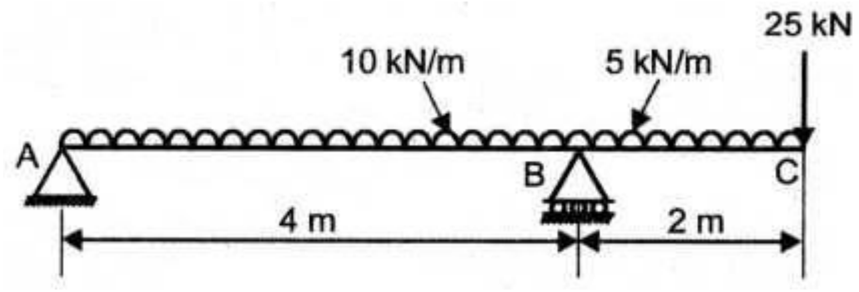
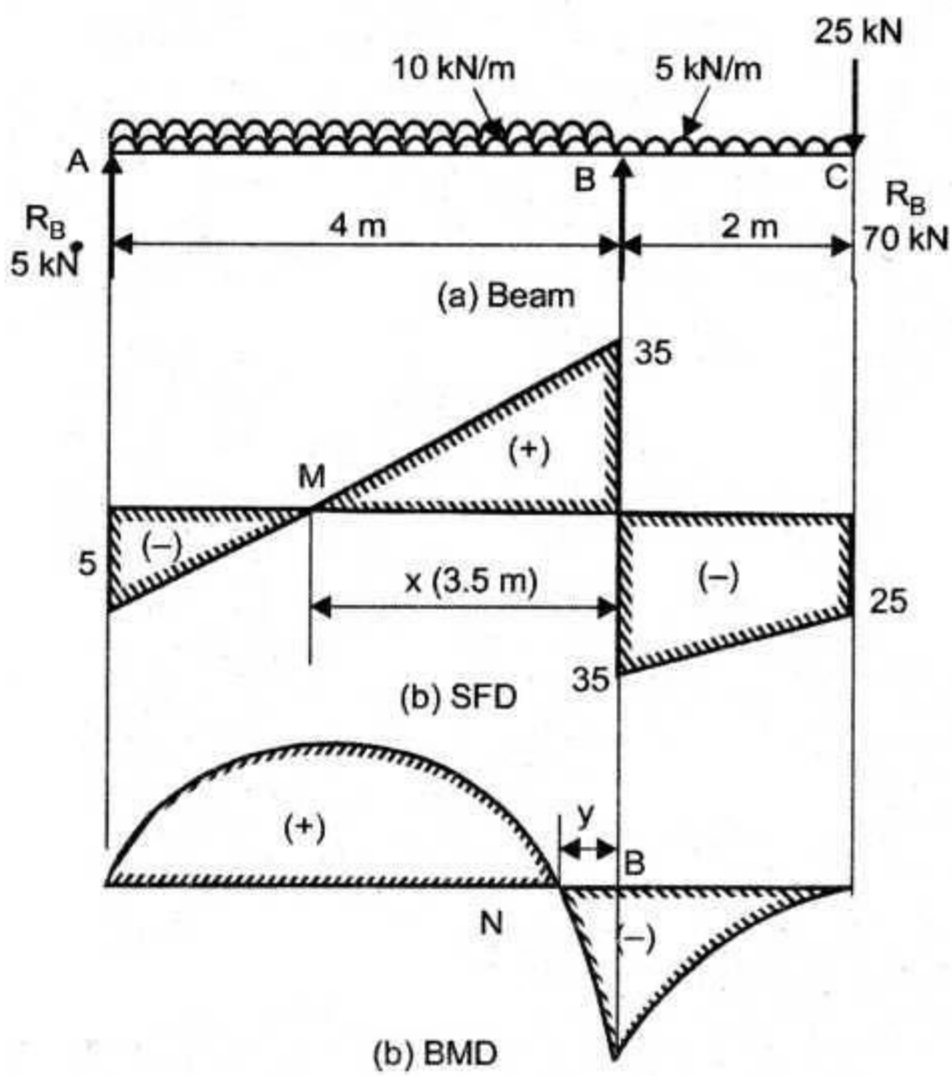
Problem

