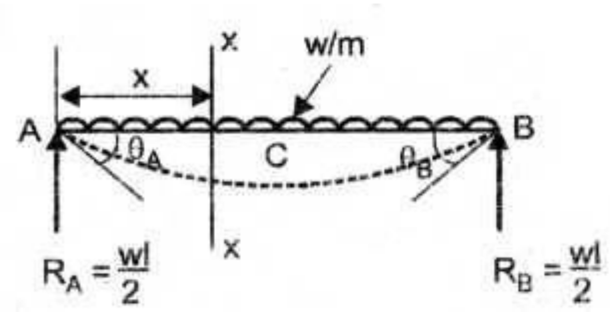
or $$y_c = \frac{Wl^2}{16EI} \times \frac{l}{3} \qquad \left(\because \frac{Wl^2}{16EI} = 0.0174;\ \ell = 5000\text{ mm}\right)$$

$$= 0.0174 \text{ x} \left(\frac{5000}{3}\right) = 29.08 \text{ mm}$$

Let us derive the expressions to find slope at supports and central deflection of a simply supported beam subjected to uniformly distributed load on its entire span.

Solution:

Consider a simply supported beam uniformly loaded w/m run on entire span as shown below.



Boundary conditions

at A, $y_A = 0$

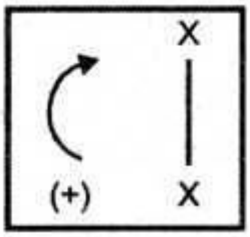
at B, $y_B = 0$

at C, $\theta_c = 0$

Due to symmetry, $R_A = R_B = \dfrac{w1}{2}$

Consider a section XX at a distance of x from A as shown in Figure.

$$(BM)_{xx} = M = \frac{wl}{2}x - \frac{wx^2}{2}$$



Using the relation $EI\dfrac{d^2y}{dx^2} = M$

$$\therefore \qquad EI\frac{d^2y}{dx^2} = \frac{w\ell}{2}x - \frac{wx^2}{2} \qquad \text{....(i)}$$

Integrating the equation (1)

$$EI\frac{dy}{dx} = \frac{w\ell}{2}\frac{x^2}{2} - \frac{wx^3}{6} + C_1$$

To find the constant C_1

Apply the Boundary condition that at x = 0, $\theta_C = 0$

$$EI\ \theta_C = \frac{w\ell}{2} \times \frac{1}{2}\left(\frac{\ell}{2}\right)^2 - \frac{w}{6}\left(\frac{\ell}{2}\right)^3 + C_1$$

$$0 = \frac{w\ell^3}{16} - \frac{w\ell^3}{48} + C_1$$

$$\therefore \qquad C_1 = \frac{wl^3}{48} - \frac{wl^3}{16}$$

$$= \frac{wl^3 - 3wl^3}{48} = \frac{-wl^3}{24}$$

substituting $C_1 = \frac{-w\ell^3}{24}$ in the above equation,

$$\boxed{EI\frac{dy}{dx} = \frac{wl}{4}x^2 - \frac{wx^3}{6} - \frac{-wl^3}{24}} \quad ...(ii)$$

known as Slope equation.

Integrating the equation (ii), we get

$$EI\,y = \frac{w\ell}{4}\frac{x^3}{3} - \frac{wx^4}{24} - \frac{-w\ell^3}{24}x + C_2$$

To Find constant C_2

Apply the Boundary condition that

atx = 0, y = 0

$\therefore EI \times 0 = 0 - 0 - 0 + C_2$

$\therefore C_2 = 0$

Substituting $C_2 = 0$ in the above equation,

$$\boxed{EI\ y = \frac{wl}{12}x^3 - \frac{wx^4}{24} - \frac{wl^3}{24}x} \qquad \text{(iii)}$$

known as Deflection equation.

To Find slope at supports

To Find θ_A, substitute x = 0 in equation (ii) i.e., in slope equation.

$$\therefore \qquad EI\,\theta_A = \frac{-wl^3}{24}$$

$$\theta_A = \frac{-wl^3}{24EI}$$ (– indicates clockwise slope)

Due to symmetry, $$\theta_B = \frac{-wl^3}{24EI}$$ (Clockwise slope)

To Find central deflection

Substitute $x = \frac{\ell}{2}$ in equation (iii), i.e., deflection equation

$$\therefore \qquad EIy_C = \frac{wl}{12}\left(\frac{l}{2}\right)^3 - \frac{w}{24}\left(\frac{l}{2}\right)^4 - \frac{wl^3}{24}\left(\frac{l}{2}\right)$$

$$= \frac{w\ell^4}{96} - \frac{w\ell^4}{384} - \frac{w\ell^4}{48}$$

$$= \frac{4w\ell^4 - w\ell^4 - 8w\ell^4}{384} = \frac{-5w\ell^4}{384}$$

$$\boxed{y_C = \frac{5wl^4}{384EI}}$$ (– indicates downward deflection)(Note that $y_c = y_{max}$).

