8. 1 kW power at 720 r.p.m. is supplied to the worm shaft. The number of starts for threads of worm are four with a 50 mm pitch circle diameter. The worm wheel has 30 teeth with a 5 mm module. The normal pressure is 20°. Calculate the efficiency of the worm gear drive and the power lost in friction. [*Ans* : 90.12% ; 98.8 W]

Problems on worm gear drive design :

9. Design a worm gear drive to transmit 8 kW at 720 r.p.m. The desired velocity ratio is 36 : 1.

10. Design a worm drive for a speed reducer to transmit 15 kW at 1440 r.p.m. of the worm shaft. The desired wheel speed is 60 r.p.m. Select suitable worm and wheel materials.

11. Design a worm gear drive to transmit 10 kW at 1440 r.p.m. with a gear ratio of 12. Use steel worm and cast iron wheel.

12. A hardened steel worm rotates at 1440 r.p.m. and transmits 12 kW to a phosphor bronze gear with a gear ratio of 15. Calculate the centre distance and module required for the drive.

13. Design a worm gear drive to transmit 15 kW from a worm at 1440 r.p.m. to the worm wheel. The speed of the worm wheel should be 40 ± 2% r.p.m.

14. Design a worm and worm wheel drive to transmit 50 kW from an electric motor running at 1440 r.p.m. to a rolling mill required to run at 100 r.p.m. Selecting case hardened alloy steel for the worm and centrifugally cast phosphor bronze for the wheel, design the details of the geared set.

If the complete heat generated is dissipated by the housing, what should be the housing area required ? The overall heat transfer coefficient for the housing can be assumed as W/m^2 °C and the temperature rise of the lubricant is restricted to 50°C.

15. Design a worm gear drive to transmit a power of 22.5 kW. The worm speed is r.p.m. and the speed of the wheel is 60 r.p.m. The drive should have a minimum efficiency of 80% and above. Select suitable materials for the worm and wheel and decide upon the dimensions of the drive.

9

Gear Box

"There is something wore than a difficulty. It is inertia.
If you try to escape difficulties, you decay."
– Herbat Casson

9.1. INTRODUCTION

2 Mark

We know that machine tools like lathe, milling machines, *etc.*, require a wide range of spindle speeds. Because a machine tool is adoptable for cutting different types of metals having different properties using varying grades of cutting tools on work-pieces of different diameters. Thus the provision of variable spindle speeds is necessary in order to meet different requirements. The various methods used for obtaining different speeds of machine tool spindle are as follows :

9.38

(i) By using a gear box mechanism,
(ii) By using a cone pulley arrangement,
(iii) By using a variable speed electric motor, and
(iv) By hydraulic operation.

Among these methods, the gear box method is very popularly used. In this chapter, we shall discuss the design of gear boxes, in detail, in the following sections.

9.2. REQUIREMENTS OF A SPEED GEAR BOXES

A speed gear box should have the following requirements :

- ✓ It should provide the designed series of spindle speeds.
- ✓ It should transmit the required amount of power to the spindle.
- ✓ It should provide smooth silent operation of the transmission.
- ✓ It should have simple construction.
- ✓ Mechanism of speed gear boxes should be easily accessible so that it is easier to carry out preventive maintenance.

9.3. THE SPEEDS IN MACHINE TOOL GEAR BOXES ARE IN GEOMETRIC PROGRESSION. WHY ?

The speeds in gear boxes can be arranged in arithmetic progression (A.P.), geometric progression (G.P.), harmonic progression (H.P), and logarithmic progression (L.P.).

2 Mark

Advantages

However, when the speeds are arranged in G.P., it has the following advantages over the other progressions.

1. The speed loss is minimum.
   i.e., Speed loss = Desired optimum speed – Available speed
2. The number of gears to be employed is minimum.
3. G.P. provides a more even range of spindle speeds at each step.
4. The layout is comparatively very compact.
5. Productivity of a machining operation, *i.e.*, surface area of the metal removed in unit time, is constant in the whole speed range.
6. G.P. machine tool spindle speeds can be selected easily from preferred numbers. Because preferred numbers are in geometric progression.

9.4. METHODS FOR CHANGING SPEED IN GEAR BOXES

The two important methods widely used are :

1. Sliding mesh gear box, and
2. Constant mesh gear box.

9.4.1. Sliding Mesh Gear Box

It is the oldest and simplest form of gear box. Sliding type gear boxes are quite commonly used in general purpose machine tools. In order to mesh gears on the main shaft with appropriate gears on the spindle shaft for obtaining different speeds, they are moved to the right or the left. It derives its name from the fact that the meshing of the gears take place by sliding of gears on each other.

9.4.2. Constant Mesh Gear Box

It derives its name from the fact that all the gears whether of the countershaft or the main shaft are in constant mesh with each other. It is also known as a ***silent or quite gear box.*** It gives a quieter operation and makes gear changing easier by employing helical gears for the constant mesh. In order to connect the required gear wheel by means of teeth on the side of the gear wheel, a separate sliding member is employed.

9.5. PREFERRED NUMBERS

Preferred numbers are the conventionally rounded off values derived from geometric series. There are five basic series, denoted as R 5, R 10, R 20, R 40 and R 80 series. The symbol 'R' is used as a tribute to French engineer Charles Renard, who introduced the preferred numbers first. Preferred numbers assist the designer in avoiding the selection of sizes in an arbitrary manner.

Each series has its own step ratio *i.e.*, series factor. The series factor for various series are given in Table 9.1.

⑦

Table 9.1. Step ratio / series factor (from data book, page no. 7.19)

Basic series	Step ratio (ϕ)
R 5	$\sqrt[5]{10} = 1.58$
R 10	$\sqrt[10]{10} = 1.26$
R 20	$\sqrt[20]{10} = 1.12$
R 40	$\sqrt[40]{10} = 1.06$
R 80	$\sqrt[80]{10} = 1.03$

The series of preferred numbers is obtained by multiplying a step ratio with the first number to get the second number. The third number is obtained by multiplying a step ratio with the second number. Similarly the procedure is continued until the series is completed. Table 9.2 shows the basic series of preferred numbers.

Table 9.2. Basic series of preferred numbers (from data book, page no. 7.20)

Basic series	Preferred numbers
R 5 (ϕ = 1.6)	1.00, 1.60, 2.50, 4.00, 6.30, 10.00
R 10 (ϕ = 1.25)	1.00, 1.25, 1.60, 2.00, 2.50, 3.15, 4.00, 5.00, 6.30, 8.00, 10.00
R 20 (ϕ = 1.12)	1.00, 1.12, 1.25, 1.40, 1.60, 1.80, 2.00, 2.24, 2.50, 2.80, 3.15, 3.55, 4.00, 4.50, 5.00, 5.60, 6.30, 7.10, 8.00, 9.00, 10.00
R 40 (ϕ = 1.06)	1.00, 1.06, 1.12, 1.18, 1.25, 1.32, 1.40, 1.50, 1.60, 1.70, 1.80, 1.90, 2.00, 2.12, 2.24, 2.36, 2.50, 2.65, 2.80, 3.00, 3.15, 3.35, 3.55, 3.75, 4.00, 4.25, 4.50, 4.75, 5.00, 5.30, 5.60, 6.00, 6.30, 6.70, 7.10, 7.50, 8.00, 8.50, 9.00, 9.50, 10.00.

Example : Machine tool spindle speeds under R 20 series is given by 100, 112, 125, 140, 160, 180 and 200 r.p.m.

9.6. STEP RATIO (OR SERIES RATIO OR PROGRESSION RATIO) (ϕ)

When the spindle speeds are arranged in geometric progression, ***then the ratio between the two adjacent speeds is known as step ratio or progression ratio.*** It is denoted by ϕ.

If $N_1, N_2, N_3, \ldots\ldots, N_n$ are the spindle speeds arranged in geometric progression, then

$$\frac{N_2}{N_1} = \frac{N_3}{N_2} = \frac{N_4}{N_3} = \ldots\ldots\ldots \frac{N_n}{N_n - 1} = \text{constant} = \phi$$

If '*n*' is the number of steps of speed, then

$$\frac{N_n}{N_1} = \phi^{n-1} \quad \text{or} \quad \boxed{\frac{N_{max}}{N_{min}} = \phi^{n-1}} \qquad \dots (9.1)$$

Note Permissible deviation = ± 10 (ϕ – 1)%.

