Problems

Let us consider a solid shaft being subjected to a torque of 1.6 k Nm. Find the necessary diameter of the shaft, if the allowable shear stress is 50 MPa. The allowable twist is 1° for every 20 diameters length of the shaft. Take C = 80 GPa.

Given:

T = 1.6 k Nm,f_s = 50 N/mm^2,θ=1°,

l = 20 d,C = 80 N/mm^2

i)***Diameter of shaft considering strength of shaft.***

(i.e., Taking the consideration that the shear stress should not exceed the permissible limit)

For a solid circular shaft,

$$T = \frac{\pi}{16} f_s D^3$$

$$1.6 \times 10^6 = \frac{\pi}{16} \times 50 \times D^3$$

$$\text{or} D = \sqrt[3]{\frac{16 \times 1.6 \times 10^6}{\pi \times 50}} = 54.63 \text{ mm Say } 55 \text{ mm}$$

ii)Diameter of shaft considering stiffness of shaft.

(i.e., Taking the consideration that the angle of twist should not exceed the permissible limit)

Using the Torsional equation,

$$\frac{T}{J} = \frac{C\theta}{\ell} \qquad \left(\because J = \frac{\pi}{32}D^4 \text{ for soild shaft}\right)$$

Or $$\frac{T}{\frac{\pi}{32}D^4} = \frac{C\theta}{20D}$$ **($\because$ 1 = 20D)**

Or $$\frac{32T}{\pi D^3} = \frac{C\theta}{20}$$ $\theta = 1° = \frac{\pi}{180}$ rad

$$D = \sqrt[3]{\frac{32T \times 20}{\pi C\theta}}$$

$$= \sqrt[3]{\frac{32 \times 1.6 \times 10^6 \times 20 \times 180}{\pi \times 80 \times 10^3 \times \pi}}$$

= 61.59 mm say 62 mm

∴ Necessary diameter of shaft is the maximum value of (i) and (ii) i.e., 62 mm.

A solid cylindrical shaft is to transmit 300 KW power at 100 rpm. If the shear stress is not to exceed 60 N/mm^2, find its diameter. What percent saving in weight would be obtained it this shaft is replaced by a hollow one whose internal diameter equals to 0.6 of the external diameter, the length, the material and the maximum shear stress being the same.

Given

Solid shaft	Hollow shaft
speed, N = 100 rpm	
P_s = 300 KW	P_h = 300 KW
f_s = 60 N/mm^2	d = 0.6 D_h
D =?	

Length is same, i.e., $1_s = 1_h$

Material is same, i.e., $C_s = C_h$;

Max shear stress is same, i.e., $(f_s)_s = (f_s)_h$

For a solid shaft,

using the equation,P = $\frac{2\pi NT}{60}$

or300 × 10^3 = $\frac{2\pi \times 100 \times T}{60}$

∴ Torque, T = $\frac{300 \times 10^3 \times 60}{2\pi \times 100}$

= 28647.8 Nm

= 28647800 Nmm

Using the equation,T = $\frac{\pi}{16} f_S D^3$

$28647800 = \frac{\pi}{16} \times 60 \times D^3$

$\text{or} D = \sqrt[3]{\frac{28647800 \times 16}{\pi \times 60}}$

SolvingD = 134.49 say l35mm.

For hollow shaft

Givend = 0.6 D_h

Since length, material and maximum shear stress values are the same, the torque transmitted by a solid shaft is equal to the torque transmitted by a hollow shaft, i.e., T_h = 28647800 N mm

But Torque transmitted by a hollow shaft

$$T_h = \frac{\pi}{16} f_s \frac{(D_h^4 - d^4)}{D_h}$$

$$\therefore 28647800 = \frac{\pi}{16} \times 60 \times \left(\frac{D_h^4 - (0.6\,D_h)^4}{D_h} \right)$$

$$= \frac{60\pi}{16} \times \left(\frac{D_h^4 - 0.1296\,D_h^4}{D_h} \right)$$

$$= \frac{60\pi}{16} \times \left(\frac{0.8704\,D_h^4}{D_h} \right)$$

$$28647800 = \frac{60\pi}{16} \times 0.8704 D_h^3$$

$$\therefore D_h = \sqrt[3]{\frac{16 \times 28647800}{60\pi \times 0.8704}}$$

= 140.86mm say 141 mm

∴Internal dia, $d = 0.6\ D_h$

= 0.6 × 141

= 84.6 mm

% Saving in weight

$$\text{Weight of shaft} = \begin{pmatrix} \text{weight} & & \text{Cross} & & \text{Length} \\ \text{density} & \times & \text{section area} & \times & \text{of} \\ \text{of shaft} & & \text{of shaft} & & \text{shaft} \end{pmatrix}$$

