Area moment method for computation of slopes and deflections in beams

Moment area method

Moment - area method can conveniently be used, when

i)The beam is cantilever

ii)The beam is simply supported and carrying symmetrical loading.

iii)The beam is fixed at both ends.

The two theorems of Moment Area Method

1.Mohr's theorem I (Moment area theorem)

2.Mohr's theorem II (Moment area theorem)

Mohr’s theorem I

Mohr's theorem I states that “ the change of slope between any two points on an elastic curve is equal to the net area of BM diagram these points divided by EI”.

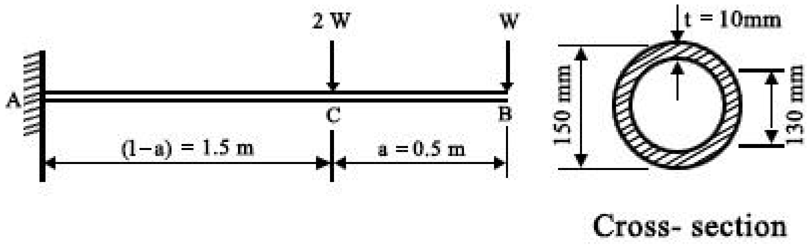
Mohr’s Theorem II.

Mohr's theorem II states that “the interrupt vertical reference line of tangents at any points on an elastic curve is equal to the moment of BM diagram between these points about the reference line divided by EI”.

Problems

A 2 meters long cantilever made up of steel tube of section 150 mm external diameter and 10 mm thick is loaded as in Figure below. If E = 200 GPa calculate,i) The value of W so that the

maximum bending stress is 150 MPa.ii) The maximum deflection for the loading.



Solution:

i) Load W

Given, $(f_b)_{max}$= 150N/mm^2

$$I_{NA} = \frac{\pi}{64}\left[D^4 - d^4\right] = \frac{\pi}{64}\left[150^4 - 130^4\right]$$

= 10825150 mm^4

$$y_{max} = \frac{150}{2} = 75 \text{ mm}$$

For the given loading,

Max BM = $(BM)_A$ at A

= (2W × 1.5) + (W × 2)

= 5 W K Nm = 5 W × 10^6 N mm....(i)

Using the relation,

$$\frac{M}{I} = \frac{f}{y}$$

$$M = \frac{fI}{y} = \frac{150 \times 10825150}{75}$$

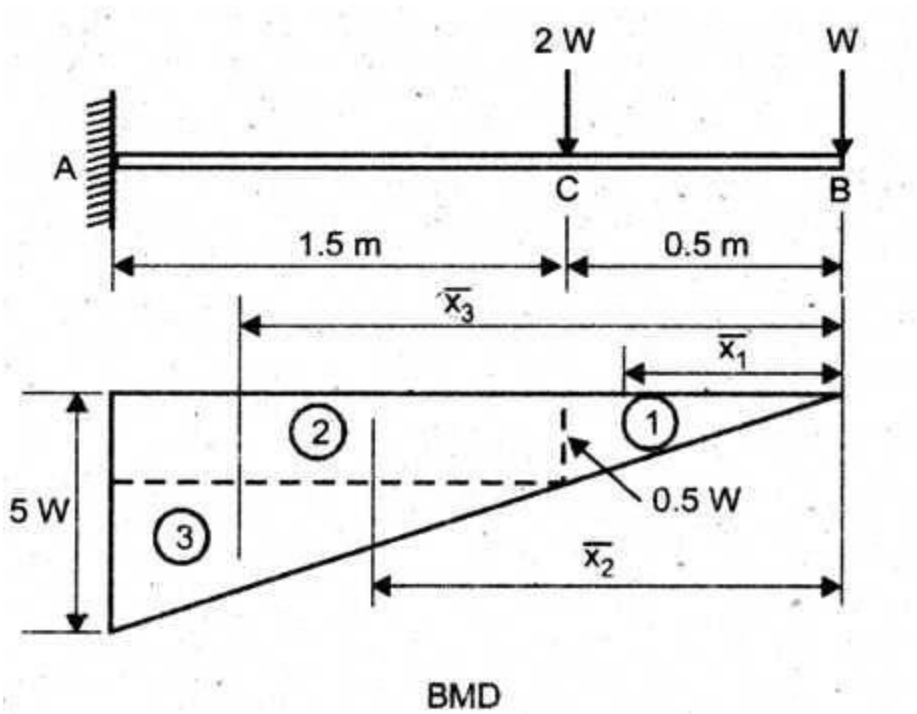
= 21650300 N mm....(ii)

Now equating (i) and (ii),

$5\ W \times 10^6 = 21650300$

Solving, $W = \dfrac{21650300}{5 \times 10^6}$ = 4.33 KN (Ans)

ii) Maximum deflection by Moment - Area Method



The Bending moment diagram is shown in Fig (b).