Let us solve about point of contraflexure and BF and BM for



Solution:



Support Reactions

$\sum V = 0 \Rightarrow 25 + (5 \times 2) + (10 \times 4) = R_a + R_b$

or $R_a + R_b = 75$ KN....(1)

Applying $\sum M_A = 0$

$(25 \times 6) + \left\{5 \times 2 \times \left(4 + \frac{2}{2}\right)\right\} + \left\{10 \times 4 \times \frac{4}{2}\right\} = R_b \times 4$

or $150 + 50 + 80 = 4R_B$

Solving $R_B = 70$ KN

Substituting R_B in equation (1), $R_A = 5$ KN

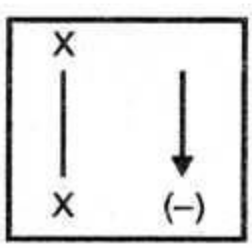
Shear Force:

$(SF)_C = -25$ KN

$(SF_b)_r = -25 - (5 \times 2) = -35$ KN

$(SF)_b = -25 - (5 \times 2) + 70 = 35$ KN

$(SF_a)_r = -25 - (5 \times 2) - (10 \times 4) + 70 = -5$ KN

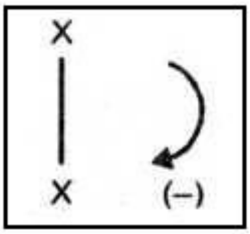


Bending Moment:

$(BM)_C = 0$

$(BM)_b = -(25 \times 2) - (5 \times 2 \times \frac{2}{2}) = -60$ k Nm

$(BM)_a = 0$



Point of Max.BM (i.e. Point of zero SF)

