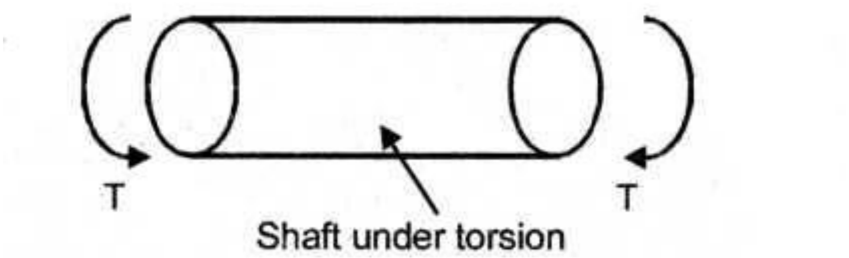
Torsion formulation stresses and deformation in circular and hollows shafts

3.1.1 Torsion

Torsion

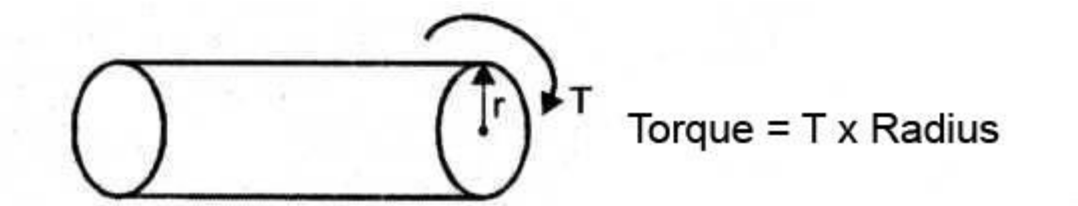
A shaft is said to be in torsion, when equal and opposite torques (i.e., Rotational moment) are applied at the two ends of the shaft.



3.1.2 Torque

Torque

Torque is a rotational moment equal to the product of the force applied tangentially at the end of the shaft and radius of shaft.



Torque transmitted by a Solid shaft

$$T_{max} = \frac{\pi}{16} f_s D^3$$

Wherefs = Maximum shear stress induced at the outer surface of the shaft.

D = Diameter of solid shaft.

Hollow shaft

T_{max} = $\frac{\pi}{16} f_s \left[\frac{D^4 - d^4}{D} \right]$

WhereD = Outer diameter of shaft

d = Inner diameter of shaft

3.1.3 Torsional equation

Torsional equation

$$\frac{T}{J} = \frac{f_s}{R} = \frac{C\theta}{\ell}$$

where,

T= Torque in N mm

J= Polar moment of inertia of cross section of shaft in mm^4.

f_s= Shear stress in N/mm^2

R= Radius of shaft in mm

C= Shear modulus of shaft material in N/mm^2

θ= Angle of twist in radians

l= Length of shaft in mm

Torsional equation

1.The material of the shaft is uniform throughout, homogeneous perfectly elastic and obeys Hooke's law.

2.Twist is uniform along the length of shaft.

3.The shaft is uniform circular cross section throughout its length.

4.Cross section of the shaft which are plane before twist remain plane after twist.

5.The stress does not exceed the limit of proportionality.

3.1.4 Polar modulus

Let us discuss about polar modulus

The ratio of polar moment of inertia to the radius of the shaft is known as polar modulus. It is denoted by the symbol Z.

$$\text{Polar Modulus} = \frac{\text{Polar moment of inertia, } J}{\text{Radius of shaft, } R}$$

$$\boxed{Z = \frac{J}{R}}$$

3.1.5 Shaft

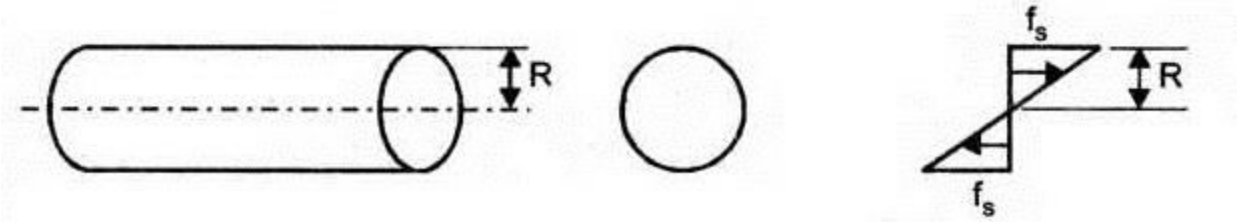
Strength of shaft:

The maximum torque (or) the maximum power that can be transmitted by a shaft is known as strength of a shaft.

