Deflection of helical springs, carriage springs

Carriage spring:

The springs which are used in vehicles to absorb shocks are known as carriage spring (or) laminate spring. A carriage spring is normally designed for maximum bending moment, hence it is analyzed for maximum bending stress.

Helical springs:

The helical springs are made up of a wire coiled in the form of a helix and are primarily intended for compressive or tensile load.

Deflection of helical spring

When a close coiled helical spring is subjected to an axial load W,

Deflection, $\delta = \frac{64\,W R^3 n}{C d^4}$

Problems

Let us consider a closely coiled helical spring of round steel wire 10 mm in diameter having 10 complete turns with a mean diameter of 12 cm is subjected to an axial load of 200 N. Determine i)the deflection of the spring,ii)maximum shear stress in the wire,iii)Stiffness of the spring. Take C = 8 × 10^4 N/mm^2

Given:

Diameter of wired = 10 mm

Mean diameter of spring, $D = 12cm = 120\ mm$

Number of turns, $n = 10$

Axial load, $W = 200\ N$

Shear modulus, $C = 8 \times 10^4\ N/mm^2$

Find f_s ?δ ?h ?

Radius of coil, $R = \frac{D}{2} = \frac{120}{2} = 60\ mm$

i)***Deflection of spring***

$$\delta = \frac{64 WR^3 n}{Cd^4} = \frac{64 \times 200 \times 60^3 \times 10}{8 \times 10^4 \times 10^4} = 34.5\ mm$$

ii)***Maximum shear stress***

$$f_s = \frac{16wR}{\pi d^3} = \frac{16 \times 200 \times 60}{\pi \times 10^3} = 61.1\ N/mm^2$$

iii)***Stiffness of spring,***

Stiffness of spring, $k = \frac{\text{load W}}{\text{Deflection } \delta}$

$$\therefore \qquad k = \frac{200}{34.5} = 5.8\ N/mm.$$

Here we see an example of close coiled helical spring is to have a stiffness of 900 N/m in compression, with a maximum load of 45N and a maximum shearing stress of 120 N/mm^2. The solid length of the spring is 45 mm. Find i)Wire diameter,ii)The mean coil Radius,iii)The number of CoilsTake C = 0.4 x 105 N/mm^2

Given:

$$k = \frac{900\,N}{m} = \frac{900}{1000} = 0.9\ N/mm$$

load,W = 45 N

Max shearing stress, f_s = 120N/mm^2

Solid length,nd = 45 mm

C = 0.4 × 10^5 N/mm^2

To Find: d ? R ? n ?

Using the equation,

Stiffness,

$$k = \frac{Cd^4}{64R^3n}$$

or

$$0.9 = \frac{0.4 \times 10^5 \times d^4}{64 \times R^3 \times n}$$

∴

$$d^4 = \frac{0.9 \times 64 \times R^3 \times n}{0.4 \times 10^5}$$

= 0.00144R^{3}n....(1)

Using the equation, $f_s = \frac{16WR}{\pi d^3}$

$$120 = \frac{16 \times 45 \times R}{\pi d^3}$$

$$\therefore \quad R = \frac{120 \times \pi d^3}{16 \times 45} = 0.5236\, d^3 \quad \text{.... (2)}$$

Substitute the value of R in equation (1),

$d^4 = 0.00144R^3n$

$= 0.00144 \times (0.5236d^3)^3n$

$= 0.00144 \times 0.5236^3 \times d^9 \times n$

$= 0.0002067\, d^9n$

$$\text{or} \quad \frac{d^9n}{d^4} = \frac{1}{0.0002067}$$

$$\text{or} \quad d^5\, n = \frac{1}{0.0002067} \quad \text{.... (3)}$$

We know, solid length = n d

i.e.,45 = n d

$$\text{or} \quad n = \frac{45}{d} \quad \text{.... (4)}$$

Substitute the value of n in equation (3)

$$d^5\left(\frac{45}{d}\right) = \frac{1}{0.0002067}$$

or $$d^4 = \frac{1}{0.0002067} \times \frac{1}{45}$$

= 107.50

$$\therefore \quad d = \sqrt[4]{107.5} = 3.22 \text{ mm}$$

Substitute the value of d in equation (4)

$$n = \frac{45}{d} = \frac{45}{3.22}$$ = 13.97say 14

Substitute the value of n in equation (2)

$R = 0.5236\ d^3 = 0.5236 \times (3.22)^3 = 17.48$ mm

∴ Mean diameter of coil,

D = 2R = 2 × 17.48 = 34.96 mm.