Referring the SFD in the region AB

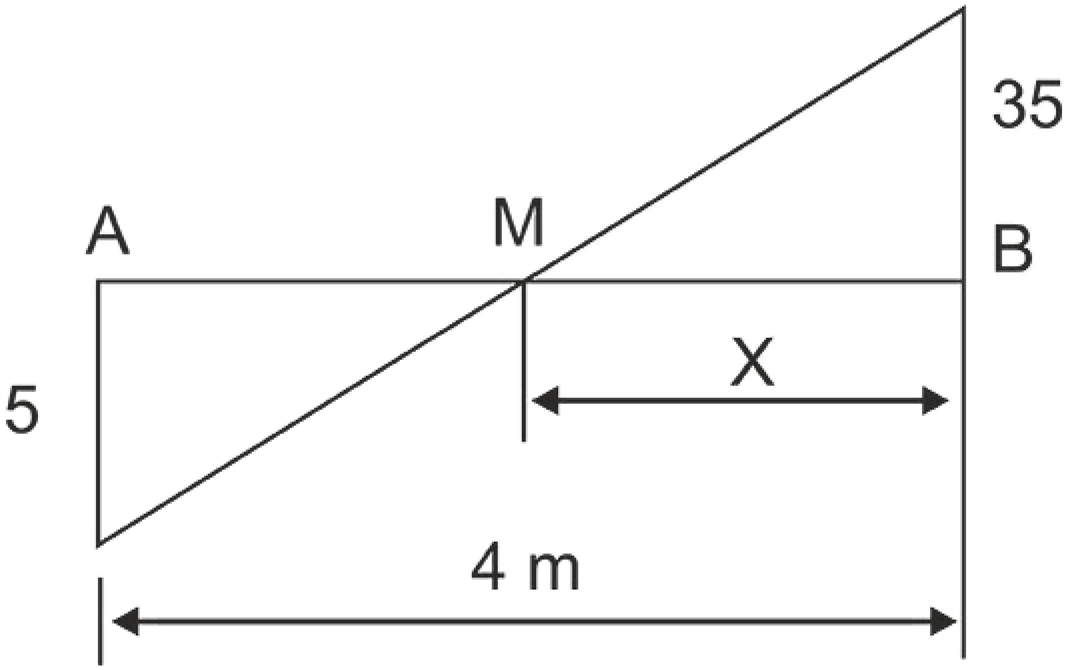
From similar triangles

$$\frac{35}{x} = \frac{5}{(4-x)}$$

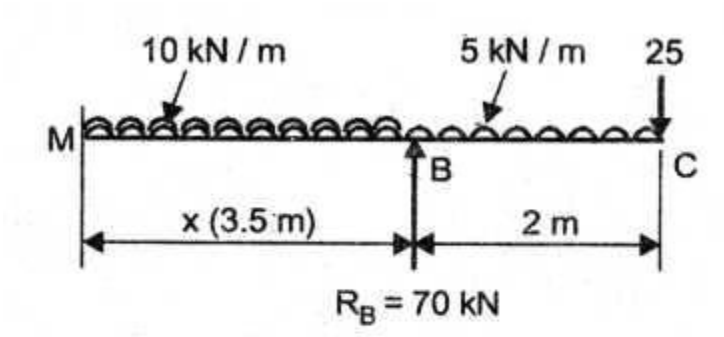
or $5x = 140 - 35x$

or $40x = 140$

$x = 3.5$ m



Magnitude of Max BM



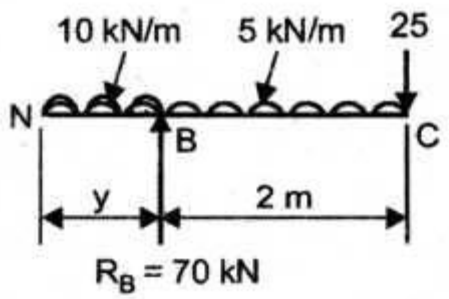
Max BM = $(BM)_m$

$= -(25 \times 5.5) - \left\{5 \times 2 \times \left(3.5 + \frac{2}{2}\right)\right\} - \left(10 \times 3.5 \times \frac{3.5}{2}\right) + (70 \times 3.5)$

$= -137.5 - 45 - 61.25 + 245$

$= 1.25$ k Nm

To find Point of zero BM



Let zero BM occurs at N $\therefore (BM)_N = 0$

$(BM)_N = -25(2 + y) + \left\{5 \times 2 \times \left(y + \frac{2}{2}\right)\right\} - \left(10 \times y \times \frac{y}{2}\right) + (70 \times y)$

$= -50 - 25y - \{10(y + 1)\} - 5y^2 + 7$

$= -50 - 25y - 10y - 10 - 2.5\, y^2 + 70\, y$

$0 = -5y^2 + 35y - 60$

or $5y^2 - 35y + 60 = 0$

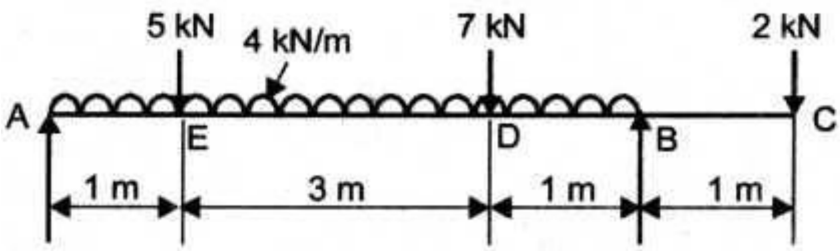
Solving, y = $\dfrac{35 \pm \sqrt{35^2 - (4 \times 5 \times 60)}}{2 \times 5}$