Stiffness of shaft:

The torque required to produce a twist of one radian per unit length of the shaft is known as stiffness of the shaft.

Stress distribution:



Longitudinal view of shaft Cross section Shear stress distribution

Torsional rigidity

Torsional Rigidity = GJ

where G = Shear modulus andJ = Polar moment of Inertia

For solid shaft,J = $\frac{\pi}{32}D^4$

Torsional Rigidity = $G\frac{\pi}{32}D^4$

Power transmitted by a shaft

Power Transmitted by a shaft

P = $\frac{2\pi NT}{60}$

WhereT = Average Torque in Nm.

N = rpm of the shaft.

P = Power transmitted in watts.

Shear stress produced in a circular shaft

Consider a solid circular shaft of Radius 'R' and length l fixed at the left end and subjected to a torque 'T' at the right end as shown below. The cross section of the shaft is shown in Figure,

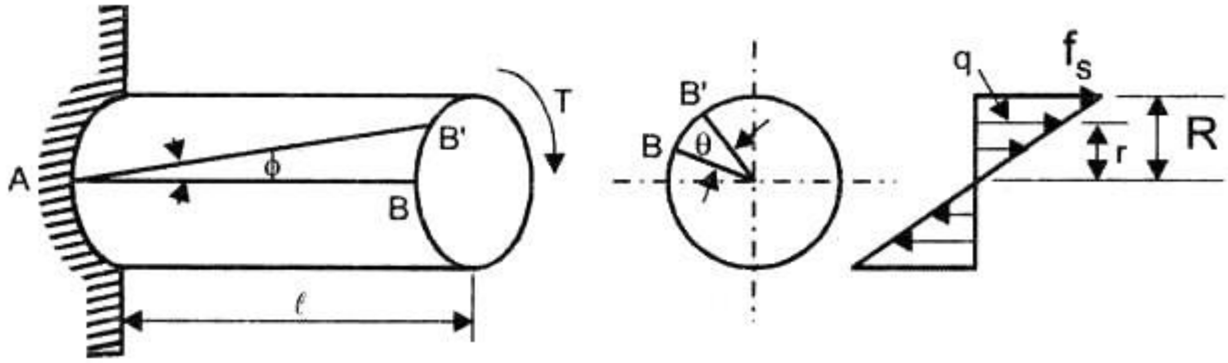


Figure:cross section of the shaft

Prior to the application of torque, all the fibers in the shaft along the length are straight and parallel to each other. After the application of clockwise torque T at the free end of shaft, all the fibers will be twisted in the clockwise direction and every cross section of the shaft is subjected to shear stress.

As a result of T, let the fiber AB on the surface of the shaft be twisted to AB' at an angle φ) (in degrees). In the cross section of the shaft, the twist from B to B^1 is measured by an angle 'θ' (in radians).

Shear strain at the outer surface= Distortion per unit length

$$= \frac{\text{Distortion at the outer surface}}{\text{Length of shaft}}$$

$$= \frac{BB'}{\ell} = \tan \phi$$

φ being very small, tan φ = φ

$$\therefore \qquad \boxed{\phi = \frac{BB'}{\ell}} \qquad \dots(1)$$

Arc BB' = OB × θ

= Rθ(∵ OB = Radius = R)