Example 9.1 ***Find the progression ratio for a 12 speed gear box having spee between 100 and 355 r.p.m. Also find the spindle speeds.***

Given Data : $n = 12$; $N_{min} = 100$ r.p.m. ; $N_{max} = 355$ r.p.m.

To find : 1. Progression ratio (ϕ), and 2. Spindle speeds.

☺ ***Solution : 1. Progression ratio (ϕ) :*** We know that,

$$\frac{N_{max}}{N_{min}} = \phi^{n-1}$$

or

$$\frac{355}{100} = \phi^{12-1} \quad \text{or} \quad \phi = (3.55)^{1/11} = \mathbf{1.122} \quad \textbf{Ans.}$$

2\. ***Spindle speeds :*** Since the calculated ϕ (= 1.12) is a standard step ratio for R 20 se Therefore the spindle speeds from R 20 series are

100, 112, 125, 140, 160, 180, 200, 224, 250, 280, 315 and 355 r.p.m. **Ans.**

Example 9.2 ***Select the spindle speeds for the following data : 12 speeds, between and 600 r.p.m.***

Given Data : $n = 12$; $N_{min} = 50$ r.p.m. ; $N_{max} = 600$ r.p.m.

To find : Spindle speeds.

☺ ***Solution :*** We know that

$$\frac{N_{max}}{N_{min}} = \phi^{n-1}$$

or

$$\frac{600}{50} = \phi^{12-1} \quad \text{or} \quad \phi = 1.253$$

$50 \times 1.253^0 = 50$
$50 \times 1.253^1 = 62.65$
$50 \times 1.253^2 = 78.5$
$50 \times 1.253^3 = 98.26$ 38
$50 \times 1.253^4 = 123$ 476
$50 \times 1.253^5 = 154$ 597
748
$50 \times 1.253^6 = 193$
242
303

We find the calculated ϕ is a standard step ratio for R 10 series. So from R 10 series spindle speeds are 50, 63, 80, 100, 125, 160, 200, 250, 315, 400, 500 and 630 r.p.m. Ans

It can be seen that the calculated N_{max} = 630 r.p.m., which is greater than the requ maximum speed. Therefore we have to check whether the deviation is within the permiss range or not.

$$\therefore \quad \text{Permissible deviation} = \pm 10\,(\phi - 1)\,\%$$
$$= \pm 10\,(1.253 - 1)\,\% = \pm 2.53\%$$

$$\text{Then,} \quad \text{Actual deviation} = (630 - 600) \times \frac{50}{600} = 2.5$$

Since the actual deviation is less than the permissible deviation, therefore we can the deviation. **Ans.**

Note If the actual deviation is more than the permissible deviation, then non-standard spe be used.

Example 9.3 ***Select spindle speeds, for 9 speed gear box, between 80 and 1285 r.p.m.***

Given Data : $n = 9$; $N_{min} = 80$ r.p.m. ; $N_{max} = 1285$ r.p.m.

To find : Spindle speeds.

☺ ***Solution :*** We know that,

$$\frac{N_{max}}{N_{min}} = \phi^{n-1}$$

or

$$\frac{1285}{80} = \phi^{9-1} \quad \text{or} \quad \phi = 1.415$$

We find ϕ = 1.415 is not a standard ratio. So let us find out whether multiples of standard ratio 1.12 or 1.06 come close to 1.415.

$$\text{We can write,} \quad 1.12 \times \underline{(1.12 \times 1.12)} = 1.405 \qquad \dots \text{(skip 2 speeds)}$$

So ϕ = 1.12 satisfies the requirement. Therefore, the spindle speeds from R 20 series, skipping 2 speeds, are given by

80, 112, 160, 224, 315, 450, 630, 900 and 1250 r.p.m. **Ans.**

Alternate solution: For the above problem, we can also write

$$1.06 \times \underline{(1.06 \times 1.06 \times 1.06 \times 1.06 \times 1.06)} = 1.418 \qquad \dots \text{(skip 5 speeds)}$$

So ϕ = 1.06 also satisfies the requirement. Therefore the spindle speeds from R 40 series, skipping 5 speeds, are given by

80, 112, 160, 224, 315, 450, 630, 900 and 1250 r.p.m. **Ans.**

It can be noted that both R 20 and R 40 series gives the same spindle speeds.

Example 9.4 ***Select the spindle speeds, 50 – 800 r.p.m., 12 speeds.***

Given Data : $N_{min} = 50$ r.p.m. ; $N_{max} = 800$ r.p.m. ; $n = 12$.

To find : Spindle speeds.

☺ ***Solution :*** We know that

$$\frac{N_{max}}{N_{min}} = \phi^{n-1}$$

or

$$\frac{800}{50} = \phi^{12-1} \quad \text{or} \quad \phi = 1.2866$$

We find ϕ = 1.2866 is not a standard ratio. So let us find out whether multiples of standard ratio 1.12 or 1.06 come close to 1.286.

We can write, $1.12 \times 1.12 = 1.2544$ and $1.24 \times 1.12 \times 1.12 = 1.4$

Then, $1.06 \times 1.06 \times 1.06 \times 1.06 = 1.26$ and $1.06 \times 1.06 \times 1.06 \times 1.06 \times 1.06 = 1.34$

So both standard ratios 1.12 and 1.06 not satisfy the requirement. It means 1.286 is not a standard ratio. Therefore the non-standard spindle speeds are obtained as

50, 50×1.286, 50×1.286^2, 50×1.268^3, ...

Thus the spindle speeds are 50, 64.3, 82.7, 106.3, 136.7, 175.8, 226.2, 290.8, 374, 481, 61.5 and 795.5 r.p.m. **Ans.**

9.7. STRUCTURAL FORMULA

Let

n = Number of speeds available at the spindle,

$p_1, p_2, p_3, \ldots\ldots$ = Stage numbers in the gear box, and

$X_1, X_2, X_3, \ldots\ldots$ = Characteristic of the stage.

Then, the structural formula is given as

$$n = \underbrace{p_1(X_1)}_{1^{st}\text{ stage}} \cdot \underbrace{p_2(X_2)}_{2^{nd}\text{ stage}} \cdot \underbrace{p_3(X_3)}_{3^{rd}\text{ stage}} \cdot \underbrace{p_4(X_4)}_{4^{th}\text{ stage}} \quad \ldots(9.2)$$

where $\boxed{X_1} = 1,\ \boxed{X_2} = p_1;\ \boxed{X_3} = p_1 \cdot p_2;\ \boxed{X_4} = p_1 \cdot p_2 \cdot p_3$

9.7.1. Preferred Structural Formulas

The Table 9.3 shows the preferred structural formulas for the different speeds of gear boxes.

Table 9.3. Preferred structural formulas

S.No.	Number of speeds		Preferred structural formula
1.	6 speeds 10	(i)	3 (1) 2 (3)
		(ii)	2 (1) 3 (2)
2.	8 speeds	(i)	2 (1) 2 (2) 2 (4)
		(ii)	4 (1) 2 (4)
3.	9 speeds 12	(i)	3 (1) 3 (3)
4.	12 speeds 14	(i)	3 (1) 2 (3) 2 (6)
		(ii)	2 (1) 3 (2) 2 (6)
		(iii)	2 (1) 2 (2) 3 (4)
5.	14 speeds	(i)	3 (1) 3 (3) 2 (5)
		(ii)	4 (1) 2 (4) 2 (6)
6.	15 speeds	(i)	3 (1) 3 (3) 2 (6)
7.	16 speeds	(i)	4 (1) 2 (4) 2 (8)
		(ii)	2 (1) 4 (2) 2 (8)
		(iii)	2 (1) 2 (2) 4 (4)
8.	18 speeds 16	(i)	3 (1) 3 (3) 2 (9)
		(ii)	3 (1) 2 (3) 3 (6)
		(iii)	2 (1) 3 (2) 3 (6)

9.8. KINEMATIC LAYOUT (OR KINEMATIC ARRANGEMENT)

The kinematic arrangement of a multi-speed gear box is shown in Fig.9.1.