$= \dfrac{35 \pm 5}{10}$

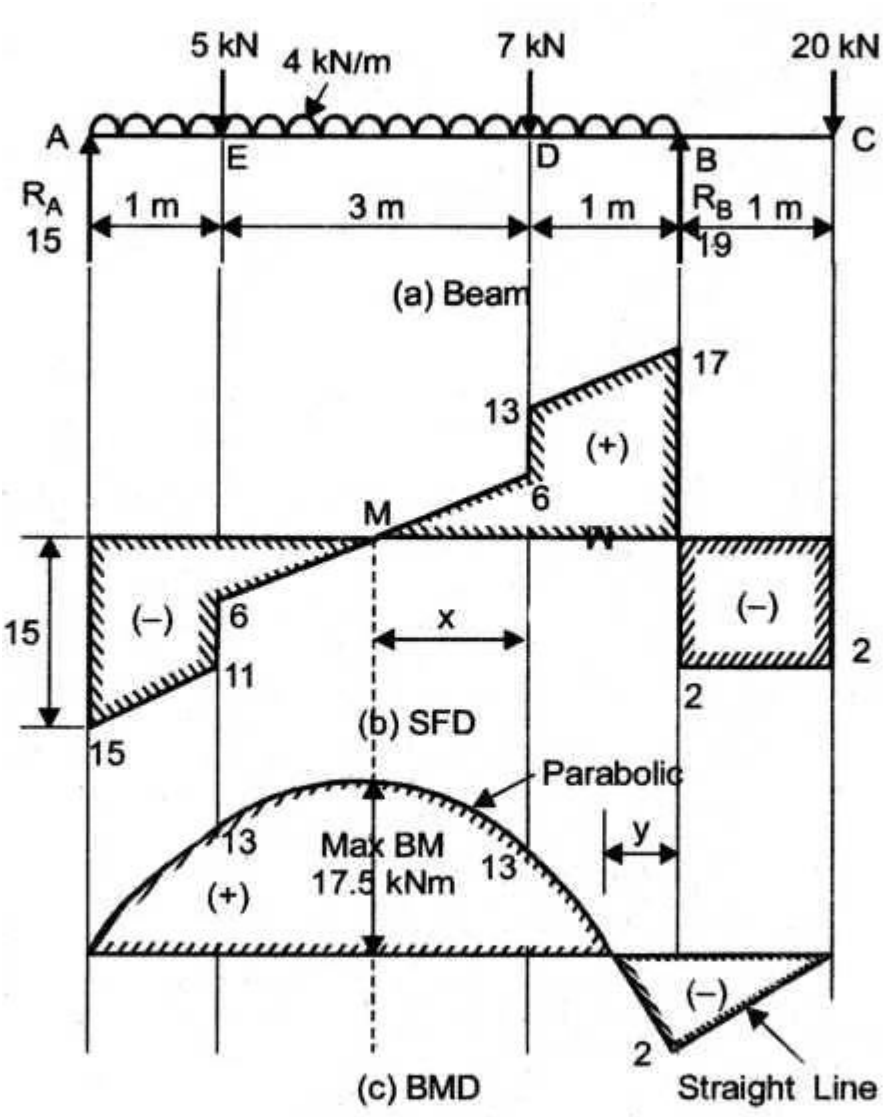
= 4m and 3m

∴ Zero BM occurs at 4m and 3m from the support B and towards the support A (i.e., at the support A itself and at the point N).

Shall we solve shear force and bending moment diagram



Solution:



Support Reactions

Applying $\sum V = 0$ ($\uparrow = \downarrow$)

$R_a + R_B = 5 + 7 + 2 + (4 \times 5)$

$= 34$ KN....(1)

Applying $\sum M_A = 0$ $(\circlearrowleft = \circlearrowright)$

$$(5 \times 1) + (7 \times 4) + (2 \times 6) + (4 \times 5 \times \frac{5}{2}) = R_B \times 5$$

or$5 + 28 + 12 + 50 = 5R_B$

Solving, $R_B = 19$ KN

Substituting R_B in equation (1),$R_A = 15$ KN

Shear Force:

$(SF)_C = -2$KN

$(SF_b)_r = -2$KN

$(SF)_b = -2 + 19 = 17$ KN

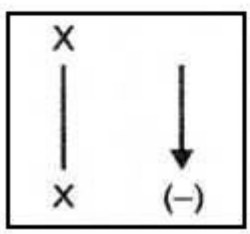
$(SF_D)_R = -2 - (4 \times 1) + 19 = 13$ KN

