Problems

Let us consider a cylindrical shell 3 m long which is closed at the ends has an internal diameter of 1 m and a wall thickness of 15 mm. Young'smodulus 200 GN/m^2 and poisson's ratio = 0.3. Calculate the circumferential and longitudinal stresses induced and also change in diameter and change in length of the shell, if it is subjected to an internal pressure of 1.5 MN/m^2.

Given:

l = 3 m = 3000 mm

d = 1 m = 1000 mm

t = l5 mm

$$E = 200\frac{GN}{m^2} = \frac{200\times10^9}{(1000)^2} = \mathbf{200 \times l0^3\ N/mm^2}$$

$$\frac{1}{m} = 0.3$$

$$P = 1.5\frac{MN}{m^2} = \frac{1.5\times10^6}{(1000)^2} = 1.5\ N/mm^2$$

Find: σ_c = ? σ_1 = ? δ_d = ? δ_1 = ?

1)Circumferential stress

Circumferential stress, $\sigma_c = \frac{pd}{2t}$

$$= \frac{1.5 \times 1000}{2 \times 15}$$

= 50 N/mm^2

2)Longitudinal stress

Longitudinal stress, $\sigma_1 = \frac{pd}{4t}$

$$= \frac{1.5 \times 1000}{4 \times 15}$$ = 25 N/mm^2

3)Change in diameter

using the relation,

Change in diameter, $\delta_d = \frac{pd^2}{2tE}\left(1 - \frac{1}{2m}\right)$

$$= \frac{1.5 \times 1000^2}{2 \times 15 \times 2 \times 10^5}\left\{1 - \left(\frac{1}{2} \times 0.3\right)\right\}$$

= 2.125 mm

4)Change in length,

using the relation,

Change in length, $\delta_1 = \frac{pd\ell}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$

$$= \frac{1.5 \times 1000 \times 3000}{2 \times 15 \times 2 \times 10^5}\left(\frac{1}{2} - 0.3\right)$$

= 0.15 mm

Here we discuss about a cylindrical thin drum 80 cm in diameter and 3m long has a shell thickness of 1 cm. If the drum is subjected to an internal pressure of 2.5 N/mm^2, determine i) Change in diameter ii) Change in length and iii) Change in volume. Take E = 2 × 10^5 N/mm^2 and Poisson's ratio = 0.25.

Given:

d = 80 cm = 800 mm;l = 3 m = 3000 mm

t = 1 cm =10 mm p = 2.5 N/mm^2

E = 2 × 10^5 N/mm^2;1/m = 0.25

Find: δ_d = ? δ_l = ? δ_V = ?

1)Change in diameter, δ_d = $\frac{pd^2}{2tE}\left(1 - \frac{1}{2m}\right)$

$$= \frac{2.5 \times 800^2}{2 \times 10 \times 2 \times 10^5}\left\{1 - \left(\frac{1}{2} \times 0.25\right)\right\}$$

= 0.35 mm

2)Change in length, $\delta_l = \frac{pd\ell}{2tE}\left(\frac{1}{2}-\frac{1}{m}\right)$

$$= \frac{2.5\times 800\times 3000}{2\times 10\times 2\times 10^5}\left[\frac{1}{2}-0.25\right]$$

= 0.357 mm

3)Change in volume, $\delta_v = V\left(2\frac{\delta d}{d}+\frac{\delta \ell}{\ell}\right)$

$$= V\left(2\times\frac{0.35}{800}+\frac{0.357}{3000}\right)$$

But, volume, $V = \frac{\pi}{4}d^2\ell$

$$= \frac{\pi}{4}\times 800^2\times 3000$$

$= 150.796 \times 10^7\ mm^3$

$\therefore \delta_v = 150.79\times 10^7\left(\frac{0.7}{800}+\frac{0.357}{3000}\right)$

$= 1507960\ mm^3$.