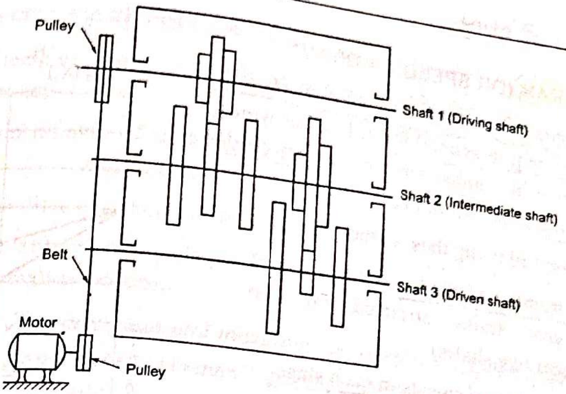


Fig. 9.1. Kinematic arrangement of a 9 speed gear box

From the Fig.9.1, it is clear that the kinematic layout shows the arrangement of gears in a gear box. The kinematic layout provides the following informations required for gear box design.

- ✓ The number of speeds available at the spindle, *i.e.*, at the driven shaft.
- ✓ The number of stages used to achieve the required spindle speeds.
- ✓ The number of simple gear trains required to obtain the required spindle speeds and their arrangement.
- ✓ The overall working principle of the gear box.
- ✓ The information required for structural formula and ray diagram.

Illustration : In Fig.9.1, the power is transmitted from driving shaft to driven shaft through a intermediate shaft. In this conventional gear box, speed changing is obtained using sliding gear mechanism. It can be seen that the number of speeds from driving shaft to intermediate shaft is 3, and that from intermediate shaft to driven shaft is 3. Then the number of spindle speeds is equal to $3 \times 3 = 9$.

The structural formula for the kinematic arrangement of gear box shown in Fig.9.1, is given by

$$n = p_1(X_1) \cdot p_2(X_2)$$

where

$p_1 = 3$ (*i.e.*, in stage 1, there are 3 speeds available)

$p_2 = 3$ (*i.e.*, in stage 2, there are 3 speeds available)

$X_1 = 1$; and $X_2 = p_1 = 3$

∴ Structural formula, $z = 3(1) \cdot 3(3)$.

where

n = Number of speeds available at the driven shaft

$= p_1 \cdot p_2 = 3 \times 3 = 9$

G.A 17

% 2.Marks

9 12

9.9. RAY DIAGRAM (OR SPEED DIAGRAM)

%

The ray diagram is a graphical representation of the drive arrangement in general form. In other words, the ray diagram is a graphical representation of the structural formula, as shown in Fig.9.2.

It provides the following data on the drive :

- ✓ The number of stages (a stage is a set of gear trains arranged on two consecutive shafts).
- ✓ The number of speeds in each stage.
- ✓ The order of kinematic arrangement of the stages.
- ✓ The specific values of all the transmission ratios in the drive.
- ✓ The total number of speeds available at the spindle.

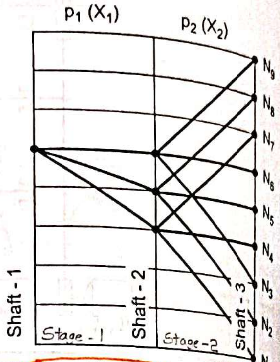


Fig. 9.2. *Ray diagram of 9 speed gear bo*

9.9.1. Procedure

Rules

- ✓ In this diagram, shafts are shown by vertical equidistant and parallel lines.
- ✓ The speeds are plotted vertical on a logarithmic scale with log ϕ as a unit.
- ✓ Transmission engaged at definite speeds of the driving and driven shafts a shown on the diagram by rays connecting the points on the shaft lines representin these speeds.
- ✓ Fig.9.2 shows the ray diagram for a 9 speed gear box, having the structur formula, $z = 3(1) \cdot 3(3)$.

9.10. BASIC RULES FOR OPTIMUM GEAR BOX DESIGN

The basic rules to be followed while designing the gear boxes are as follows :

1. The transmission ratio (i) in a gear box is limited by

$$\frac{1}{4} \leq i \leq 2. \quad \text{Refer Fig.9.3.}$$

In other words,

$$i_{min} = \frac{N_{min}}{N_{input}} \geq \frac{1}{4} \quad \text{and}$$

$$i_{max} = \frac{N_{max}}{N_{input}} \leq 2 \qquad \ldots (9.3a)$$

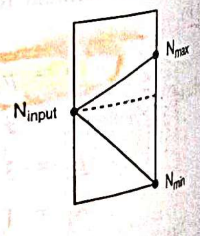


Fig. 9.3.

2. For stable operation, the speed ratio at any stage should not be greater than 8.

In other words,

$$\frac{N_{max}}{N_{min}} \leq 8 \qquad \ldots (9.3$$

---

3. *In all stages except in the first stage,* $N_{max} \geq N_{input} > N_{min}$.
4. *The sum of teeth of mating gears in a given stage must be the same for same module in a sliding gear set.*
5. *The minimum number of teeth on smallest gear in drives should be greater than or equal to 17.* Assume → 20
6. *The minimum difference between the number of teeth of adjacent gears must be 4.*
7. *Gear box should be of minimum possible size. Both radial as well as axial dimensions should be as small as possible.*

X:

Example 9.5 ***The minimum and maximum speed of a six speed gear box are to be 160 and 500 r.p.m. Construct the kinematic arrangement and the ray diagram of the gear box.***

Given Data : $n = 6$; $N_{min} = 160$ r.p.m. ; $N_{max} = 500$ r.p.m.

To find : Construction of the kinematic arrangement and the ray diagram.

☺ Solution :

Selection of spindle speeds :

We know that,

$$\frac{N_{max}}{N_{min}} = \phi^{n-1}$$

or

$$\frac{500}{160} = \phi^{6-1} \quad \text{or} \quad \phi = 1.256$$

We find $\phi = 1.256$ is not a standard ratio. So let us find out whether multiples of standard tio 1.12 or 1.06 come close to 1.256.

We can write,

$$1.12 \times \underline{1.12} = 1.254 \qquad \ldots \text{(skip one speed)}$$

So $\phi = 1.12$ satisfies the requirement. Therefore the spindle speeds from R 20 series, kipping one speed, are given by

160, 200, 250, 315, 400 and 500 r.p.m.

Structural formula : For 6 speeds, the preferred structural formula = 3 (1) 2 (3).
1st stage 2nd stage

Ray diagram :

Procedure :

- ✓ Since there are 3 shafts in kinematic layout, draw 3 vertical equidistant lines to represent shafts.
- ✓ Since there are 6 spindle speeds, draw 6 horizontal equidistant lines to represent speeds. Then mark the speeds on the horizontal lines, as shown in Fig.9.4(a).
- ✓ From the structural formula, it is clear that there are two stages. In the second stage, *i.e.*, in 2 (3), 2 represents the number of speeds available in a stage and (3) represents the steps or intervals between those two speeds.

- ✓ Locate the first point A on the lowest speed *i.e.*, at 160 r.p.m., on the last shaft. After 3 steps above, locate the second point B at 315 r.p.m. These two points A and B are the two output speeds.
- ✓ To locate the input speed point from the preceding shaft (*i.e.*, shaft 2), the following requirements should be met. That is,

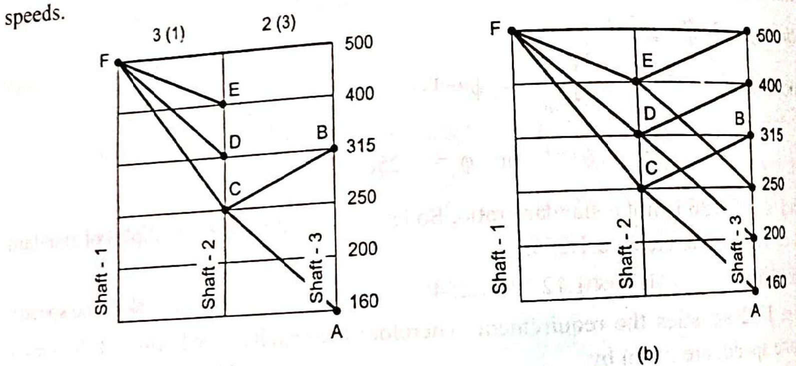
$$\frac{n_{max}}{n_{input}} \geq \frac{1}{4} \quad \text{and} \quad \frac{n_{max}}{n_{input}} \leq 2$$

Locate point C at the input speed of 250 r.p.m.

At point C, we get
$$\frac{N_{min}}{N_{input}} = \frac{160}{250} \geq \frac{1}{4} \quad \text{and}$$

$$\frac{N_{max}}{N_{input}} = \frac{315}{250} \leq 2$$

Thus the requirements are satisfied. Note that the above conditions are met for other inp speeds.