∴ Weight of solid shaft

$$w_s = \rho \frac{\pi}{4} D^2 l$$

Weight of hollow shaft

$$W_h = \rho \frac{\pi}{4}(D_h^2 - d^2)l$$

Both shafts are of same length and same material. Therefore, density and length are the same. And also weight of solid shaft is more than weight of hollow shaft for the same length.

∴ Percentage saving in weight

$$= \frac{w_s - W_h}{w_s} \times 100$$

$$= \frac{\left(\frac{\pi}{4} D^2 \rho l\right) - \left(\frac{\pi}{4}(D_h^2 - d^2)\rho l\right)}{\frac{\pi}{4} D^2 \rho l} \times 100$$

$$= \frac{D^2 - (D_h^2 - d^2)}{D^2} \times 100$$

(∵ D = 135 mm D_h = 141mn)

$$= \frac{135^2 - (141^2 - 84.6^2)}{135^2} \times 100$$

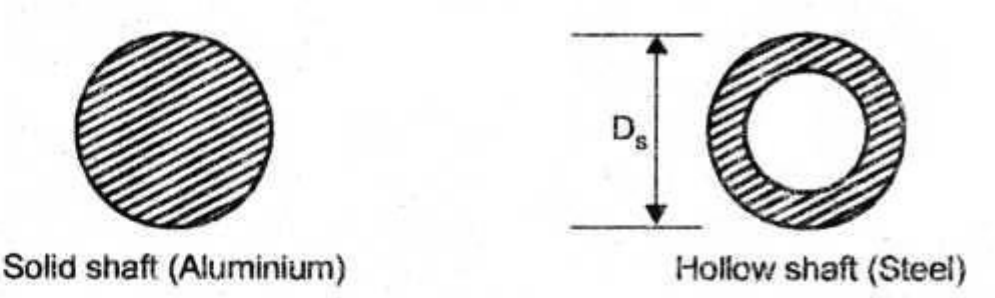
(d_h = 84.6 mm)

$$= \frac{18225 - 12723}{18225} \times 100$$

= 30.18%

Here we see an example of a solid aluminium shaft 1 m long and 100 mm diameter is replaced by a hollow steel shaft of the same length and same external diameter. The angle of twist per unit torsional moment over total length is same for both the shafts. If modulus of rigidity of steel is thrice that of aluminium, find the inner diameter of steel shaft.

Given



1_a=1000 mm1_s= 1000 mm

D_a = 100 mmD_s = D_a= 100 mm

C_s = 3 C_a

It is given

$$\left(\frac{\theta}{T}\right)_{Solid} = \left(\frac{\theta}{T}\right)_{hollow}$$

C_{Steel} = 3 $C_{aluminium}$

Find the diameter of shaft.

Using the torsional equation,

$$\frac{T}{J} = \frac{C\theta}{\ell}$$

orTl = CθJ

or $$\frac{\theta}{T} = \frac{\ell}{CJ}$$

$$\left(\frac{\theta}{T}\right)_{solid} = \left(\frac{\theta}{T}\right)_{hollow}$$

It is given

$$\text{or} \quad \left(\frac{\ell}{CJ}\right)_{soild} = \left(\frac{\ell}{CJ}\right)_{hollow}$$

$$\text{or} \quad \frac{\ell_a}{C_a J_a} = \frac{\ell_S}{C_S J_S}$$

$$\text{or} \quad \frac{1000}{C_a \times \frac{\pi}{32} D_a^4} = \frac{1000}{3C_a \times \frac{\pi}{32}(D_S^4 - d^4)} \qquad (C_S = 3C_a)$$

$$\text{or} \quad \frac{1}{D_a^4} = \frac{1}{3 \times (D_s^4 - d^4)}$$

$$\text{or} \quad \frac{1}{100^4} = \frac{1}{3 \times (100^4 - d^4)}$$

or$100^4 = (3 \times 100^4) - 3d^4$

or$3d^4 = (3 \times 10^8) - 10^8$

$3d^4 = 2 \times 10^8$

Solving,d = $\sqrt[4]{\frac{2 \times 10^8}{3}}$

= 90.36mmsay 91 mm.