$(BM)_c = -W \times 0.5 = -0.5\ W$ K Nm

$(BM)_A = -(W \times 2) - (2W \times 1.5) = -5$ W K Nm

Divide the BMD in three parts,

$A_1 = \frac{1}{2} \times 0.5 \times 0.5W = 0.125$ W

$A_2 = 1.5 \times 0.5$ W $= 0.75$ W

$A_3 = \frac{1}{2} \times 1.5 \times (5W - 0.5W) = 3.375$ W

$$\overline{x_1} = \frac{2}{3} \times 0.5 = 0.333 \text{ m}$$

$$\overline{x_2} = 0.5 + \left(\frac{1.5}{2}\right) = 1.25 \text{ m}$$

$$\overline{x_3} = 0.5 + \left(\frac{2}{3} \times 1.5\right) = 1.5 \text{ m}$$

Total moment of BMD about B,

$$A x = A_1 \overline{x_1} + A_2 \overline{x_2} + A_3 \overline{x_3}$$

$= (0.125 \text{ W} \times 0.333) + (0.75 \text{ W} \times 1.25)$

$+ (3.375 \text{ W} \times 1.5)$

$= 6.0416$W

Using Mohr's theorem II,

$$y_{B-A} = \frac{\text{Area x } \overline{X}}{EI}$$

But $y_{B\_A} = y_B - y_A$

$$= \frac{1}{EI}\left[\text{Area of BMD } \times \bar{X}\right] = \frac{1}{EI}\left[6.0416\ W\right]$$

$$\therefore\ y_B = \frac{6.0416 \times 4.33}{EI}$$ ($\because$ W = 4.33 KN)

Substitute E = 200 GPa = 2×10^5 N/mm^2;

I = 10825150 mm^4

$$\therefore\ y_B = \frac{6.0416 \times 4.33 \times 10^{12}}{2 \times 10^5 \times 10825150}$$

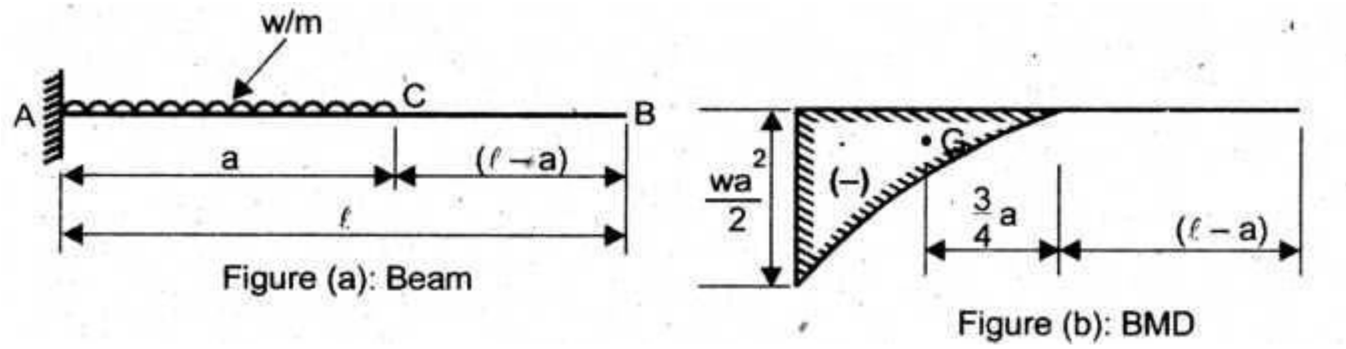
(unit of $A\bar{X}$ is KNm^3 = 1×10^{12} Nmm^3)

= 12mm (downward).

Here we solve the slope deflection of moment area method of a cantilever beam of length 'l' subjected to UDL of w/m run over length of 'a' from fixed end.

Solution:

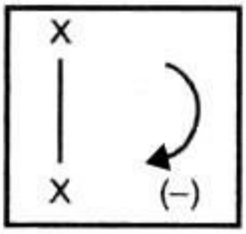
The cantilever beam with UDL over a span of 'a' from fixed end is shown below.



Bending moment diagram for the given beam is shown in figure (b).

$(BM)_B = 0$ $(BM)_c = 0$

$(BM)_A = -w \times a \times \frac{a}{2} = -\left(\frac{Wa^2}{2}\right)$



Variation of BM diagram from C to A is parabolic.

Area of BM diagram $= \frac{1}{3} \text{ x base x height}$

$$= -\left(\frac{1}{3} \text{ x a x } \frac{wa^2}{2}\right) = -\frac{wa^3}{6}$$

Distance of the centroid of Parabolic (i.e., BMD) from B,

$$\overline{x} = (l - a) + \frac{3a}{4}$$

To Find slope at free end (i.e., ϑ_A)

Using moment area theorem I,

$$\theta_{BA} = \frac{\text{Area}}{EI}$$

But $\theta_{BA} = \theta_B - \theta_A$

$\therefore \vartheta_B - \vartheta_A = \frac{1}{EI}$ [Area of BMD between A and B]

or $$\theta_B = \frac{1}{EI}\left[-\frac{wa^3}{6}\right]$$ $(\because \vartheta_A = 0)$

$$\boxed{\theta_B = \frac{-wa^3}{6EI}}$$ clockwise slope

To Find deflection at free end

Using Moment area theorem II,

$$y_{BA} = \frac{\text{Area x X}}{EI}$$

But $y_{BA} = y_B - y_A$

$\therefore y_B - y_A = \frac{1}{EI}\left[\text{Area of BMD} \times \overline{x}\right]$

$$= \frac{1}{EI}\left[\left(\frac{-wa^3}{6}\right) \times \left\{(\ell - a) + \frac{3a}{4}\right\}\right]$$

$$= \frac{-wa^3}{6EI}(\ell - a) - \left\{\frac{wa^3}{6EI} \times \frac{3a}{4}\right\}$$

$$= \frac{-wa^3}{6EI}(\ell - a) - \frac{wa^4}{8EI}$$

$$= -\left[\frac{-wa^3}{6EI}(\ell - a) + \frac{wa^4}{8EI}\right]$$