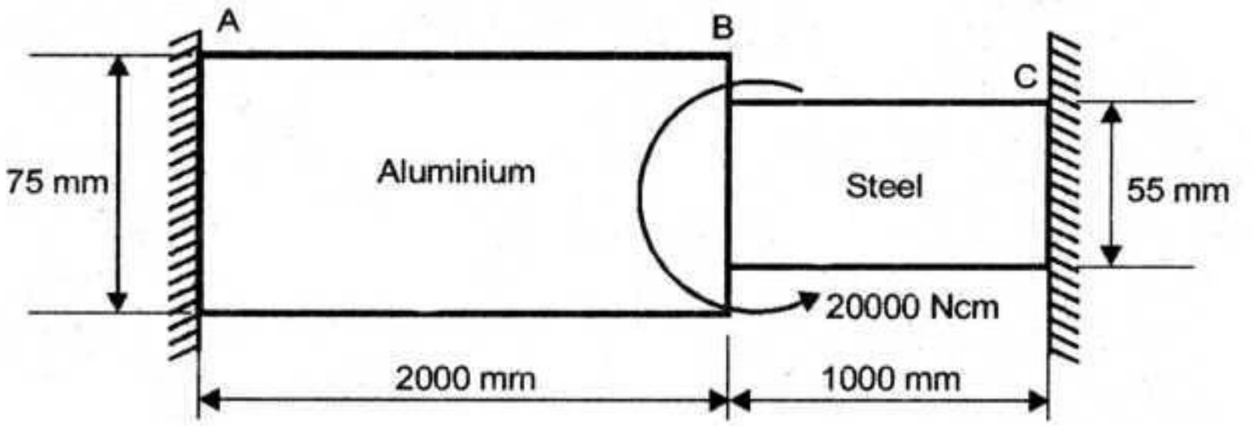
## Deflection in shafts fixed at the both ends

**Let us see an example of two solid shafts AB and BC of aluminium and steel respectively are rigidly fastened together at B and attached to two rigid supports at A and C. Shaft AB is 75 mm in diameter and 2 m in length. Shaft BC is 55 mm in diameter and 1 m in length. A torque of 20000 N cm is applied at the Junction B. Compute the maximum shearing stresses in each material. What is the angle of twist at the junction? Take modulus of rigidity of the materials as 0.3 x $10^5$ N/mm$^2$ and 0.9 × 105 N/mm$^2$ respectively for aluminium and steel.**




*Solution:*

**Shaft AB (Aluminium)**

$l_1$ = 2000 mm$D_1$ = 75mm$C_1$ = 0.3 × $10^5$ N/mm$^2$

Shaft BC (Steel)

$l_2$ = 1000 mm$D_2$ = 55 mm$C_2$ = 0.9 × $10^5$ N/mm$^2$

Torque at the Junction = 20000 N cm = 20 × $10^5$ N mm

Conditions to be followed:

i) $\theta_1 = \theta_2$ ii) $T_1 + T_2$ = Total Torque of $20 \times 10^5$ N mm

Using the equation, $$\frac{T}{J} = \frac{C\theta}{\ell} \quad \text{or} \quad \theta = \frac{T\ell}{CJ}$$

$$\therefore \quad \theta_1 = \frac{T_1 \ell_1}{C_1 J_1} \quad \text{and} \quad \theta_2 = \frac{T_2 l_2}{C_2 J_2}$$

or $$\theta_1 = \frac{T_1 \times 2000}{\frac{\pi}{32} \times 75^4 \times 0.3 \times 10^5}$$

$$\theta_2 = \frac{T_2 \times 1000}{0.9 \times 10^5 \times \frac{\pi}{32} \times 55^4}$$

Now equating both,

$$\frac{T_1 \times 2000 \times 32}{\pi \times 75^4 \times 0.3 \times 10^5} = \frac{T_2 \times 1000 \times 32}{0.9 \times 10^5 \times \pi \times 55^4}$$

$$\text{or} \quad T_1 = \frac{75^4 \times 0.3}{55^4 \times 0.9 \times 2} T_2$$

or $T_1 = 0.576\ T_2$

Substituting in equation, $T_1 + T_2 = 20 \times 10^5$

i.e., $0.576\ T_1 + T_2 = 20 \times 10^5$

solving, $T_2 = \dfrac{20 \times 10^5}{1.576} =$ 126900 N mm

$\therefore T_1 = 0.576 \times 126900 = 73100$ **N mm**

**To Find Shear Stress**

Using the equation, $\frac{T}{J} = \frac{fs}{R}$

**For Shaft AB**

$$(f_s)_1 = \frac{T_1 \times R_1}{J_1}$$

$$= \frac{73100 \times \left(\frac{75}{2}\right)}{\left(\frac{\pi}{2}\right) \times 75^4} = 0.882 \text{ N/mm}^2$$

**For Shaft RC**

$$(f_s)_2 = \frac{T_2 \times R_2}{J_2} = \frac{126900 \times \left(\frac{55}{2}\right)}{\left(\frac{\pi}{32}\right) \times 55^4} = 3.884 \text{ N/mm}^2$$