Let us consider a solid circular shaft transmits 75 KW power at 200 rpm. Calculate the shaft diameter, if the twist in the shaft is not to exceed 1 ° in 2m length of shaft and shear stress is

limited to 50 N/mm^2. Take modulus of rigidity, C = 1 × 105 N/mm^2.

Given:

P = 75 KW,N = 200rpm,

θ = 1°,l = 2000 mm

f_s = 50N/mm^2,C = 1 × $l0^5$ N/mm^2. Solid shaft dia?

Using the equation, P = $\frac{2\pi NT}{60}$

$$T = \frac{60\ P}{2\pi N}$$

$$= \frac{60 \times 75 \times 10^3}{2\pi \times 200}$$

= 3582.8Nm

= 3582.8 × 10^3 N mm

ii)***Diameter of shaft considering the strength of shaft.***

Using the equation, T = $\frac{\pi}{16} f_s D^3$

Diameter, $D = \sqrt[3]{\frac{16T}{\pi f_s}}$

$$= \sqrt[3]{\frac{16 \times 3582.8 \times 10^3}{\pi \times 50}}$$

= 71.47 say 72 mm

ii)Diameter of shaft considering the stiffness of shaft.

Using the equation, $\frac{T}{J} = \frac{C\theta}{\ell}$ (0=1° = $\frac{\pi}{180}$ rad)

$$\frac{T}{\frac{\pi}{32}D^4} = \frac{C\theta}{\ell}$$

Substituting the values,

$$\frac{32 \times 3582.8 \times 10^3}{\pi \times D^4} = \frac{1 \times 10^5 \times \pi}{2000 \times 180}$$

or $\frac{32 \times 3582.8 \times 10^3 \times 2000 \times 180}{1 \times 10^5 \times \pi^2} = D^4$

orD = $\sqrt[4]{\frac{32 \times 3582.8 \times 10^3 \times 2000 \times 180}{1 \times 10^5 \times \pi^2}}$

= 80.43 mmsay 81 mm

∴ Diameter of the shaft is the larger value of (i) and (ii) i.e., 81 mm.

A hollow shaft of diameter ratio $3/8$ (internal dia to outer dia) is to transmit 375 kW power at 100 rpm. The maximum torque being 20 % greater than the mean torque. The shear stress is not to exceed 60 N/mm^2 and twist in a length of 4m not to exceed 2°. Calculate the external and internal diameters which would satisfy both the above conditions. Assume modulus of rigidity, C = 0.85 × 10^5 N/mm^2.

Given:

$\frac{d}{D} = 3/8$, P = 375 kW = 375 × 10^{3}w,

N = 100 rpmT_{max} = 20 % greater than T_{mean} = 1.2 T_{mean}

f_s = 60 N/mm^2.θ = 2° = 2 × $\frac{\pi}{180}$ rad

l = 4m = 4000 mm;C = 0.85 × 10^5 N/mm^2

Find d ? D?

Using the relation, P = $\frac{2\pi NT}{60}$

or $$T = \frac{60P}{2\pi N} = \frac{60 \times 375 \times 10^3}{2\pi \times 100}$$ = 35810 Nm

$T_{max} = 1.2T_{max}$

$\therefore T_{max} = 1.2 \times 35810$ Nm

= 42972 Nm

= 42972×10^3 N mm

i)***Diameter of the shaft considering the strength of shaft (i.e., shear stress not to exceed 60 N/mm^2)***

For a hollow shaft, using the relation

$$T = \frac{\pi}{16} f_s \left(\frac{D^4 - d^4}{D} \right)$$

or $42972 \times 10^3 = \frac{\pi}{16} \times 60 \left(\frac{D^4 - \left(\frac{3}{8}D\right)^4}{D} \right)$