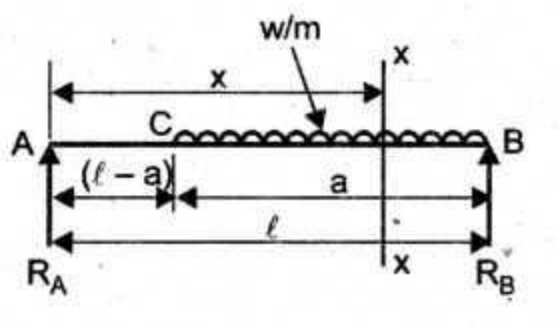
Here we see a horizontal beam of uniform section and length 'l' rests on supports at its ends. It carries a uniformly distributed load of w per unit run for a distance 'a' from the right end. Calculate the value of 'a' for which the maximum deflection will occur at the left end of the uniformly distributed load. If the maximum deflection is expressed by $\frac{wl^4}{kEI}$' find the value of k.

Given:

Y_{max} occurs at C Determine, a?

Solution:

If $y_c = y_{max} = \frac{wl^4}{kEI}$ determine k.



Step 1: Support Reactions

Applying$\sum V = 0$ (↑=↓)

$R_A + R_B = w_a$....(i)

Applying $\Sigma M_A = 0$ (↺=↻)

$R_B \times l = w \times a \times \left(l - \frac{a}{2}\right)$

$$= wa \quad l - \frac{a}{2}$$

$$R_B \text{ x } l = wal - \frac{wa^2}{2}$$

$$\boxed{R_B = wal - \frac{wa^2}{2l}}$$

$R_A = w_a - R_B$

$$= \quad wa - \left(wa - \frac{wa^2}{2\ell} \right) = \frac{wa^2}{2\ell}$$

Consider a section XX at a distance of x from A.

$(BM)_{xx} = M$

$$= \quad (R_A \times x) - \left\{ w \times \{x - (\ell - a)\} \times \frac{(x - (\ell - a))}{2} \right\}$$

$$= \quad R_A x - w \frac{(x - (\ell - a))^2}{2}$$

$$= \quad \frac{wa^2}{2\ell} x - w \frac{(x - (\ell - a))^2}{2}$$

or $$EI \frac{d^2y}{dx^2} = \frac{wa^2}{2\ell} x - \frac{w}{2}(x - (\ell - a))^2 \qquad .... \quad (1)$$

Integrating the equation (1)

$$EI \frac{dy}{dx} = \frac{wa^2x^2}{2\ell \times 2} - \left\{ \frac{w}{2}\left(x - (\ell - a)^3\right) \times \frac{1}{3} \right\} + C_1 \quad ....(2)$$

Integrating the equation (2), we get

$$EI\, y \quad = \quad \frac{wa^2x^3}{12\ell} - \frac{w}{4}\left(x - (\ell - a)^4\right) + C_1 x + C_2 \quad ....(3)$$

To find C_1 and C_2

Apply the Boundary conditions at x = 0, y = 0 in equation (1)

$C_2 = 0$

atx = l, y = 0 in equation (1)

$$0 = \frac{wa^2}{12\ell}(\ell^3) - \frac{w}{24}(\ell - (\ell - a))^2 + (C_1 \times \ell)$$

Solving, $C_1 = -\frac{wa^2 l}{12} + \frac{wa^4}{24l}$

Substituting C_1 in equation (2), we get

$$EI\frac{dy}{dx} = \frac{wa^2x^2}{4l} - \frac{wa^2x^2}{12} + \frac{wa^4}{24l} + \left|\frac{w}{6}\{x - (l - a)\}^2\right.$$

It is given y_{max} occurs at C

At maximum deflection, $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 0$ at x = (l – a)

Substituting in the above equation,

$$0 = \frac{wa^2}{4\ell}(\ell - a)^2 - \frac{wa^2\ell}{12} + \frac{wa^4}{24\ell}$$