$(SF)_D = -2 - (4 \times 1) - 7 + 19 = 6$ KN

$(SF_e)_r = -2 - 7 - (4 \times 4) + 19 = -6$ KN

$(SF)_e = -2 - 7 - (4 \times 4) - 5 + 19 = -11$ KN

$(SF)_A = -15$ KN



Bending Moment:

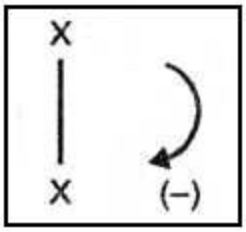
$(BM)_C = 0$

$(BM)_B = -(2 \times 1) = -2$ k Nm

$(BM)_D = -(2 \times 2) - (4 \times 1 \times \frac{1}{2}) + (19 \times 1) = 13$ k Nm

$(BM)_e = -(2 \times 5) - (7 \times 3) - (4 \times 4 \times \frac{4}{2}) + (19 \times 4) = 13$ k Nm

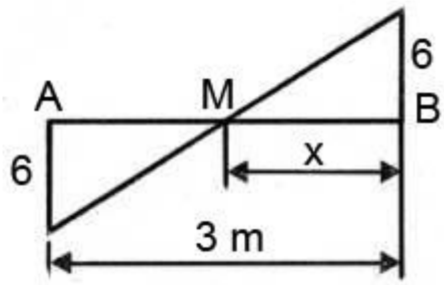
$(BM)_A = 0$



Maximum Bending Moment:

Let the Bending moment is maximum at M (i.e., zero shear force).Referring the SFD, zero shear force occurs in the region ED at point M, at a distance of x m from D.

From similar triangles,

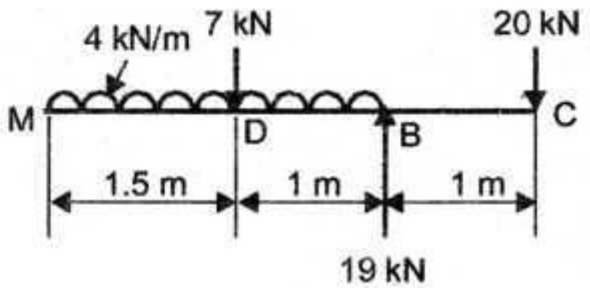


$$\frac{6}{x} = \frac{6}{(3 - x)}$$

or$6x = 18 - 6x$

or$12x = 18$(or)$x = \frac{18}{12} = 1.5$ m

To find the maximum bending moment, consider the right side of beam up to the point M as shown below.



Max BM = $(BM)_m$

$$= -(2 \times 3.5) - (7 \times 1.5) - (4 \times 2.5 \times \frac{2.5}{2}) + (19 \times 2.5)$$

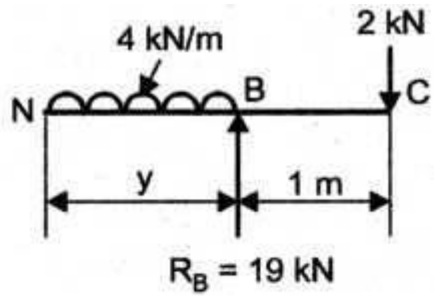
$(BM)_m = -7 - 10.5 - 12.5 + 47.5$

= 17.5 k Nm

To find the point of zero BM

Referring the BMD, zero Bending moment occurs in the region BA, at a distance of ‘y’ meter from B. Let the point be N.

Consider the right side of beam up to the point N.



Now$(BM)_n = 0$

$$\text{Burt}(BM)_n = -2(1 + y) - \left(4 \times y \times \frac{y}{2}\right) + (19 \times y)$$