Let us slove a cylindrical shell which is 1.5 m diameter and 4 m long closed at both the ends is subjected to an internal pressure of 3 N/mm^2. Maximum circumferential stress is not to exceed

150 N/mm^2. Find changes in diameter, length and volume of the cylinder. E = 2 × 10^{5}N/mm^2, Poisson's ratio = 0.25.

Given:

d = 1500 mml = 4000 mm;p = 3N/mm^2

E = 2 × 10^5 N/mm^{2}1/m = 0.25

Maximum principal stress, ie, σ_c = 150 N/mm^2

Find: δ_d = ?δ_l = ?δ_v = ?

It is given, σ_c = 150 N/mm^2

using the relation, $\sigma_c = \frac{pd}{2t}$

or150 = $\frac{3\times1500}{2\times t}$

Solving, t = 15 mm

Change in dia,δ_d = $\frac{pd^2}{2tE}\left(1-\frac{1}{2m}\right)$

$$= \frac{3\times1500^2}{2\times15\times2\times10^5}\left(1-\frac{1}{2}\times0.25\right)$$

= 0.984 mm

Change in length, δ_l = $\frac{pd\ell}{2tE}\left(\frac{1}{2}-\frac{1}{m}\right)$

$$= \frac{3\times1500\times4000}{2\times15\times2\times10^5}\left[\frac{1}{2}-0.25\right]$$

= 0.75 mm

Change in volume,

$$\delta_v = V\left[\frac{pd}{2Et}\left(\frac{5}{2}-\frac{2}{m}\right)\right]$$

$$= \left\{\frac{\pi}{4}\times1500^2\times4000\right\}\left[\frac{3\times1500}{2\times15\times2\times10^5}\left(\frac{5}{2}-2\times0.25\right)\right]$$

= 10602875 mm^3

A cylinder has an internal diameter of 230 mm, wall thickness 5 mm and is 1 m long. It is found to change in internal volume by 12 × 10^{-6} $m^3$when filled with a liquid at a pressure 'P'. Taking E = 200 GPa and Poisson's ratio = 0.25 determine the stresses in the cylinder, the changes in its length and internal diameter.

Given:

$\delta_v = 12 \times 10^{-6}\ m^3 = 12 \times 10^{-6}\ (1000)^3 = 12 \times 10^3\ mm^3$

using the relation,

$$\frac{\delta v}{V} = \frac{pd}{2Et}\left(\frac{5}{2} - \frac{2}{m}\right)$$

$$= \frac{p \times 230}{2 \times 5 \times 2 \times 10^5}\left(\frac{5}{2} - 2 \times 0.25\right)$$

$$\frac{12 \times 10^3}{\frac{\pi}{4} \times 230^2 \times 1000} = \frac{460p}{2 \times 5 \times 2 \times 10^5}$$

Solving, p = $\frac{12 \times 10^3 \times 2 \times 5 \times 2 \times 10^5}{\frac{\pi}{4} \times 230^2 \times 1000 \times 460}$

= 1.256 N/mm^2

1)Circumferential stress, σc= $\frac{pd}{2t}$

$$= \frac{1.256 \times 230}{2 \times 5}$$ = 28.88 N/mm^2

2)Longitudinal stress,$\sigma 1$ = $\frac{pd}{4t}$

$$= \frac{1.256 \times 230}{4 \times 5}$$ = 14.44 N/mm^2

3)Change in diameter,δd = $\frac{pd^2}{2tE}\left(1 - \frac{1}{2m}\right)$

$$= \frac{1.256 \times 230^2}{2 \times 5 \times 2 \times 10^5}\left(1 - \frac{1}{2} \times 0.25\right)$$

= 0.029 mm

4)Change in length,δ1 = $\frac{pd\ell}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$

$$= \frac{1.256 \times 230 \times 1000}{2 \times 5 \times 2 \times 10^5}\left(\frac{1}{2} - 0.25\right)$$

= 0.036 mm

Problems

Let us discuss about a cylindrical shell 2m long and 90 cm internal diameter and 12 mm metal thickness is subjected to an internal pressure of 1.6 N/mm2. Determine (a) Maximum intensity of shear stress (b) Changes in the dimensions of the shell. Take E = 2.1 × 10^5 N/mm^2 and 1/m = 0.3.

