Conjugate beam and strain energy

Conjugate beam

Conjugate beam is an imaginary beam of length equal to that of the original beam, loaded with $\frac{M}{EI}$ diagram.

Slope and deflection of conjugate beam method

i)The slope at any section of the given beam is equal to the shear force at the corresponding section of the conjugate beam.

ii)The deflection at any section for the given beam is equal to the bending moment at the corresponding section of the conjugate beam.

Strain Energy

In a molecule, strain energy is released when the constituent atoms are allowed to rearrange themselves in a chemical reaction interference are reduced. The external work done on an elastic member in causing it to distort from its unstressed state is transformed into strain energy which is a form of potential energy.

4.5 Maxwell’s reciprocal theorems

Maxwell’s reciprocal theorem

The influence coefficients for corresponding forces and displacements are symmetric.

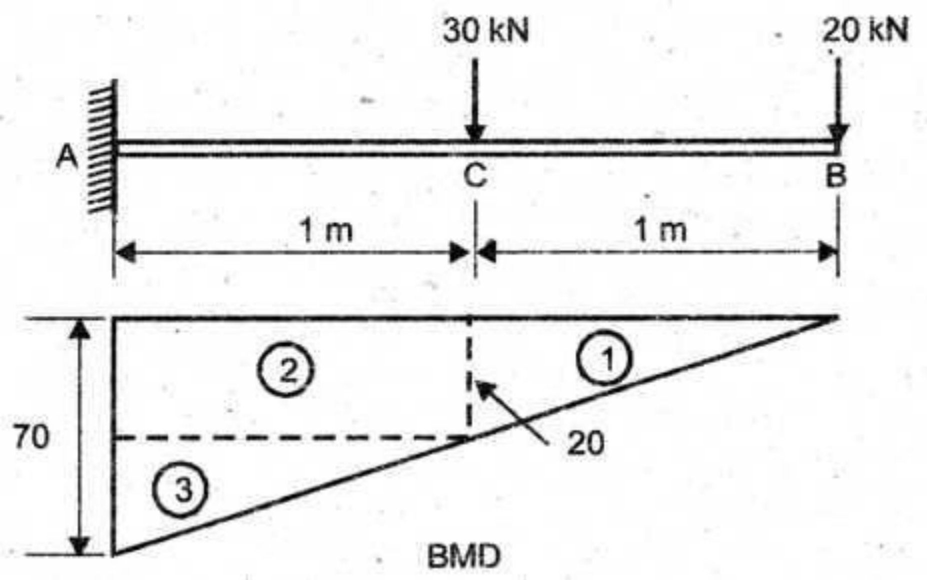
$C_{ij}=C_{ji}$

In other words, the displacement at point i due to a unit load at another point j is equal to the displacement at j due to a unit load at i, provided that the displacements and forces "correspond," i.e.,they are measured in the same direction at each point.

Problems

Let us solve a Cantilever AB, 2 m long is carrying a load of 20 KN at free end and 30 KN at a distance 1 m from the free end. Find the slope and deflection at the free end. Take E = 200 G pa and I = 150 × 106 mm^4.

Solution:



Bending moment diagram is shown in Figure. Divide the BMD into three portions, and locate the Centroid of each portion from the free end B.

$(BM)_c = -(20 \times 1) = -20$ K Nm

$(BM)_A = -(20 \times 2) = -(30 \times 1) = -70$ K Nm

$$A_1 = \left(\frac{1}{2} \times 1 \times 20\right) = 10\ KNm^2$$

$A_2 = (1 \times 20) = 20\ KNm^2$

$A_3 = \frac{1}{2} \times 1 \times (70 - 20) = 25\ KNm^2$

$$\overline{x_1} = \frac{2}{3} \times 1 = 0.67\ m$$

$$\overline{x_2} = 1 + \left(\frac{1}{2}\right) = 1.5\ m$$

$$\overline{x_3} = 1 + \left(\frac{2}{3} \times 1\right) = 1.67\ m$$

i) Slope at Free end

Using Mohr's theorem I,

$$\theta_{B-A} = \frac{(Area)_{B-A}}{EI}$$

is$\boldsymbol{\vartheta B - \vartheta A} = \frac{(A_1 + A_2 + A_3)}{EI}$ $(\because \theta_A = 0)$

or $\theta_B = \frac{(10 + 20 + 25)}{EI}$

$$\theta_B = \frac{55}{EI}$$

(Note that the unit of 55 is KNm^2; Hence multiply by 10^9 to convert into N mm).

$$\therefore \quad \theta_B = \frac{55 \times 10^9}{2 \times 10^5 \times 150 \times 10^6}$$

= 1.833 × 10^{-3} rad

$= 1.833 \times 10^{-3} \times \frac{180}{\pi}$ degrees

= 0.105°

ii) Deflection at Free end

Using Mohr's theorem II,

$$y_{B-A} = \frac{(A\overline{x})_{B-A}}{EI}$$

But $A x = A_1\overline{x_1} + A_2\overline{x_2} + A_3\overline{x_3}$

$$= \frac{(10 \times 0.67) + (20 \times 1.5) + (25 \times 1.67)}{EI}$$

$$= \frac{78.45}{EI}$$

$$\therefore \quad y_{B-A} = \frac{78.45}{EI}$$

($\therefore$ $y_A = 0$; unit of 78.45 is $kNm^3 = 78.45 \times 10^{12}$ Nmm^3)

or $$y_B = \frac{78.45 \times 10^{12}}{2 \times 10^5 \times 150 \times 10^6}$$

= 2.615 mm