(a) (b)

Fig. 9.4. Ray diagram for 6 speed gear box

- ✓ In the first stage, *i.e.*, in 3(1), 3 represents the number of speeds available in thi stage and (1) represents the step between those speeds. The lowest speed is at C which is already located. Now locate points D and E on the 2nd shaft, above poi C, in a single step interval. For these three output speeds in the first stage, th input should be from shaft 1.
- ✓ Input speed can be located anywhere on shaft 1 satisfying the ratio requirement In this case, the point F is located at 500 r.p.m.
- ✓ In the second stage, we find input speed at C gives two output speeds at A and B Similarly, input speeds at D and E should give two output speeds. That is, fro point D, draw lines parallel to CA and CB. Then from point E, draw lines para to CA and CB, as shown in Fig.9.4(b).

Thus the completed ray diagram is shown in Fig.9.4(b).

Kinematic arrangement : The kinematic arrangement for 6 speed gear box is drawn, as shown in Fig.9.5.

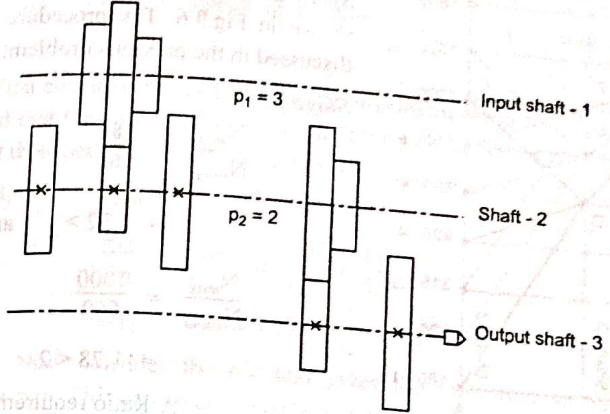


Fig. 9.5. Kinematic arrangement for 6 speed gear box

Example 9.6 ***A nine speed gear box, used as a head stock gear box of a turret lathe, is to provide a speed range of 180 r.p.m. to 1800 r.p.m. Using standard step ratio, draw the speed diagram, and the kinematic layout. Also find and fix the number of teeth on all gears.***

Given Data : $n = 9$; $N_{min} = 180$ r.p.m. ; $N_{max} = 1800$ r.p.m.

To find : Construction of speed diagram and kinematic layout.

☺ ***Solution :***

Selection of spindle speeds :

We know that,
$$\frac{N_{max}}{N_{min}} = \phi^{n-1}$$

or
$$\frac{1800}{180} = \phi^{9-1} \quad \text{or} \quad \phi = 1.333$$

We find $\phi = 1.333$ is not a standard ratio. So let us find out whether multiples of standard ratio 1.12 or 1.06 come close to 1.333.

We can write,

Then
$$1.12 \times 1.12 = 1.2544 \quad \text{and} \quad 1.12 \times 1.12 \times 1.12 = 1.405$$

$$1.06 \times \underline{(1.06 \times 1.06 \times 1.06 \times 1.06)} = 1.338$$

... (skip 4 speeds)

So $\phi = 1.06$ satisfies the requirement. Therefore the spindle speeds from R 40 series, skipping 4 speeds, are given by

180, 236, 315, 425, 560, 750, 1000, 1320 and 1800 r.p.m.

8 7 7
6 4
5 1

Structural formula : For 9 speeds, the preferred structural formula = 3 (1) 3 (3).

Speed diagram : The speed diagram is drawn as shown in Fig.9.6. The procedure is the same as discussed in the previous problem.

Stage 2:

$$\frac{N_{min}}{N_{input}} = \frac{180}{560}$$

$$= 0.32 > \frac{1}{4} \text{ ; and}$$

$$\frac{N_{max}}{N_{input}} = \frac{1000}{560}$$

$$= 1.78 < 2$$

∴ Ratio requirements are satisfied.

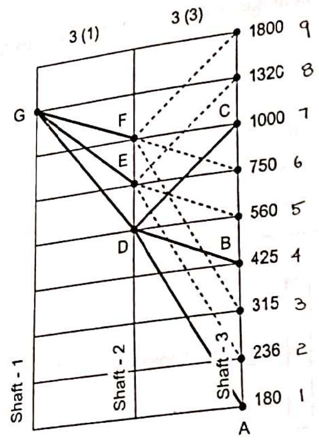


Fig. 9.6. Speed diagram for 9 speed gear box

Kinematic arrangement : The kinematic arrangement for 9 speed gear box is drawn, as shown in Fig.9.7.

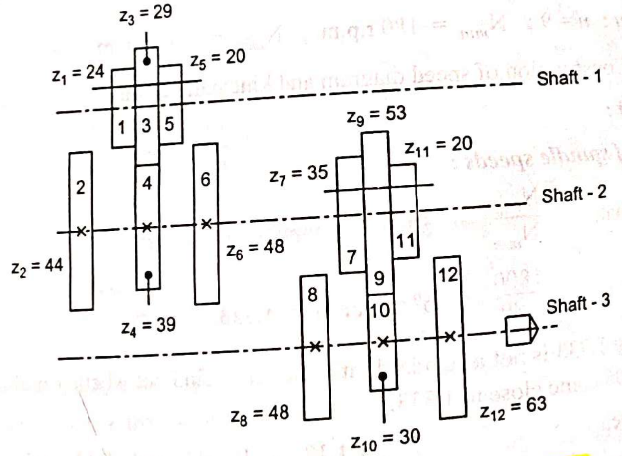


Fig. 9.7. Kinematic arrangement for 9 speed gear box

Calculation of number of teeth :

Let $z_1, z_2, z_3, \ldots\ldots\ldots z_{12}$ = Number of teeth of the gears 1, 2, 3, ... 12 respectively, and

$N_1, N_2, N_3, \ldots\ldots N_{12}$ = Speeds of the gears 1, 2, 3, ..., 12 respectively.

Second stage :

First pair : First consider the ray that gives the maximum speed reduction. From the speed diagram, we find that the speed is reduced from 560 r.p.m. to 180 r.p.m. (Refer Fig.9.6). We may assume that this speed reduction is achieved by using the gears 11 and 12.

We know that, $z_{min} \geq 17$. Therefore assume $z_{11} = 20$ (driver)

$$\therefore \quad \frac{z_{11}}{z_{12}} = \frac{N_{12}}{N_{11}} \text{ or } \frac{20}{z_{12}} = \frac{180}{560} ;$$

$$\text{or} \quad z_{12} = 62.22 \approx 63$$

Second pair : Now consider the ray that gives the minimum speed reduction from 560 r.p.m. to 425 r.p.m. This can be achieved by using the gears 7 and 8.

$$\therefore \quad \frac{z_7}{z_8} = \frac{N_8}{N_7} = \frac{425}{560} \text{ or } z_7 = 0.76\,z_8 \quad \ldots \text{(i)}$$

We know that the centre distance between the shafts are fixed and same. Therefore, the sum of number of teeth of mating gears should be equal. So we can write

$$z_7 + z_8 = z_{11} + z_{12} = 20 + 63 = 83 \quad \ldots \text{(ii)}$$

On solving equations (i) and (ii), we get

$$z_8 = 47.16 \approx 48 \text{ and } z_7 = 83 - 48 = 35$$

Third pair : Now consider the ray that gives the speed increase from 560 r.p.m. to 1000 r.p.m. This can be achieved by using the gears 9 and 10.

$$\therefore \quad \frac{z_9}{z_{10}} = \frac{N_{10}}{N_9} = \frac{1000}{560} \text{ or } z_9 = 1.786\,z_{10} \quad \ldots \text{(iii)}$$

$$\text{Also,} \quad z_9 + z_{10} = z_{11} + z_{12} = 20 + 63 = 83 \quad \ldots \text{(iv)}$$

On solving equations (iii) and (iv), we get

$$z_{10} = 29.79 \approx 30 \text{ and } z_9 = 83 - 30 = 53$$

First stage :

First pair : Consider the maximum speed reduction from 1320 r.p.m. to 560 r.p.m. This can be achieved by gears 5 and 6.