Given:

l = 2000 mmd = 900 mm

t = 12 mmp = 1.6 N/mm^2

E = 2.1 × 10^5 N/$mm^2$1/m = 0.3

Find: Maximum intensity of shear stress and δ_d= ?δ_1 = ?

1)Maximum intensity of shear stress,

$$\tau_{max} = \frac{pd}{8t}$$

$$= \frac{1.6 \times 900}{8 \times 12}$$ = 15 N/mm^2

2)Change in diameter, δ_d = $\frac{pd^2}{2tE}\left(1-\frac{1}{2m}\right)$

$$= \frac{1.6 \times 900^2}{8 \times 12 \times 2.1 \times 10^5}\left[1-\left(\frac{1}{2} \times 0.3\right)\right]$$

= 0.218 mm

3)Change in length, δ_l = $\frac{pd\ell}{2tE}\left(\frac{1}{2}-\frac{1}{m}\right)$

$$= \frac{1.6 \times 900 \times 2000}{2 \times 12 \times 2.1 \times 10^5}\left(\frac{1}{2}-0.3\right)$$

= 0.114 mm

We have a cylinder of 200 mm internal diameter and 50 mm thickness carries a fluid at a pressure of 10 MN/m^2. Calculate the maximum and minimum intensities of circumferential stress across the section. Also sketch the radial stress distribution and circumferential stress distribution across the section.

Given :

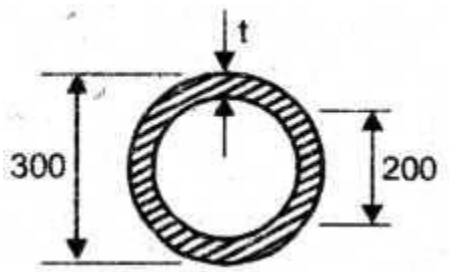
Inner dia, d = 200 mm

thickness, t = 50 mm

∴ Outer dia, D = d + 2t

= 200 + (2 × 50)

= 300 mm



Fluid pressure, p = 10 MN/m^2

$$= \frac{10 \times 10^6}{(1000)^2} = 10 \text{ N/mm}^2$$

Find: $(f_c)_{max}$ and $(f_c)_{min}$

Let r_1 = inner radius = $\frac{d}{2} = \frac{200}{2}$ = 100 mm

r_2 = outer radius = $\frac{D}{2} = \frac{300}{2}$ = 150 mm

Lame's equations are:

$f_r = \frac{b}{r^2} - a$ and $f_c = \frac{b}{r^2} + a$

where f_r and f_c are radial stress and circumferential stress respectively.

'a' and 'b' are the Lame's constants.

To find Lame's constants

Apply the boundary conditions

It is given $f_r = 10$ N/mm^2 at inner surface of shell

i.e., at r = 100 mm, $f_r = 10$ N/mm^2

∴ at r= 150 mm,$f_r = 0$

(outer surface)

Substitute these boundary conditions in f_r equation,

$$f_r = \frac{b}{r^2} - a$$

$$10 = \frac{b}{100^2} - a \quad \text{....(i)}$$

and $$0 = \frac{b}{150^2} - a \quad \text{....(ii)}$$

Solve equations (i) and (ii),

Subtracting equation (2) from equation (1),

$$10 = \frac{b}{100^2} - \frac{b}{150^2} = \frac{b}{10000} - \frac{b}{22500}$$

$$10 = \frac{2.25b - b}{22500}$$

Solving, b = $\frac{22500 \times 10}{1.25}$ = 180,000

Substituting the value of 'b' in equation (i),

$$10 = \frac{b}{100^2} - a$$

$$10 = \frac{180000}{100^2} - a$$

Solving, a = $\frac{180000}{100^2}$ – 10 = 8

∴ Lame's equations are

$f_r = \frac{180000}{r}$ – 8 and f_c = $\frac{180000}{r^2} + a$

To Find Circumferential (or Hoop) stress

i)At Inner surface (i.e., at r = 100 mm)