Substituting BB' in equation (1)

$$\boxed{\phi = \frac{R\theta}{\ell}} \qquad \dots(2)$$

We know shear modulus = $\frac{\text{Shear Stress}}{\text{Shear Strain}}$

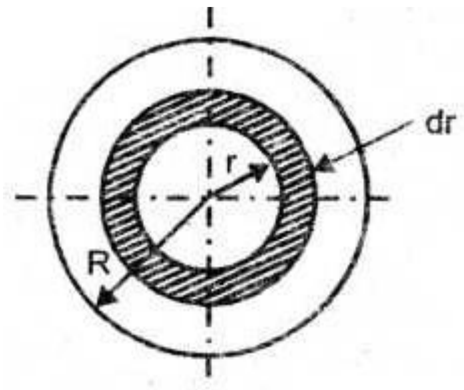
$$= \frac{\text{Shear Stress at the outer surface}}{\text{Shear Strain at the outer surface}}$$

$$C = \frac{f_s}{\left(\frac{R\theta}{\ell}\right)} \qquad \left(\begin{array}{l} f_s = \text{Shear stress} \\ C = \text{Shear modulus} \end{array}\right)$$

$$C = \frac{f_s \times l}{R\theta}$$

or $\boxed{\frac{f_s}{R} = \frac{C\theta}{\ell}}$ $\qquad \dots(3)$

In the above equation Torsional shear stress f_s may be expressed in terms of shear modulus and angle of twist. Similarly the torsional shear stress may be expressed in terms of torque as below.



Consider a solid circular shaft of Radius R as shown in Figure

LetT = Maximum Torque the solid shaft can Transmit.

f_s = max shear stress at the outer surface of shaft.

q = Shear stress @ radius r.

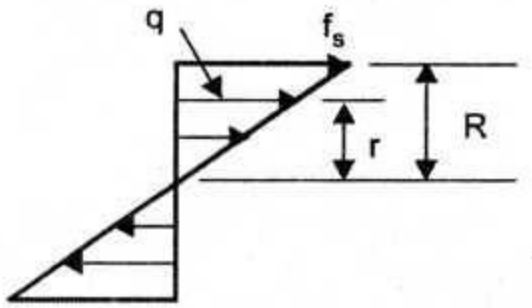
Consider an elementary circular ring of thickness 'dr' at a radial distance 'r' as shown in Figure (c)Area of the ring, dA = 2πrdr

The distribution of shear stress is shown in figure.

From similar triangles,

$$\frac{f_s}{R} = \frac{q}{r}$$

or $$\boxed{q = \frac{f_s}{R} r}$$



∴ Shear stress at the radius $r, q = \frac{f_s}{R} r$

$$\therefore \quad \left.\begin{array}{l}\text{Turning force on the} \\ \text{elementary circular ring}\end{array}\right\} = \begin{bmatrix}\text{Shear stress} \\ \text{on the ring}\end{bmatrix} \times \begin{bmatrix}\text{Area of} \\ \text{the ring}\end{bmatrix}$$

= q × dA

$$= \left(\frac{f_s}{R} r\right)(2\pi r dr) = \frac{f_s}{R} 2\pi r^2 dr$$

$$\therefore \quad \left.\begin{array}{l}\text{Turning moment} \\ \text{on the ring (dT)}\end{array}\right\} = \begin{bmatrix}\text{Turning} \\ \text{force}\end{bmatrix} \times \begin{bmatrix}\text{Radial} \\ \text{distance} \\ \text{of ring}\end{bmatrix}$$

$$= \frac{f_s}{R} 2\pi r^2 dr \times r = \frac{f_s}{R} r^2 (2\pi r dr)$$

$$dT = \frac{f_s}{R} r^2 dA \qquad (\because 2\pi r dr = dA)$$

∴ Total Torque T that can be transmitted by the solid shaft is determined by integrating dT between the limit zero and Radius R.

$$\therefore \quad T = \int_0^R dT = \int_0^R \frac{f_s}{R} r^2 dA = \frac{f_s}{R} \int_0^R r^2 dA$$

But $\int r^2 dA$ = Polar moment of inertia of circle, denoted by J

$$\therefore \quad T = \frac{f_s}{R} J$$

or $$\boxed{\frac{T}{J} = \frac{f_s}{R}} \qquad \text{.... (4)}$$

Now, combining the equations (3) and (4), we may write

$$\frac{T}{J} = \frac{f_s}{R} = \frac{C\theta}{\ell}$$

Which is known as the Torsional equation.