Assume $z_5 = 20$ (driver)

$$\therefore \quad \frac{z_5}{z_6} = \frac{N_6}{N_5} = \frac{560}{1320} = 0.4242$$

$$z_6 = \frac{z_5}{0.4242} = \frac{20}{0.4242} = 47.14 \approx 48.$$

or

Second pair : Consider the speed reduction from 1320 r.p.m. to 750 r.p.m. This can be achieved by gears 1 and 2.

$$\therefore \quad \frac{z_1}{z_2} = \frac{N_2}{N_1} = \frac{750}{1320} \quad \text{or} \quad z_1 = 0.57\, z_2 \qquad \text{... (v)}$$

$$\text{and} \quad z_1 + z_2 = z_5 + z_6 = 20 + 48 = 68 \qquad \text{... (vi)}$$

On solving equations (v) and (vi), we get

$$z_2 = 43.3 \approx 44 \quad \text{and} \quad z_1 = 68 - 44 = 24$$

Third pair : Finally consider the speed reduction from 1320 r.p.m. to 1000 r.p.m. This can be achieved by gears 3 and 4.

$$\therefore \quad \frac{z_3}{z_4} = \frac{N_4}{N_3} = \frac{1000}{1320} \quad \text{or} \quad z_3 = 0.76\, z_4 \qquad \text{... (vii)}$$

$$\text{and} \quad z_3 + z_4 = z_5 + z_6 = 20 + 48 = 68 \qquad \text{... (viii)}$$

On solving equations (vii) and (viii), we get

$$z_4 = 38.64 \approx 39 \quad \text{and} \quad z_3 = 68 - 39 = 29$$

Example 9.7 ***For the data of the above problem, calculate the percentage deviation of the obtainable speeds from the calculated ones.***

Given Data : Refer Example 9.6.

To find : Percentage deviation of the speeds.

☺ ***Solution :*** It is understood from the kinematic arrangement (Fig.9.7) that the combinations of gears 1 and 2 (in the first stage), gears 7 and 8, gears 9 and 10, and gears 11 and 12 (in the second stage) are to give three output speeds. Then, the combinations of gears 3 and 4 (in the first stage), gears 7 and 8, gears 9 and 10, and gears 11 and 12 (in the second stage) provide the next three output speeds. Similarly when gears 5 and 6 are engaged, we obtain three more speeds. Thus we can achieve totally 9 output speeds.

Calculation of output speeds : Let N_I and N_O = Input and output speeds of the gears.

From the ray diagram (Fig.9.6), input speed N_I = 1320 r.p.m.

$$N_{01} = N_I \times \frac{z_1}{z_2} \times \frac{z_7}{z_8} = 1320 \times \frac{24}{44} \times \frac{35}{48} = 525 \text{ r.p.m.}$$

$$N_{02} = N_I \times \frac{z_1}{z_2} \times \frac{z_9}{10} = 1320 \times \frac{24}{44} \times \frac{53}{30} = 1272 \text{ r.p.m.}$$

$$N_{03} = N_I \times \frac{z_1}{z_2} \times \frac{z_{11}}{z_{12}} = 1320 \times \frac{24}{44} \times \frac{20}{63} = 228.57 \text{ r.p.m.}$$

$$N_{04} = N_I \times \frac{z_3}{z_4} \times \frac{z_7}{z_8} = 1320 \times \frac{29}{39} \times \frac{35}{48} = 715.7 \text{ r.p.m.}$$

$$N_{05} = N_I \times \frac{z_3}{z_4} \times \frac{z_9}{z_{10}} = 1320 \times \frac{29}{39} \times \frac{53}{30} = 1734 \text{ r.p.m.}$$

$$N_{06} = N_I \times \frac{z_3}{z_4} \times \frac{z_{11}}{z_{12}} = 1320 \times \frac{29}{39} \times \frac{20}{63} = 311.6 \text{ r.p.m.}$$

$$N_{07} = N_I \times \frac{z_5}{z_6} \times \frac{z_7}{z_8} = 1320 \times \frac{20}{48} \times \frac{35}{48} = 401 \text{ r.p.m.}$$

$$N_{08} = N_I \times \frac{z_5}{z_6} \times \frac{z_9}{z_{10}} = 1320 \times \frac{20}{48} \times \frac{53}{30} = 971.66 \text{ r.p.m.}$$

$$N_{09} = N_I \times \frac{z_5}{z_6} \times \frac{z_{11}}{z_{12}} = 1320 \times \frac{20}{48} \times \frac{20}{63} = 174.6 \text{ r.p.m.}$$

Calculation of % deviation :

S.No.	Obtainable speed (N_{obt}, r.p.m.)	Calculated speed (N_{cal}, r.p.m.)	% deviation = $\frac{N_{obt} - N_{cal}}{N_{cal}} \times 100$
1.	174.6	180	−3
2.	228.57	236	−3.15
3.	311.6	315	−1.08
4.	401	425	−5.65
5.	525	560	−6.25
6.	715.7	750	−4.57
7.	971.66	1000	−2.83
8.	1272	1320	−3.64
9.	1734	1800	−3.66

Example 9.8 ***A gear box is to be designed to provide 12 output speeds ranging from 160 to 2000 r.p.m. The input speed of motor is 1600 r.p.m. Choosing a standard speed ratio, construct the speed diagram and the kinematic arrangement.***

Given Data : $n = 12$; N_{min} = 160 r.p.m. ; N_{max} = 2000 r.p.m. ; N_{input} = 1600 r.p.m.

To find : Construction of the speed diagram and the kinematic arrangement.

☺ ***Solution :***

Selection of spindle speeds :

We know that,

$$\frac{N_{max}}{N_{min}} = \phi^{n-1}$$

$$\frac{2000}{160} = \phi^{12-1} \text{ or } \phi = 1.258$$

or $\quad 1.12 \times \underline{1.12} = 1.254$ ... (skip one speed)

We can write,

So $\phi = 1.12$ satisfies the requirement. Therefore the spindle speeds from R 20 series, skipping one speed, are given by

160, 200, 250, 315, 400, 500, 630, 800, 1000, 1250, 1600 and 2000 r.p.m.

Structural formula : For 12 speeds, the preferred structural formula

$= 3\ (1) \quad 2\ (3) \quad 2\ (6)$

1st stage 2nd stage 3rd stage

Speed diagram (or Ray diagram) :

Procedure :

- ✓ Since there are 4 shafts, draw 4 vertical equidistant lines to represent shafts.
- ✓ Since there are 12 spindle speeds, draw 12 horizontal equidistant lines.
- ✓ From the structural formula, it is clear that there are three stages. In the third stage, *i.e.*, in 2 (6), 2 represents the number of speeds available in that stage and (6) represents the steps or intervals between these two speeds.
- ✓ Locate the first point A on the lowest speed *i.e.*, at 160 r.p.m., on the last shaft. After 6 steps above, locate the second point B at 630 r.p.m. These are the two output speeds.
- ✓ Locate the input speed at any point on the preceding shaft (*i.e.*, shaft 2), meeting the ratio requirements. We find, the input speed 400 r.p.m. at point C satisfies the ratio requirements.
- ✓ In the second stage, there are two speeds. Lowest speed is at C, which is already located. Now locate point D on the 3rd shaft, above point C, in a three step interval. For these two output speeds in the second stage, the input should be from shaft 2. We find, the input speed 630 r.p.m. at point E on shaft 2 satisfies the ratio requirements.
- ✓ In the first stage, there are three speeds. Lowest speed is at E, which is already located. Now locate points F and G on the shaft 2, above point E, in a single step interval.
- ✓ Input speed can be located anywhere on shaft 1 meeting the ratio requirements. But in this problem, given that, input speed is at 1600 r.p.m.
- ✓ In stage 2, we find input speed at E gives two output speeds at C and D. Similarly, input speeds at F and G, should give two output speeds. This can be achieved by drawing lines parallel to EC and ED, from points F and G, as shown in Fig.9.8.
- ✓ Now for stage 3, to get the output speeds to all the input speeds in shaft 3, draw lines parallel to CA and CB. Thus we have located all the input and the output speeds. The completed ray diagram is shown in Fig.9.8.

Stage 3 : $\dfrac{N_{min}}{N_{input}} = \dfrac{160}{400} = 0.4 > \dfrac{1}{4}$; and

$\dfrac{N_{max}}{N_{input}} = \dfrac{630}{400} = 1.57 < 2.$

Stage 2 : $\dfrac{N_{min}}{N_{input}} = \dfrac{400}{630}$

$= 0.63 > \dfrac{1}{4}$; and

$\dfrac{N_{max}}{N_{input}} = \dfrac{800}{630}$

$= 1.27 < 2.$

Stage 1 : $\dfrac{N_{min}}{N_{input}} = \dfrac{630}{1600}$

$= 0.39 > \dfrac{1}{4}$; and

$\dfrac{N_{max}}{N_{input}} = \dfrac{1000}{1600}$

$= 0.625 < 2.$

∴ Ratio requirements are satisfied.