(Note that $T = T_{max}$ and $\frac{d}{D} = \frac{3}{8}$ or $d = \frac{3}{8}$ **D)**

or $\frac{42972 \times 10^3 \times 16}{\pi \times 60} = \frac{D^4 - \left(\frac{3}{8}D\right)^4}{D}$

$$= \frac{D^4}{D}\left\{1 - \left(\frac{81}{4096}\right)\right\}$$

$3649426 = D^3 \times \frac{4015}{4096}$

or $D = \sqrt[3]{\frac{3649426 \times 4096}{4015}}$

= 154.98 mmsay 155 mm

***ii)*Diameter of the shaft considering the stiffness of shaft (i.e., angle of twist not to exceed 2°)**

Using the relation, $\frac{T}{J} = \frac{C\theta}{\ell}$

Substitute $T = T_{max}$ = 42972000 N mm

$J = \frac{\pi}{32} (D^4 - d^4)$

$C = 0.85 \times 10^3$

$\theta = 2° = \frac{2 \times \pi}{180}$ rad = 0.0349 rad

l = 4m = 4000 mm

$$\therefore \quad \frac{42972000 \times 32}{\pi(D^4 - d^4)} = \frac{0.85 \times 10^5 \times 0.0349}{4000}$$

or
$$D^4 - d^4 = \frac{42972000 \times 32 \times 4000}{\pi \times 0.85 \times 10^5 \times 0.0349}$$
$$= 590.2 \times 10^6$$

or
$$D^4 - \left(\frac{3}{8}D\right)^4 = 590.2 \times 10^6$$

or
$$D^4 - \left(\frac{81}{4096}D^4\right) = 590.2 \times 10^6$$

or
$$D^4\left(1 - \frac{81}{4096}\right) = 590.2 \times 10^6$$

or
$$D^4\left(\frac{4015}{4096}\right) = 590.2 \times 10^6$$

or
$$D = \sqrt[4]{\frac{590.2 \times 10^6 \times 4096}{4015}}$$

= 156.6 mm say 157 mm

Ans: The diameter of the shaft to satisfy both the conditions are the greater of values (i) and (ii) i.e., 157 mm.

∴External dia. D = 157 mm

Internal dia,d = $\frac{3}{8} D = \frac{3}{8} \times 157$

= 59 mm

A hollow shaft is to transmit 300 KW at 80 rpm. If the shear stress is not to exceed 60 MN/m^{2}and internal diameter is 0.6 of the external diameter, find the external and internal diameters assuming that the maximum torque is 1.4 times the mean.

Given:

PowerP = 300 KW = 300×10^3 Watts

Speed,N = 80rpm

Shear stress,f_s = 60 MN/m^2 = $\frac{60 \times 10^6}{10^6}$ = 60 N/mm^2

Internal diameter= 0.6 × External diameter

i.e.,d = 0.6 D

$T_{max} = 1.4 \times T_{mean}$

Find D ?d?

Using the relation, P = $\frac{2\pi N T_{mean}}{60}$

$\therefore T_{mean} = \frac{60\,P}{2\pi N}$

$= \frac{60 \times 300 \times 10^3}{2\pi \times 80}$ = 35809 Nm

= 35809×10^3 N mm

i)Diameter of shaft based on strength of shaft.

Using the relation,T = $\frac{\pi}{16} f_s \left(\frac{D^4 - d^4}{D} \right)$

or1.4 × 35809 × 10^3 = $\frac{\pi}{16} \times 60 \left[\frac{D^4 - (0.6D)^4}{D} \right]$

or50132600 = $11.78 \left(\frac{D^4 - 0.1296 D^4}{D} \right)$

or $\frac{D^4}{D}(1 - 0.1296)$ = 4255738

orD^3 (0.8704) = 4255738

∴D = $\sqrt[3]{\frac{4255738}{0.8704}}$

= 169.72 say l70 mm.

∴ External diameter, D =170 mm

Internal diameter, d = 0.6 D

= 0.6 × l70 = 102 mm.

A solid shaft is subjected to a torque of 45 k Nm. If angle of twist is 0.5 degree per meter length of the shaft and shear stress is not to exceed 90 MN/m2 find,i)Suitable diameter of shaft,ii)Find maximum shear stress and the angle of twist per meter length. Modulus of rigidity = 80 GN/m2.