substitute r = 100 in f_c equation

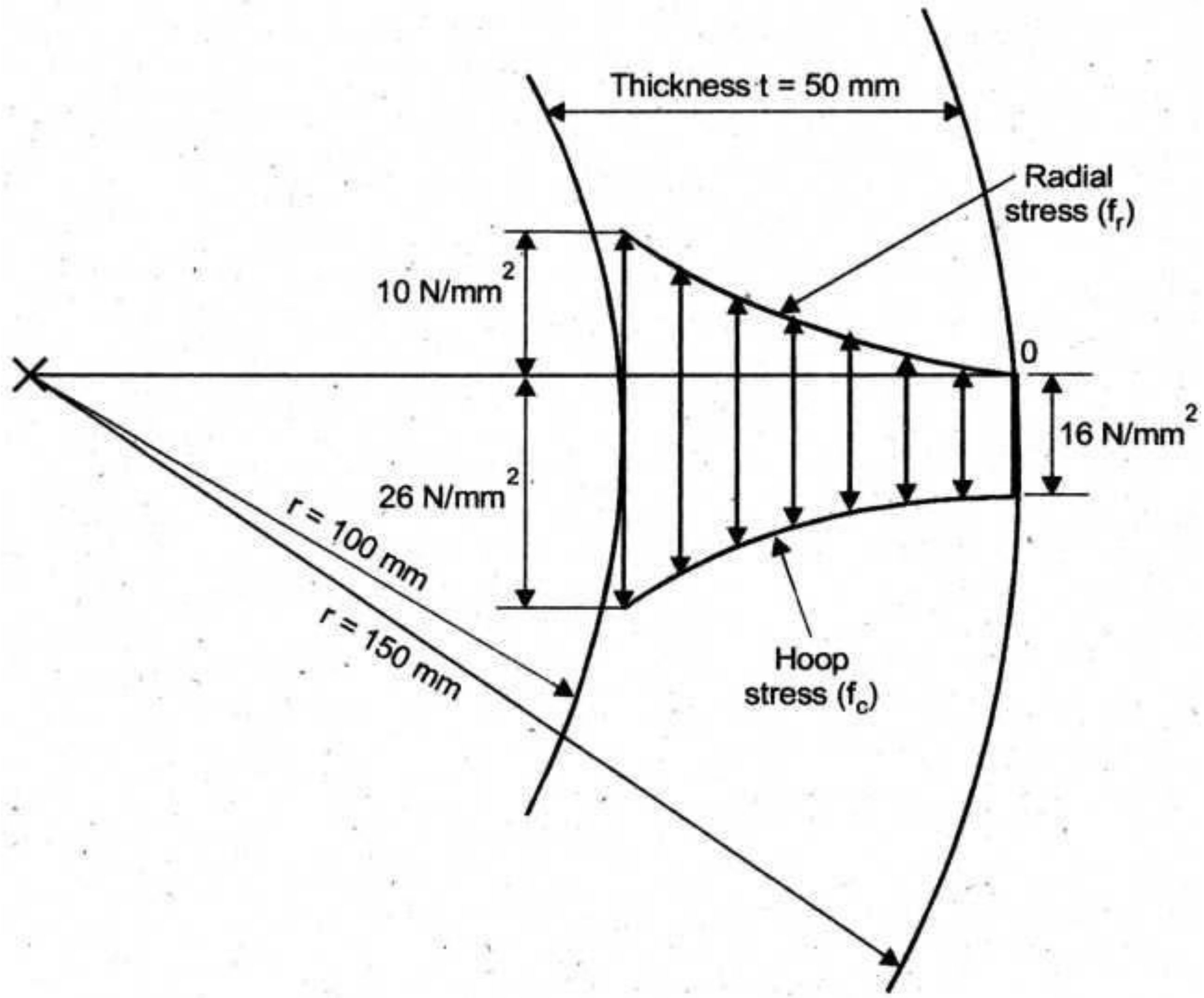
$\therefore\ f_c = \frac{180000}{100^2} + 8$ = 26 N/mm^2

ii)At outer surface (i.e., at r = 150 mm)

substitute r = 150 in f_c equation

$$\therefore f_c = \frac{180000}{150^2} + 8 = 16 \text{ N/mm}^2$$

The distribution of radial stress and Hoop stress across the section are shown below.