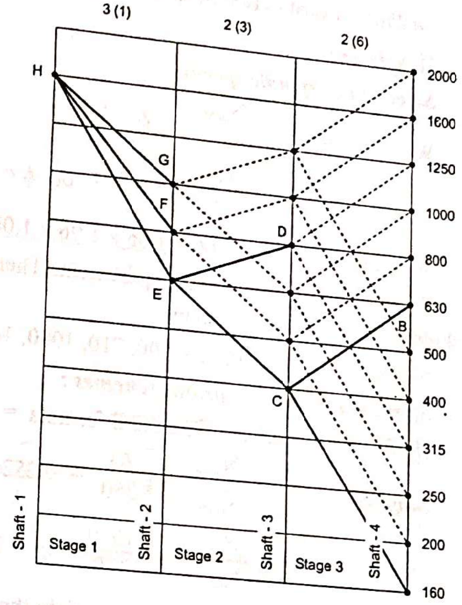


Fig. 9.8. Ray diagram for 12 speed gear box

Kinematic arrangement : The kinematic arrangement for 12 speed gear box is drawn, as shown in Fig.9.9.

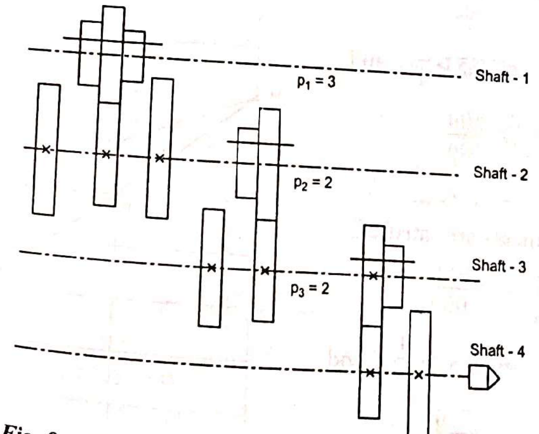


Fig. 9.9. Kinematic arrangement for 12 speed gear box

Example 9.9 ***A machine tool gear box is to have 12 speeds, with the output speeds ranging from 63 r.p.m. to 2800 r.p.m. Draw the speed diagrams for 2 × 2 × 3, 3 × 2 × 2, 3 × 4 and 4 × 3 schemes. Among these schemes which is better and why ?***

Given Data : $n = 12$; $N_{min} = 63$ r.p.m. ; $N_{max} = 2800$ r.p.m.

To find : Construction of speed diagrams for various schemes.

☺ **Solution :**

Selection of spindle speeds :

We know that, $\frac{N_{max}}{N_{min}} = \phi^{n-1}$

or $\frac{2800}{63} = \phi^{12-1}$ or $\phi = 1.412$

We can try, $1.06 \times (\underline{1.06 \times 1.06 \times 1.06 \times 1.06 \times 1.06}) = 1.418$ ... (skip 5 speeds)

So $\phi = 1.06$ satisfies the requirement. Therefore the spindle speeds from R 40 series, skipping 5 speeds, are given by

63, 90, 125, 180, 250, 355, 500, 710, 1000, 1400, 2000 and 2800 r.p.m.

Speed diagrams for various schemes :

(i) 2 × 2 × 3 scheme : Structural formula = 2 (1) 2 (2) 3 (4). Refer Fig.9.10.

Stage 3 : $\frac{N_{min}}{N_{input}} = \frac{63}{250} = 0.252 > \frac{1}{4}$; and

$\frac{N_{max}}{N_{input}} = \frac{1000}{250} = 4 > 2$

In this stage, it is not possible to satisfy the ratio requirements. So treat as an exceptional case.

Stage 2 : $\frac{N_{min}}{N_{input}} = \frac{250}{500}$

$= 0.5 > \frac{1}{4}$; and

$\frac{N_{max}}{N_{input}} = \frac{500}{500}$

$= 1 < 2.$

∴ Ratio requirements are satisfied.

Stage 1 : $\frac{N_{min}}{N_{input}} = \frac{500}{1000}$

$= 0.5 > \frac{1}{4}$; and

$\frac{N_{max}}{N_{input}} = \frac{710}{1000}$

$= 0.71 < 2.$

∴ Ratio requirements are satisfied.

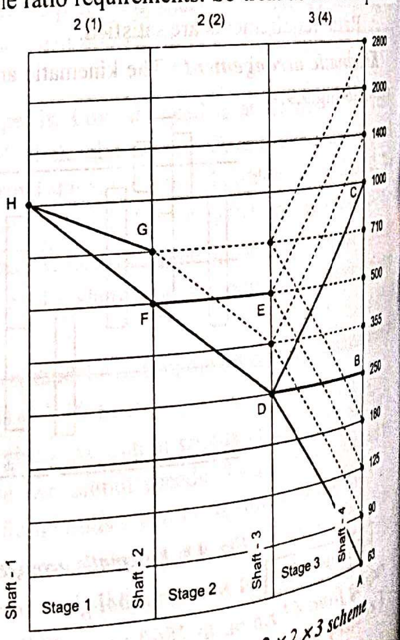


Fig. 9.10. Ray diagram for 2 × 2 × 3 scheme

(ii) 3 × 2 × 2 scheme : Structural formula = 3 (1) 2 (3) 2 (6). Refer Fig.9.11.

Stage 3 : $\frac{N_{min}}{N_{input}} = \frac{63}{250}$

$= 0.252 > \frac{1}{4}$; and

$\frac{N_{max}}{N_{input}} = \frac{500}{250}$

$= 2 \leq 2$

Stage 2 : $\frac{N_{min}}{N_{input}} = \frac{250}{500}$

$= 0.5 > \frac{1}{4}$; and

$\frac{N_{max}}{N_{input}} = \frac{710}{500}$

$= 1.42 < 2$

Stage 1 : $\frac{N_{min}}{N_{input}} = \frac{500}{1400} = 0.36 > \frac{1}{4}$; and

$\frac{N_{max}}{N_{input}} = \frac{1000}{1400} = 0.71 < 2$

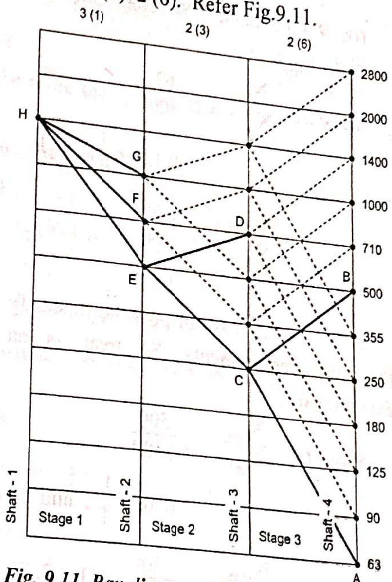


Fig. 9.11. Ray diagram for 3 × 2 × 2 scheme

∴ Ratio requirements are satisfactory.

(iii) 3 × 4 Scheme : Refer Fig.9.12.

Structural formula = 3 (1) 4 (3)

Stage 2 : $\frac{N_{min}}{N_{input}} = \frac{63}{710} < \frac{1}{4}$; and

$\frac{N_{max}}{N_{input}} = \frac{1400}{710} = 1.97 < 2.$

In this stage, it is not possible to satisfy the ratio requirements. So treat as an exceptional case.

Stage 1 : $\frac{N_{min}}{N_{input}} = \frac{710}{2000}$

$= 0.35 > \frac{1}{4}$; and

$\frac{N_{max}}{N_{input}} = \frac{1400}{2000}$

$= 0.7 < 2.$

∴ Ratio requirements are satisfied.

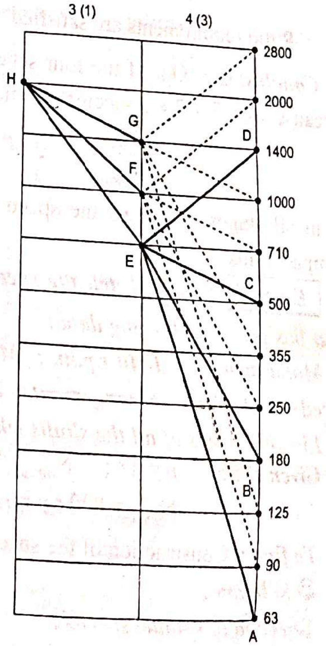


Fig. 9.12. Ray diagram for 3 × 4 scheme

(iv) ***4 × 3 Scheme :*** Structural formula = 4 (1) 3 (4).