$= -2 - 2y - 2y^2 + 19y$

$0 = -2y^2 + 17y - 2$

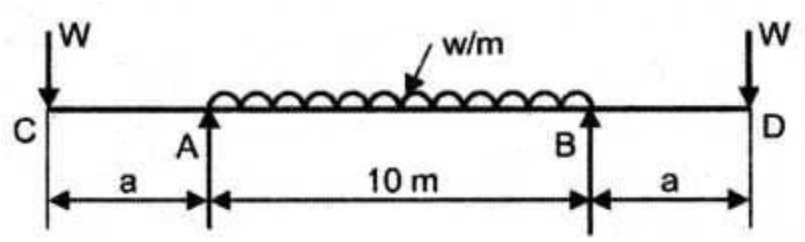
or $2y^2 - 17y + 2 = 0$

Solving, $y = \dfrac{17 \pm \sqrt{17^2 - (4 \times 1 \times 2)}}{2 \times 2}$

$= 8.44$ and 0.06 m

Taking the possible value, $y = 0.06$ m

Shall we discuss about bending moment diagram

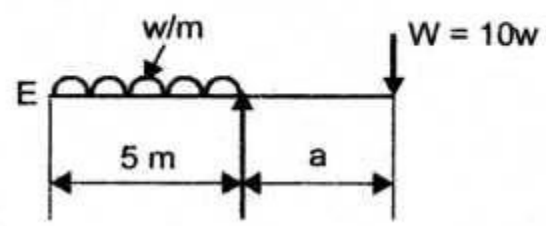


W = 10 w

Solution:

To find overhanging length 'a':

It is given that the bending moment at the middle of the beam is zero. Hence consider right side of the beam up to the midpoint and equate the bending moment value at midpoint to zero. Let E be the midpoint of beam.



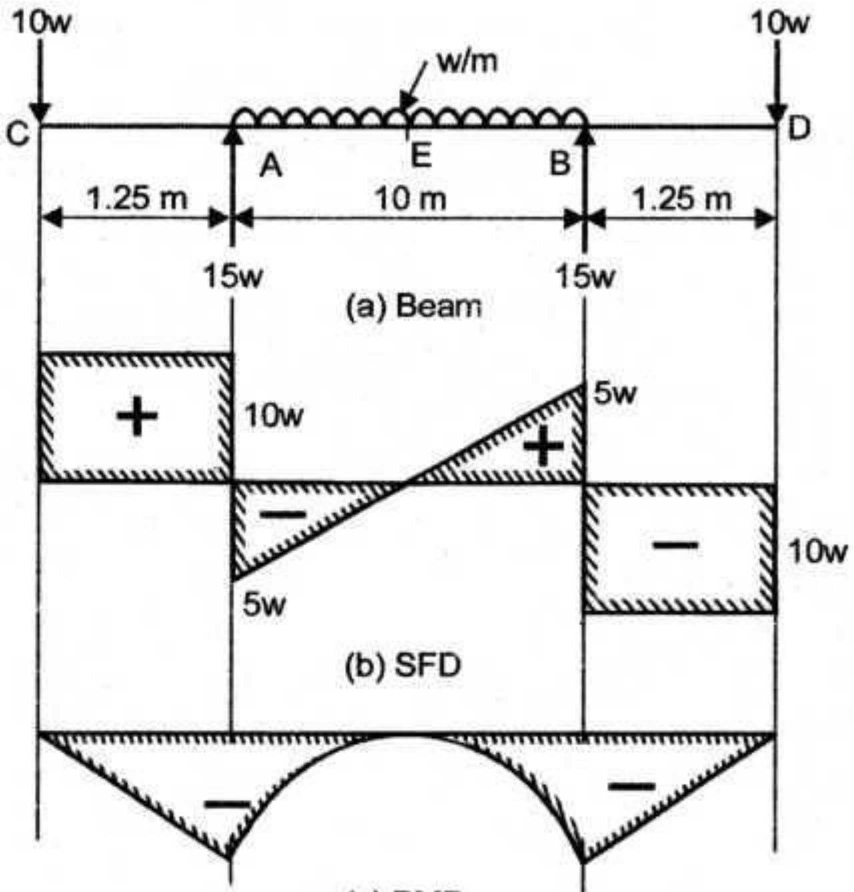
$(BM)_e = -10w(5 + a) - \left(w \times 5 \times \dfrac{5}{2}\right) + (R_B \times 5)$

