Stepped shafts

Stepped shafts are widely applied in machine and automotive industry. A lot of these steps have toothed wheel rims or worm windings. Products of this kind are mainly manufactured by means of machining from semi-products obtained in metal forming processes (e.g.forging, extrusion, rolling).

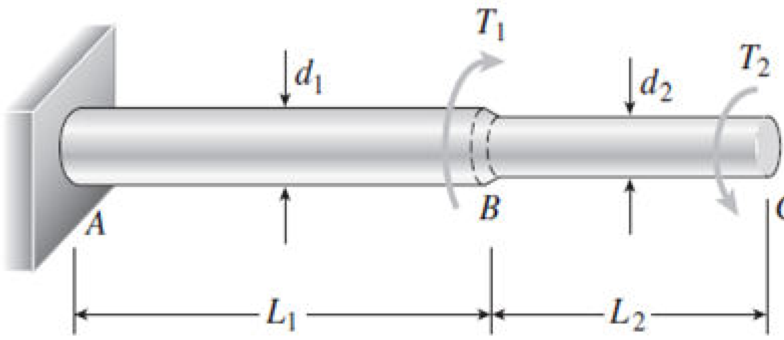
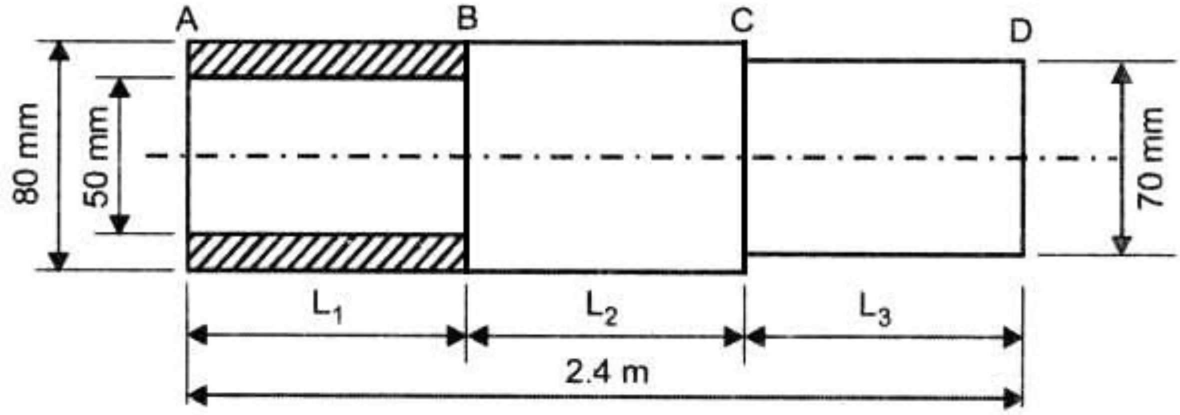


Fig.: stepped shafts

Let's consider a steel shaft ABCD having a total length of 2.4 m consists of three lengths having different sections as follows:AB is hollow having outside and inside diameters of 80 mm and 50 mm respectively and BC and CD are solid, BC having a diameter of 80 mm and CD a diameter of 70 mm. If the angle of twist is the same for each section, determine the length of each section and the total angle of twist if the maximum shear stress in the hollow portion is 50 N/mm2. Take C = 8.2 × l0^4 N/mm^2.

Solution:

The stepped shaft is shown below:



Shaft 1

Shaft AB

L_1= length;D_1 = 80 mmd_1 = 50 mm$(f_s)_1$ = 50 N/mm^2

Shaft 2

Shaft BC

length = L_2D_2 = 80 mm

Shaft 3

Shaft CD

length= L_3D_3= 70 mm

It is given angle of twist θ is same for each section.

i.e, $\theta_1 = \theta_2 = \theta_3$

$C = 8.2 \times 10^4$ N/mm^2

Polar Moment of Inertia of various shafts

$$J_1 = \frac{\pi}{32}\left(D_1^4 - d_1^4\right) = \frac{\pi}{32}\left(80^4 - 50^4\right) = 340.9 \times 10^4 \text{ mm}^4$$

$$J_2 = \frac{\pi}{32} \times D_2^4 = \frac{\pi}{32} \times 80^4 = 235.8 \times 10^4 \text{ mm}^4$$

$$J_3 = \frac{\pi}{32} \times D_3^4 = \frac{\pi}{32} \times 70^4 = 235.8 \times 10^4 \text{ mm}^4$$

Using the Torsion equation, $\frac{T}{J} = \frac{C\theta}{l}$

$$\theta = \frac{T\ell}{JC}$$

$$\therefore \theta_1 = \frac{T_1 \ell_1}{J_1 C_1}; \qquad \theta_2 = \frac{T_2 \ell_2}{J_2 C_2}; \qquad \theta_3 = \frac{T_3 \ell_3}{J_3 C_3}$$

But$C_1 = C_2 = C_3$

$T_1 = T_2 = T_3$ and

$\theta_1 = \theta_2 = \theta_3$

$$\therefore \quad \frac{\ell_1}{J_1} = \frac{\ell_2}{J_2} = \frac{\ell_3}{J_3}$$

or $$\frac{\ell_1}{340.9 \times 10^4} = \frac{\ell_2}{402.4 \times 10^4} = \frac{\ell_3}{235.8 \times 10^4}$$

or $$\frac{\ell_1}{340.9} = \frac{\ell_2}{402.4} = \frac{\ell_3}{235.8}$$

or $$\ell_1 = \frac{340.9}{235.8}\ell = 1.44\ \ell_3$$

or $$\ell_2 = \frac{402.4}{235.8}\ell_3 = 1.71\ \ell_3$$

But $l_1 + l_2 + l_3 = 2400$ mm

Substituting l_1 and l_3 in terms of l_3,

$1.44\ l_3 + 1.71\ l_3 + l_3 = 2400$

Solving, $l_3 = 578.35$ mm

or $l_1 = 1.44 \times 578.35 = 832.75$ mm

$l_2 = 1.71 \times 578.35 = 988.80$ mm

To Find angle of twist, θ

Using the equation, $$\frac{fs}{R} = \frac{C\theta}{\ell}$$

For shaft AB: $$\frac{(fs)_1}{\left(\frac{D_1}{2}\right)} = \frac{C \times \theta_1}{\ell_1}.$$

Substituting the known values,

$$\theta_1 = \frac{(fs)_1 \times \ell_1}{\left(\frac{D_1}{2}\right) \times C} = \frac{50 \times 832.75}{\left(\frac{80}{2}\right) \times 8.2 \times 10^4}$$

= 0.01269 radians = 0.7273°

Total angle of twist = $\theta_1 + \theta_2 + \theta_3$

$= 3\ \theta_1 = 3 \times 0.727 = 2.1819°$