Refer Fig.9.13.

Stage 2 : $\frac{N_{min}}{N_{input}} = \frac{63}{500}$

$= 0.125 < \frac{1}{4}$; and

$\frac{N_{max}}{N_{input}} = \frac{1000}{500}$

$= 2 \leq 2$

In this stage, it is not possible to satisfy the ratio requirements. So treat as an exceptional case.

Stage 1 : $\frac{N_{min}}{N_{input}} = \frac{500}{1400}$

$= 0.36 > \frac{1}{4}$; and

$\frac{N_{max}}{N_{input}} = \frac{1400}{1400} = 1 < 2.$

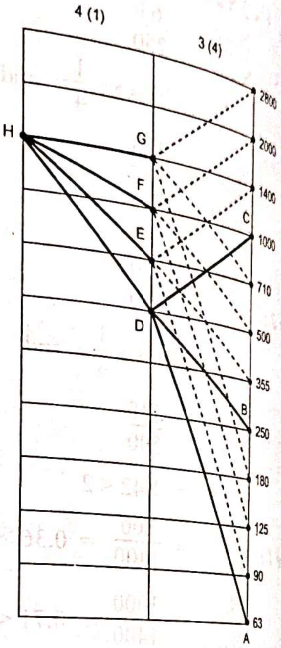


Fig. 9.13. Ray diagram for 4 × 3 scheme

∴ Ratio requirements are satisfied.

Conclusion : Out of the four schemes, 3 × 2 × 2 scheme is better than other schem... Because, only 3 × 2 × 2 scheme satisfies the ratio requirements. *i.e.,*

$$\frac{N_{max}}{N_{input}} < 2 \; ; \quad \frac{N_{min}}{N_{input}} > \frac{1}{4} \; ; \text{ and } \frac{N_{max}}{N_{min}} \leq 8,$$

in all stages. It means the speed reduction in all stages is minimum which results i... compact units.

Example 9.10 ***Sketch the speed diagram and the kinematic layout for an 18 spe... gear box for the following data :***

Motor speed = 1440 r.p.m. ; Minimum output speed = 16 r.p.m. ; Maximum outp... speed = 800 r.p.m. ; Arrangement = 2 × 3 × 3.

List the speeds of all the shafts when the output speed is 16 r.p.m.

Given Data : $n = 18$; $N_{input} = 1440$ r.p.m. ; $N_{min} = 16$ r.p.m. ;

$N_{max} = 800$ r.p.m. ; 2 × 3 × 3.

To find : Construction of the speed diagram and the kinematic layout.

☺ ***Solution :***

Selection of spindle speeds :

We know that, $\frac{N_{max}}{N_{min}} = \phi^{n-1}$

---

or $\frac{800}{16} = \phi^{18-1}$ or $\phi = 1.258$

We can write, $1.12 \times \underline{1.12} = 1.254$

So $\phi = 1.12$ satisfies the requirement. Therefore the spindle speeds from R 20 series, ... (skip one speed) skipping one speed, are given by

16, 20, 25, 31.5, 40, 50, 63, 80, 100, 125, 160, 200, 250, 315, 400, 500, 630 and 800 r.p.m.

Structural formula : Given that, 2 × 3 × 3 (*i.e.,* $p_1 \cdot p_2 \cdot p_3$)

We know that, Structural formula = $p_1(X_1)\; p_2(X_2)\; p_3(X_3)$

where $X_1 = 1$; $X_2 = p_1 = 2$; $X_3 = p_1 \cdot p_2 = 2 \times 3 = 6$

∴ Structural formula = 2 (1) 3 (2) 3 (6)

Speed diagram : The speed diagram is drawn, as shown in Fig.9.14, using the procedure discussed in Example 9.8.

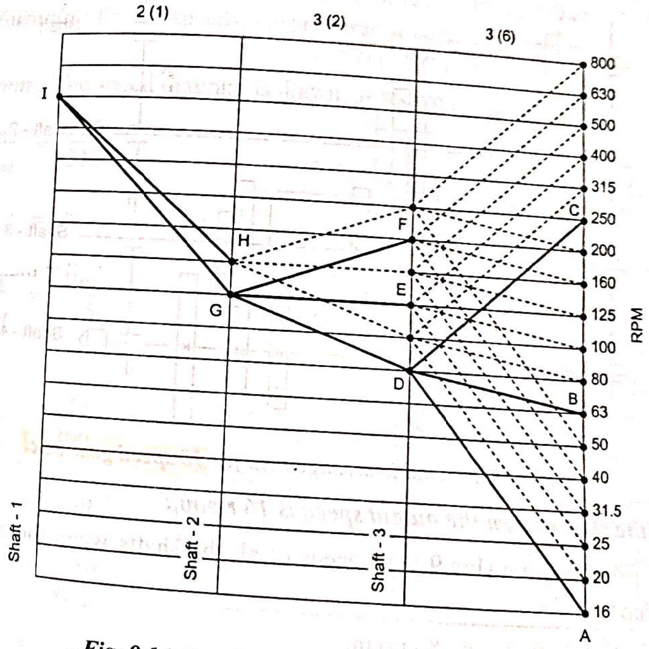


Fig. 9.14. Ray diagram for 18 speed gear box

Stage 3 : $\frac{N_{min}}{N_{input}} = \frac{16}{18} = 0.2 < \frac{1}{4}$; and

$\frac{N_{max}}{N_{input}} = \frac{250}{80} = 3.125 > 2$

∴ In this stage, ratio requirements are not satisfied. So it can be treated as an exceptional case.

In Fig.9.16, the ticked (✓) four speeds are overlapping speeds.

Note From the Fig.9.16, we find that 400 r.p.m. is achieved through two different routes, IFC and IHJB. It means that two different sets of gears are used to achieve the same speed. In the same manner the speed overlaps at speeds 500, 630 and 800 r.p.m. also.

Kinematic arrangement : The kinematic arrangement for 14 speed gear box is drawn, as shown in Fig.9.17.

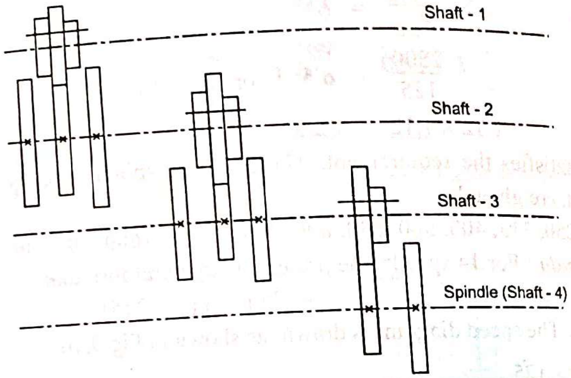


Fig. 9.17. Kinematic layout for 14 speed gear box

DESIGN OF GEAR BOX

$\Psi \rightarrow$ Psi

9.12. DESIGN PROCEDURE FOR GEAR BOX

① ***Selection of spindle speeds :***

✓ Determine the progression ratio (ϕ) using the relation

Phi?

$$\phi^{n-1} = \frac{N_{max}}{N_{min}}$$

✓ For the calculated ϕ, select the standard spindle speeds using the series of preferred numbers, from Table 9.2. (7.20) D.D

② ***Construct the ray diagram,*** as discussed earlier.

③ ***Construct the kinematic arrangement*** for the given gear box, as discussed earlier.

④ ***Calculation of number of teeth on all gears :*** Calculate the number of teeth of all gears engaged in all stages of the gear box.

⑤ ***Select the suitable material,*** consulting Tables 9.4 and 9.5.

⑥ ***Calculation of module :***

✓ Calculate the torque for the gear which has the lowest speed using the relation

$$T = \frac{P \times 60}{2\pi N}$$

✓ Calculate the tangential force on the gear in terms of module using the relation (refer Fig.9.18),

$$F_t = \frac{T}{r} = \frac{2T}{z \times m} \quad \dots (9.5)$$

$$\left[\because T = F_t \times r \text{ and } r = \frac{z \cdot m}{2}\right]$$

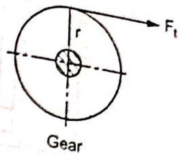


Fig. 9.18.