Note that fr is compressive and fc is tensile.

Here we calculate the thickness of metal necessary for a cylindrical shell of internal diameter of 160 mm to withstand an

internal pressure of 25 MN/m^2 if maximum permissible tensile stress is 125 MN/m^2.

Given:

Inner radiusr = $\frac{160}{2}$ = 80 mm

Inner pressure, P = 25 MN/m^2 = $\frac{25 \times 10^6}{(1000)^2}$ = 25 N/mm^2

Maximum permissible tensile stress,

i.e., $(f_c)_{max}$ = 125 MN/m^2 ($\because$ f_c is tensile nature)

Lame's equations are:

$$f_r = \frac{b}{r^2} - a \quad \text{and} \quad f_c = \frac{b}{r^2} + a$$

Boundary Conditions

Internal pressure, i.e., f_r at radius = 80 mm is 25 N/mm^2 Maximum tensile stress i.e., (f_c) at radius = 80 mm is 125 N/mm^2 Substituting these values in Lame's equations,

$$25 = \frac{b}{80^2} - a \qquad \text{.... (i)}$$

$$125 = \frac{b}{80^2} + a \qquad \text{.... (ii)}$$

Add equations (1) and (2),

$$150 = \frac{b}{80^2} + \frac{b}{80^2} = \frac{b}{6400} + \frac{b}{6400}$$

$$150 = \frac{2b}{6400}$$

solving, $b = \frac{6400 \times 150}{2} = 480000$

substituting the value of 'b' in equation (i),

$$25 = \frac{b}{80^2} - a$$

$$25 = \frac{480000}{80^2} - a$$

solvinga $= \frac{480000}{80^2} - 25 = 50$

To Find thickness

We know, fluid pressure (i.e., radial stress, f_r is zero at outer radius)

Let outer radius = r_2

$\therefore$ at $r = r_2$, $f_r = 0$

Substituting these values in f_r equation,

$$f_r = \frac{b}{r^2} - a$$

$$\text{or} \qquad 0 = \frac{480000}{r_2^2} - 50$$

solving, $r_2^2 = \frac{480000}{50}$

$$\text{or } r_2 = \sqrt{\frac{480000}{50}}$$ = 97.97 mm

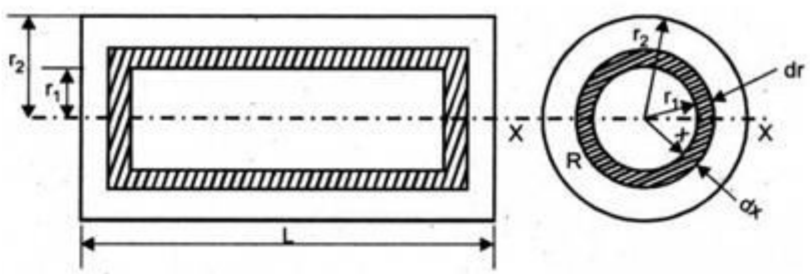
∴ Thickness of shell = Outer radius – Inner radius

= 97.97 – 80

= 17.97 mm say 18 mm

Let us derive the radial stress and hoop stress in a thick cylindrical shell subjected to an internal fluid pressure.

A thick cylinder subjected to a internal fluid pressure is shown below:

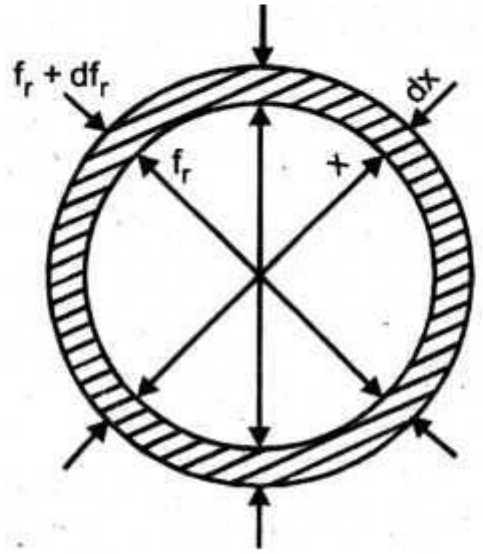


Let,r_2 = External radius of the cylinder

r_1 = Internal radius of the cylinder, and

L = Length of cylinder

Consider an elementary ring of the cylinder or radius x and thickness dx as shown in Figure