But due to symmetry,

$$R_a = R_b = \frac{\text{Total load}}{2}$$

$$= \frac{W + W + (w \times 10)}{2}$$

$$= \frac{10w + 10w + 10w}{2} = 15\ w$$



(a) Beam
(b) SFD
(c) BMD

SubstitutingR_B = 15 w, we get

$$(BM)_E = -\ 10w\ (5 + a) - (w \times 5 \times \frac{5}{2}) + (15w \times 5)$$

$= -50w - 10\,wa - 12.5\,w + 75\,w$

Or $0 = -50\,w - 12.5\,w + 75\,w - 10\,wa$

$(\because (BM)_e = 0)$

Or $10\,wa = 12.5\,w$

or $a = \frac{12.5}{10} = 1.25$ m

∴ Overhanging length on either side to have BM at midpoint zero is 1.25 m.

Shear Force*:*

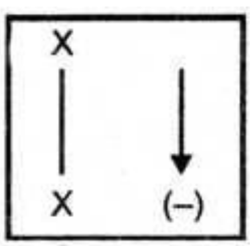
$(SF)_D = -10\,w$ KN

$(SF)_b = -10\,w + 15\,w = 5\,w$ KN

$(SF_a)_r = -10\,w - (w \times 10) + 15\,w = -5\,w$ KN

$(SF)_a = -10\,w - (w \times 10) + 15\,w + 15\,w = 10\,w$ KN

$(SF)_C = 10\,w$ KN



Bending Moment*:*

$(BM)_d = 0$

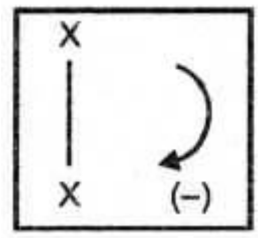
$(BM)_b = (-10\,w \times 1.25) = -12.5\,w$

$(BM)_e = 0$ (It is given in the problem)

$$(BM)_a = -(10\,w \times 11.25) - \left(w \times 10 \times \frac{10}{2}\right) + (15\,w \times 10)$$

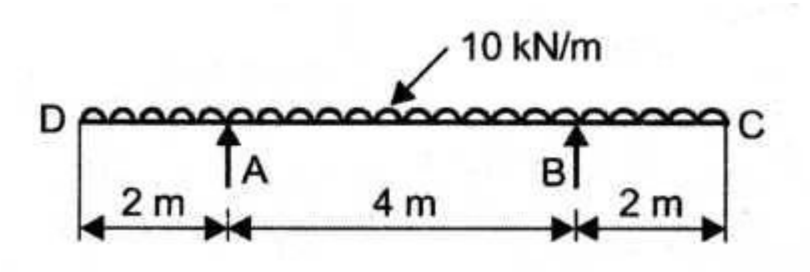
$= -12.5\,w$

$(BM)_C = 0$



Shear Force and Bending moment diagrams are shown in figure (b) and Figure (c) respectively.

Let us learn about point of contraflexure



Solution:

Support Reactions

Applying $\sum V = 0$ ($\uparrow$ +)

$V_a + V_b = 10 \times 8 = 80KN$

Applying $\sum M_a = 0$ (↻ +)

$$\left(10 \times 6 \times \frac{6}{2}\right) - (V_B \times 4) - \left(10 \times 2 \times \frac{2}{2}\right) = 0$$

or $180 - 20 = 4\,V_B$

or $V_B = \frac{160}{4} = 40$ KN(↑)

$\therefore V_A = 80 - V_B = 80 - 40 = 40$ KN (↑)

Shear Force:

$(SF)_C = 0$

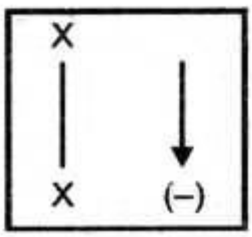
$(SF_b)_r = -10 \times 2 = -20$ KN

$(SF)_b = -20 + 40 = 20$ KN

$(SF_a)_r = -(10 \times 6) + 40 = -20$ KN

$(SF)_A = -20 + 40 = 20$ KN

$(SF)_D = 0$



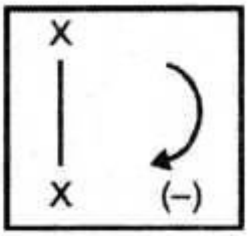
Bending moment:

$(BM)_C = 0$

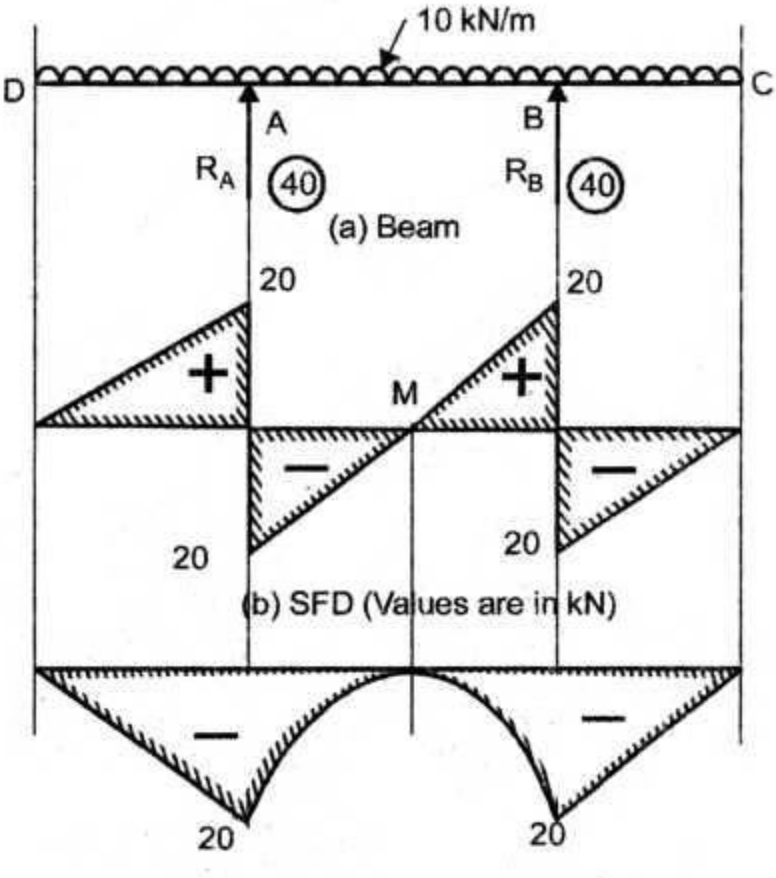
$(BM)_b = -\left(10 \times 2 \times \frac{2}{2}\right) = -20$ K Nm

$(BM)_a = -\left(10 \times 6 \times \frac{6}{2}\right) + (40 \times 4) = -20$ K Nm

$(BM)_d = 0$



The shear force and bending moment diagrams for the beam are shown below:



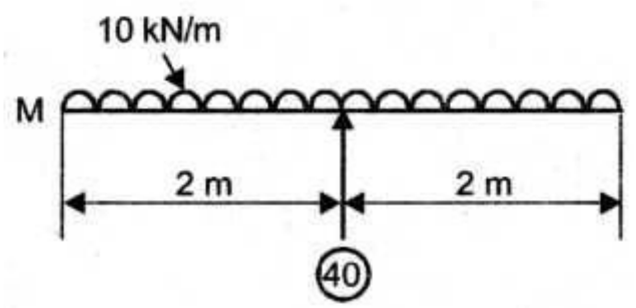
(c) BMD (values are in kNm)

Location of zero shear force (or) maximum bending moment:

From the shear force diagram, shear force is zero at mid span. Let it be M i.e., at a distance of 4 m from the right end C.

To Find Maximum Bending Moment

Consider the right side part of the beam up to the mid point M, as shown below:



$(BM)_m$ = Maximum Bending Moment

$$= -\left(10 \times 4 \times \frac{4}{2}\right) + (40 \times 2) = 0$$

The Bending moment diagram is showing Hogging (i.e. Negative) bending moment on the entire length of beam with zero bending moment at mid-span.

Referring the Bending moment diagram, there is a point of contraflexureat mid span (i.e., zero Bending moment