Given:

Torque,T = 45kNm = 45×10^6Nmm

Angle of twist,θ = 0.5° = 0.5 × $\pi/180$ rad = 8.726×10^3 rad

Shear stress,f_s = 90 MN/m^2= 90 N/mm^2

Shear modulus,C = 80 GN/m^2 = $\frac{80 \times 10^9}{10^6}$ = 80×10^3 N/mm^2

Length = 1 m = 1000 mm

i)***Diameter of shaft based on strength of shaft***

Using the relation,T = $\frac{\pi}{16} f_s D^3$

or45×10^6 = $\frac{\pi}{16}$ × 90 × D^3

orD = $\sqrt[3]{\frac{16 \times 45 \times 10^6}{\pi \times 90}}$

= 136.55say 137mm

ii)***Diameter of shaft based on stiffness of shaft***

Using the relation $\frac{T}{J} = \frac{C\theta}{\ell}$

or $$\frac{T}{\frac{\pi}{32}D^4} = \frac{C\theta}{\ell}$$

orD = $$\sqrt[4]{\frac{32T \times 1}{\pi \times C\theta}}$$

$$= \sqrt[4]{\frac{32 \times 45 \times 10^6 \times 1000 \times 180}{\pi \times 80 \times 10^3 \times 0.5 \times \pi}}$$

= 160 mm.

∴ Diameter of the shaft is the greater of values (i) and (ii) i.e., 160 mm

iii)*Find maximum shear stress*

Using the relation $\frac{T}{J} = \frac{f_s}{R}$

or $$\frac{45 \times 10^6}{\frac{\pi}{32} \times 160^4} = \frac{f_8}{\left[\frac{160}{2}\right]}$$ (Substitute D = 160mm)

∴f_s = 55.95N/mm^2

iv)Find angle of twist per meter length

(Substitute D = 160 mm)

Using the relation, $\frac{T}{J} = \frac{C\theta}{\ell}$

or $$\frac{45 \times 10^6}{\frac{\pi}{32} \times 160^4} = \frac{80 \times 10^3 \times \theta}{1000}$$

Solving, $\theta = 8.742 \times 10^{-3}$ rad

$= 8.742 \times 10^{-3} \times \left(\frac{180}{\pi}\right)$ degrees

$= 0.5°$.

Let us consider the diameter of a sold shaft to transmit 90 KW at 160 rpm such that the shear stress is limited to 60 N/mm^2. The maximum torque is likely to exceed the mean torque by 20%. Also find the permissible length of the shaft, if the twist is not to exceed 1 degree over the entire length. Take rigidity modulus as 0.8 × 10^5 N/mm^2.

Given:

P = 90 KW = 90×10^3 Watts,

N = 160 rpm, f_s = 60 N/mm^2,

$T_{max} = 1.2\ T_{mean}$

$\theta = 1° = \frac{1 \times \pi}{180}$ rad ;

C = 0.8×10^5 N/mm^2.

Find l?

Using the relation, $P = \frac{2\pi N T_{mean}}{60}$

$$\therefore T_{mean} = \frac{60P}{2\pi N} = \frac{60 \times 90 \times 10^3}{2\pi \times 160}$$

= 5374.2Nm

= 5374.2×10^3 N mm.

i)***Diameter of shaft***

Using the relation, $T = \frac{\pi}{16} f_S D^3$

or $D = \sqrt[3]{\frac{16T}{\pi \times f_S}}$

$$= \sqrt[3]{\frac{16 \times 5374.2 \times 10^3 \times 1.2}{\pi \times 60}}$$

= 81.80 say 82 mm

ii)***Permissible length, l***

Using the relation, $\frac{T}{J} = \frac{C\theta}{l}$

$$\frac{T}{\frac{\pi}{32} D^4} = \frac{C\theta}{l}$$

or

Substituting the values,

$$\frac{1.2 \times 5374.2 \times 10^3}{\frac{\pi}{32} \times 82^4} = \frac{0.8 \times 10^5 \times \pi}{180 \times l}$$

$$l = \frac{0.8 \times 10^5 \times \pi \times 82^4 \times \pi}{32 \times 5374.2 \times 10^3 \times 180 \times 1.2}$$

= 961 mm