Let,f_r = Radial pressure on the inner surface of the ring

$f_r + df_r$ = Radial pressure on the outer surface of the ring

f_c = Hoop stress induced in the ring

Take a longitudinal section x-x and consider the equilibrium of half of the ring of Figure Bursting force = $f_r(2xL) – (f_r + df_r) \times 2(x + dx) \bullet L$

$= 2L [f_r \bullet x – (f_r \bullet x + f_r \bullet dx + xdf_r + df_r \bullet dx)]$

$= 2L[–f_r \bullet dx – x \bullet df_r]$

(Neglecting $df_r \bullet dx$ which is a small quantity) .... (i)

$= – 2L (f_r dx + x df_r]$

Resisting force = Hoop stress × Area on which it acts

$= f_c \times 2dx \bullet L$....(ii)

Equating the resisting force to the bursting force, we get

$f_c \times 2dx \bullet L = -2L (f_r \bullet dx + x \bullet df_r)$

or $f_c = -f_r - x\dfrac{df_r}{dx}$ **....(iii)**

The longitudinal strain at any point in the section is constant and is independent of the radius, this means that cross-sections remain plane after straining and this is true for sections, remote from any end fixing. As longitudinal strain is constant, longitudinal stress will also be constant.

Let f_2= Longitudinal stress

Hence at any point at a distance x from the center, three principal stresses are acting: They are:

i)the radial compressive stress, f_r

ii)the hoop (or circumferential) tensile stress, f_c

iii)the longitudinal tensile stress, f_2

The longitudinal strain (e_2) at this point is given by,

$$e_2 = \frac{f_2}{E} - \frac{f_x}{mE} + \frac{f_r}{mE}$$

But longitudinal strain is constant.

$$\therefore \frac{f_2}{E} - \frac{f_c}{mE} + \frac{f_r}{mE} = \text{constant}$$

But f_2 is also constant, and for the material of the cylinder E and m are constant,

$\therefore f_c - f_r = \text{constant}$

= 2a where 'a' is constant

$\therefore f_c = f_r + 2a$....(iv)

Equating the two values of f_c given by equations (iii) and (iv), we get,

$$f_r + 2a = -f_r - x\frac{df_r}{dx}$$

$$\text{or} \quad x\cdot\frac{dx_x}{dx} = -f_r - f_r - 2a = -2f_r - 2a$$

$$\text{or} \quad \frac{df_r}{dx} = \frac{2f_r}{x} - \frac{2a}{x} = \frac{-2(f_r + a)}{x}$$

$$\text{or} \quad \frac{df_r}{(f_r + a)} = -\frac{2dx}{x}$$

Integrating the above equation, we get

$\log_e (f_r + a) = -2 \log_e x + \log_e b$

where $\log_e b$ is a constant of integration.

The above equation can also be written as

$$\log_e (f_r + a) = -\log_e x^2 + \log_e b = \log_e \frac{b}{x^2}$$

$\therefore$ fr + a = $\frac{b}{x^2}$

Or fr = $\frac{b}{x^2}$ – a....(v)

Substituting the value of fr in equation (iv), we get

f_c = $\frac{b}{x^2}$ – a + 2a = $\frac{b}{x^2}$ + a....(vi)

The equation (v) gives the radial pressure f_r and equation (vi) gives the hoop stress at any radius x. These two equations are called Lame's equations. The constants 'a' and 'b' are obtained from boundary conditions, which are:

i)at x = r_1, f_r = p_0 or the pressure of fluid inside the cylinder, and

ii)at x = r_2, f_r = 0 or atmosphere pressure

After knowing the values of 'a' and 'b', the hoop stress can be calculated at any radius.