Here we discuss about a close coiled helical spring of 5mm wire diameter with 25 number of coils. Its mean diameter is 80 mm. If maximum shear stress is limited to 150 N/mm^2. Determine i)Maximum load that spring can carry ii)Deflection due to maximum load,Take G = 8 × 10^4 N/mm^2

Given:

d = 5 mm;n = 25;D = 80 mm, ∴ R = 40 mm

$f_s = 150 N/mm^2$, $G = 8 \times 10^4\ N/mm^2$

W = ?d = ?

i) Using the equation $f_s = \frac{16WR}{\pi d^3}$

or $W = \frac{\pi d^3 f_s}{16R} = \frac{\pi \times 5^3 \times 150}{16 \times 40} = 92.04\ N$

ii) Using the equation, $\delta = \frac{64wR^3n}{Cd^4} = \frac{64 \times 92.04 \times 40^3 \times 25}{8 \times 10^4 \times 5^4}$

= 188.49 mm

Problems

A closely coiled helical spring having 12 coils of wire diameter 16 mm and made with coil diameter 250 mm is subjected to an axial load of 300 N. Find axial deflection, strain energy stored and torsional shear stress. Modulus of rigidity = 80 GN/m^3.

Given:

n = 12;d = 16mm;D = 250mm∴ R = 125 mm,

W = 300N;G = 80×10^3 N/mm^2

*i)**Axial deflection***

Using the equation, $\delta = \frac{64WR^3n}{Cd^4} = \frac{64 \times 300 \times 125^3 \times 12}{8 \times 10^4 \times 16^4}$

= 85.83 mm

*ii)**Strain energy stored***

Using the equation, $U = \frac{32W^2R^3n}{cd^4}$

$$= \frac{32 \times (300)^2 \times (125)^3 \times 12}{80 \times 10^3 \times 16^4}$$

= 12874 N mm

iii) Torsional shear stress

Using the equation, $f_s = \frac{16WR}{\pi d^3}$

$$= \frac{16 \times 300 \times 125}{\pi \times 16^3}$$

= 46.63 N/mm^2.

Let's consider that a close coiled helical spring is required to absorb 2250 joules of energy. Determine the diameter of the wire, the mean coil diameter of the spring and the number of coils necessary if, i)the maximum shear stress is not to exceed 400 MPa, ii)the maximum compression of the spring is limited to 250 mm and iii)the mean diameter of the spring is eight times the wire diameter. For the spring material, rigidity modulus is 70 GPa.

Given:

U = 2250 joules = 2250 Nmf_s = 400 M Pa = 400 N/mm^2

δ = 250mm,D = 8d or R = 4d,

C = 70 GPa = 70 × 10^3 N/mm^2,

d = ?D = ?n = ?

Diameter of spring wire

Using the relation,

$$f_s = \frac{16WR}{\pi d^3}$$

$$\text{or} \qquad 400 = \frac{16 \times W \times 4d}{\pi d^3} \qquad (\because R = 4d)$$

$$\text{or} \qquad W = \frac{400 \times \pi d^2}{16 \times 4}$$

W = 19.63d^2....(1)

Using the relation,

$$U = \frac{32\, W^2 R^3\, n}{C d^4}$$

$$\text{or} \quad 2250 \times 10^3 = \frac{32 \times W^2 \times (4d)^3 n}{Cd^4} \qquad (\because U = 2250 \times 10^3)$$

$$= \frac{32 W^2 \times 64 d^3 n}{Cd^4}$$

$$2250 \times 10^3 = \frac{32 W^2 \times 64 n}{Cd}$$

$$= \frac{32 (19.63 d^2)^2 \times 64 n}{70 \times 10^3 \times d} = \frac{11.27\, d^4 \times 64 n}{d}$$

$2250 \times 10^3 = 721.5\, d^3 n$....(2)

Using the equation,

$$\delta = \frac{64 W R^3 n}{Cd^4}$$

$$\text{or} \quad 250 = \frac{64 \times 19.63 d^2 \times R^3 n}{70 \times 10^3 \times d^4}$$

$$250 = \frac{64 \times (19.63 d)^2 \times (4d)^3 n}{70 \times 10^3 \times d^4} = \frac{1.148 d^5 n}{d^4}$$

$250 = 1.148$ dn....(3)

$$\frac{\text{Equation (2)}}{\text{Equation (3)}} = \frac{721.5 d^3 n}{1.148 dn} = \frac{2250 \times 10^3}{250}$$

or$628.48 d^2 = 9000$

$$\therefore \qquad d = \sqrt[2]{\frac{9000}{628.48}} = 3.78 \text{ mm}$$

ii)Mean coil diameter

Mean coil Diameter, D = 8 d= 8 × 3.78 = 30.24 mm

iii) Number of coils required

Substitute the value of d in equation (iii)

250 = 1.148 × 3.78 × n

$$\therefore \qquad n = \frac{250}{1.148 \times 3.78}$$ = 57.6say 58 Nos.

Let us discuss about two dose-coiled concentric helical springs of same length are wound out of the same wire of 10 mm diameter support a load of 1000 N. The inner spring consists of 20 turns of mean diameter 16 cm and the outer spring has 18 turns of mean diameter 20 cm. Calculate the maximum shear stress in each spring.

Given:

Outer spring	**Inner spring**
d = 10 mm	d = 10 mm
n = 18	n = 20
D = 200 mm	D = 160 mm
W = 1000 N	
f_S = ?	