Simplifying, we get $7a^2 - 12l\,a + 4l^2 = 0$

$$a = l\left[\frac{12 \pm \sqrt{144 - 4 \times 4 \times 7}}{2 \times 7}\right]$$

Solving, $$= l\left[\frac{12 \pm 5.657}{14}\right]$$

= 0.453 l

Substitutinga = 0.453 l we get

$$C_1 = \frac{-w\,(0.453\,l)^2\,l}{12} + \frac{w(0.453\,l)^4}{24l}$$

=– 0.01535 wl^3

:. substitute x = (l – a) = 0.547 l and

C_1 = – 0.01535 wl^3 in equation (3)

$y_c = y_{max}$

$$= \frac{1}{EI}\left[\frac{w\,(0.453\,\ell)^2}{12\,\ell}(0.547\,\ell)^3 - \left\{0.01535w\ell^2(0.547\,\ell)\right\}\right]$$

$$= \frac{w\ell^4}{EI}[0.0028 - 0.0084]$$

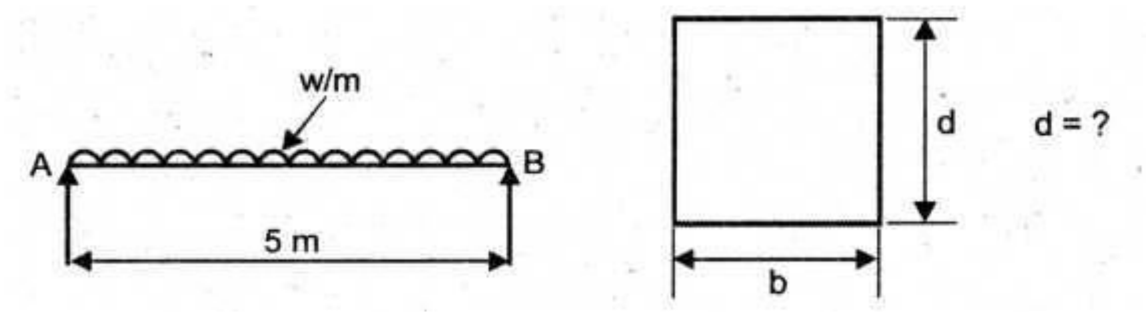
$$= \frac{-0.0056w\ell^4}{EI} \quad = -\frac{w\ell^4}{178.65EI}$$

(– sign indicates downward deflection)

$y_c = y_{max}$ $= -\frac{wl^4}{178.65EI}$ Hence k = 178.65.

We shall discuss here about a beam of length 5m and of uniform rectangular section is supported at its ends and carries uniformlydistributed load over the entire length. Calculate the depth of the section if the maximum permissible bending stress is 8 N/mm^2 and central deflection is not to exceed 10 mm. Take the value of E = 1.2 × 104 N/mm^2

Given:



$\Delta_{max} = y_{center} = 10$ mm;

$E = 1.2 \times 10^4$ N/mm^2

$(f_b)_{max} = 8$ N/mm^2; 1 = 5m = 5000 mm

Solution:

Here the unknown data are udl = w, width b and depth d

Total load on the beam (W) = udl × span of Beam

∴W = w × l = wl

Max BM,
$$M = \frac{wl^2}{8} = \frac{Wl}{8} \quad \text{....(i)}$$

Using the relation
$$\frac{M}{I} = \frac{f}{y}$$

$$M = \frac{fI}{y} = \frac{8 \times I}{\frac{d}{2}} (\because f = f_b = 8 \text{ N/mm}^2;\ y = \frac{d}{2})$$

$$\therefore \qquad M = \frac{16I}{d} \qquad \text{....(ii)}$$

Equating (i) and (ii)

$$\frac{W\ell}{8} = \frac{16I}{d} \text{ or } W = \frac{128I}{dl}$$

Using the relation, $y_C = \frac{5wl^4}{384EI}$

$$= \frac{5(wl)l^3}{384EI} = \frac{5W\ell^3}{384EI} \qquad (\because W = w\ell)$$

$$\therefore \qquad y_C = \frac{5}{384EI}\left(\frac{128I}{d\ell}\right) . \ell^3$$

$$= \frac{5}{384} \times \frac{1}{E} \times \frac{128}{d} \times \ell^2$$

or $10 = \frac{5}{384} \times \frac{1}{1.2 \times 10^4} \times \frac{128}{d} \times (5000)^2$

($\because y_c$ = 10 mm)

Solving, $d = \frac{5 \times 128 \times (5000)^2}{384 \times 10 \times 1.2 \times 10^4}$

= 347.2say 348 mm