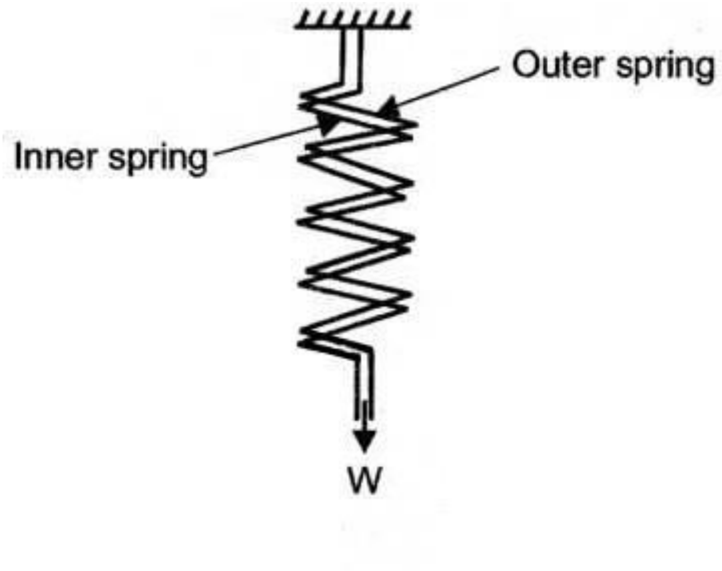

Figure:Spring

LetW_i = Load carried by inner spring

W_0 = Load carried by outer spring

f_{Si} = Max. shear stress in Inner spring

f_{So} = Max. shear stress in Outer spring

Since both the springs are wound concentrically,

Total load = Load carried by outer spring + Load carried by Inner spring

i.e.$W = W_o + W_i$.

or$1000 = W_o + W_i$

Due to concentric, deflections are also equal

i.e.,$d_o = d_i$

or $$\frac{64 W_o R_o^3 n_o}{C_o d_o^4} = \frac{64 W_i R_i^3 n_i}{C_i d_i^4}$$

or $64\ W_o \times 100^3 \times 18 = 64 \times W_i \times 80^3 \times 20$

($\because$ $C = C_i$; $d_o = d_i$)

or $18 \times 10^6\ W_o = 1024 \times 10^4\ W_i$

or $$W_i = \frac{18 \times 10^6}{1024 \times 10^4} W_o$$

$= 1.7578\ W_o$.... (ii)

Substitute W_i. in equation (i)

i.e.,$1000 = W_o + W_i$

or$1000 = W_o + (1.7578\ W_o) = 2.7578\ W_o$

$\therefore W_o = \frac{1000}{2.7578} = 362.6N$

$\therefore W_i = 1.7578 \times 362.6 = 637.4$ N

Max. shear stress induced

Outer spring

$$f_{So} = \frac{16 W_o R_o}{\pi d_o^3}$$

$$= \frac{16 \times 362.6 \times 100}{\pi \times 10^3}$$

= 184.67 N/mm^2

Inner spring

$$f_{Si} = \frac{16 W_i R_i}{\pi d_i^3}$$

$$= \frac{16 \times 637.4 \times 80}{\pi \times 10^3}$$

= 259.7 N/mm^2.

Here we slove an open coil helical spring made of 10 mm diameter wire and of mean diameter 10 cm has 12 coils, angle of helix being 15°. Determine the axial deflection and the intensities of bending and shear stress under a load of 500 N. Take C as 0.8 × 10^5 N/mm^2 and E as 2 × 10^{5}N/mm^2.

$$\text{i) } \delta = \frac{64\ WR^3 n \sec\alpha}{d^4}\left[\frac{\cos^2\alpha}{C} + \frac{2\sin^2\alpha}{E}\right]$$

ii) Bending stress, $f_b = \frac{M \times y}{I}$, where M = WR sin α

iii) Shear stress, $f_s = \frac{16T}{\pi d^3}$, where T = WR cos α

d = 10mm;D = 10 cm=100mm;∴ R= 50 mm

$n = 12; \alpha = 15°; W = 500N;$

$C = 0.8 \times 10^5\ N/mm^2$

Find:

δ, f_b and f_s

i)Deflection of spring

$$\delta = \frac{64\ WR^3 n \sec\alpha}{d^4}\left[\frac{\cos^2\alpha}{C} + \frac{2\sin^2\alpha}{E}\right]$$

$$= \frac{64 \times 500 \times 50^3 \times 12 \times \sec 15}{10^4}\left[\frac{\cos^2 15}{0.8 \times 10^5} + \frac{2\sin^2 15}{2 \times 10^5}\right]$$

= 61.5 mm (Ans)

ii)Bending Stress

$M = WR \sin \alpha = 500 \times 50 \times \sin 15 = 6470 Nmm$

$$y = \frac{10}{2} = 5\ mm;\ I = \frac{\pi}{64}d^4 = \frac{\pi}{64} \times 10^4 = 490.62\ mm^4$$

$$\therefore f_b = \frac{My}{I} = \frac{6470 \times 5}{490.62}$$ = 65.9 N/mm^2(Ans)

iii)Shear stress

$T = WR \cos \alpha$

$= 500 \times 50 \times \cos 15° = 24148\ N\ mm$

$$f_s = \frac{16T}{\pi d^3} = \frac{16 \times 24148}{\pi \times 10^3}$$

$$= 123 \text{ N/mm}^2$$