✓ Now calculate the module using the relation

$$m = \sqrt{F_t / \psi_m \times M} \quad \dots (9.6)$$

where ψ_m = Ratio between the face width and module $= \frac{b}{m} = 10$,

M = Material constant, from Table 9.4.

Table 9.4. Material constant (M)

Material	Material constant (M)
C45	30
15 Ni 2 Cr 1 Mo 15	80
40 Ni 2 Cr 1 Mo 28	100

⑦ ***Calculation of centre distance in all stages :*** Calculate the centre distance in each stage by using the relation

$$a = \left(\frac{z_x + z_y}{2}\right) m \quad \dots (9.7)$$

where z_x and z_y = Number of teeth on the gear pair in engagement in each stage.

⑧ ***Calculation of face width :*** $b = 10 \times m$ $b = \psi . m$

⑨ ***Calculation of distance between the bearings i.e., length of shafts :*** Calculate the distance between the bearings by using the following assumptions, (refer Fig.9.19) :

✓ Give 10 mm clearance between the gear and the bearing on both sides.

✓ Take the distance between the adjacent groups of gears as 20 mm.

✓ Take the total length for two pairs gear group as $4b$ and for three pairs gear group as $7b$, as shown in Fig.9.19.

✓ Assume the width of the bearings as 25 mm.

∴ Distance between the bearings is given by

$$L = 25 + 10 + 4b \text{ (or } 7b) + 20 + 7b \text{ (or } 4b) + 10 + 25 \quad \dots (9.8)$$

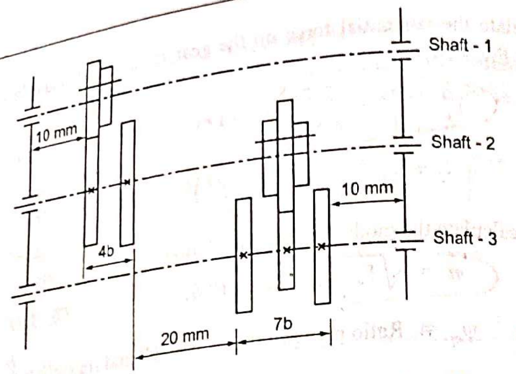


Fig. 9.19.

(10) *Design of shafts :*

(i) Design of spindle i.e., output shaft : Design the output shaft for maximum bending moment by considering the shaft as simply supported on bearings (refer Fig.9.20).

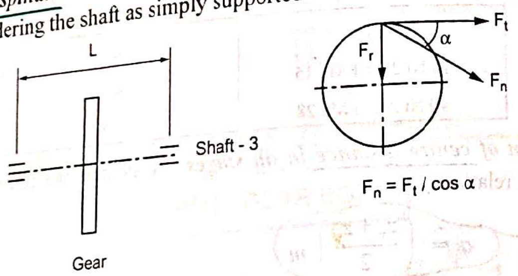


Fig. 9.20.

✓ Calculate the maximum bending moment due to normal load (F_n) using the relation

$$M = \frac{F_n \times L}{4}$$ ... (9.9)

where F_n = Normal load on gear $= \frac{F_t}{\cos\alpha}$

✓ Calculate the equivalent torque using the relation

$$T_{eq} = \sqrt{M^2 + T^2}$$ ... (9.10)

where T = Torque on the spindle $= \frac{P \times 60}{2\pi N_{low}}$

✓ Calculate the diameter of the spindle using the relation

$$d_s = \left[\frac{16 \times T_{eq}}{\pi[\tau]}\right]^{\frac{1}{3}} \qquad \left[\because [\tau] = \frac{16\,T_{eq}}{\pi d_s^3}\right]$$ ... (9.11)

where $[\tau]$ = Permissible shear stress, from Table 9.5.

Table 9.5. Permissible shear stress [τ], N/mm²

S.No.	Shaft material	[τ], N/mm²
1.	C14 (as supplied)	25
2.	C45 (case hardened)	30
3.	Low carbon alloy steel (case hardened)	40
4.	40 Ni 2 Cr 1 Mo 28 (hardened and tempered)	55

(ii) Design of other shafts : Determine the diameter of the input and intermediate shafts using the relation $T = 0.2\, d_s^3 [\tau]$ ... (9.12)

Example 9.12 *Design a 12 speed gear box for an all geared headstock of a lathe. Maximum and minimum speeds are 600 r.p.m. and 25 r.p.m. respectively. The drive is from an electric motor giving 2.25 kW at 1440 r.p.m.*

Sem

16 Mark

Model

Given Data : n = 12 ; N_{max} = 600 r.p.m. ; N_{min} = 25 r.p.m. ; P = 2.25 kW; N_{input} = 1440 r.p.m.

To find : Design the 12 speed gear box.

(9.8) Rules

$\frac{N_{Max}}{N_{Min}} \leq 8$

PSG (5.57)

Solution :

1

9 → 12
12 → 14
18 → 16

1. Selection of spindle speeds :

We know that $\phi^{n-1} = \frac{N_{max}}{N_{min}}$

phi

or $\phi^{12-1} = \frac{600}{25}$ = 24

$\frac{600}{25} = 24$

$\phi^{11} = 24$

$\phi = 24^{1/11}$ (or) $24^{0.09}$

$\phi = 1.33$

or Progression Ratio ϕ = 1.335

$\phi \Rightarrow$

1.6	R5
1.25	R10
1.12	R20
1.06 ✓	R40

9.3

7.20 PSG

$(5 \times \sqrt{1.335} = 1.059)$

We can write, 1.06 × (1.06 × 1.06 × 1.06 × 1.06) = 1.338 ... (skip 4 speeds).

120

So ϕ = 1.06 satisfies the requirement. Therefore the spindle speeds from R 40 series, skipping four speeds, are given as

25 × 1.335²

25, 33.5, 45, 60, 80, 106, 140, 190, 250, 335, 450 and 600 r.p.m.

25 × 1.335

2. Ray diagram : The ray diagram is constructed, as shown in Fig.9.21.

N_1 : 25 = 25

N_2 : 25 × 1.334 = 33

N_3 : 33 × 1.334 = 44

N_4 : 44 × 1.334 = 59

N_5 : 59 × 1.334 = 79

N_6 : 79 × 1.334 = 105

N_7 : 105 × 1.334 = 14

(or)

N_8 140 × 1.334 = 187

N_9 187 × 1.334 = 249

N_{10} 249 × 1.334 = 332

N_{11} 332 × 1.334 = 443

N_{12} 600

9.8

method

25

$25 \times 1.335^0 = 25$

$25 \times 1.335^1 = 33.5$

$25 \times 1.335^2 = 44.7$

$25 \times 1.335^3 = 59.9$

$25 \times 1.335^4 = 80.0$

$25 \times 1.335^5 = 107$

$25 \times 1.335^6 = 143$

$25 \times 1.335^7 = 191$

$25 \times 1.335^8 = 256$

$25 \times 1.335^9 = 340$

$25 \times 1.335^{10} = 439$

$25 \times 1.335^{11} = 615$

n → No of speed at spindle
$P_1 P_2 P_3$ → stage no in gear box
$x_1 x_2 x_3$ → characteristics of stage

$n = \underbrace{P_1(x_1)}_{\text{Stage 1}} \cdot \underbrace{P_2(x_2)}_{\text{Stage 2}} \cdot \underbrace{P_3(x_3)}_{\text{Stage 3}}$

25
35.5
45
60
80
106
140
190
250
335
430
600

9.8 **Structural formula :** (3)(1) (2)(3) (2)(6)
$P_1\ x_1\quad P_2 x_2\quad P_3\ x_3$
3 6 12
Design see 9.6

Stage ③ : $\dfrac{N_{min}}{N_{input}} = \dfrac{25}{80}$

$= 0.31 > \dfrac{1}{4}$ and

$\dfrac{N_{max}}{N_{input}} = \dfrac{140}{80}$

$= 1.75 < 2$

Stage ② : $\dfrac{N_{min}}{N_{input}} = \dfrac{80}{140}$

$= 0.57 > \dfrac{1}{4}$ and

$\dfrac{N_{max}}{N_{input}} = \dfrac{190}{140}$

$= 1.36 < 2$

Stage ① : $\dfrac{N_{min}}{N_{input}} = \dfrac{140}{450} = 0.311 > \dfrac{1}{4}$ and

$\dfrac{N_{max}}{N_{input}} = \dfrac{250}{450} = 0.56 < 2$

$\therefore$ Ratio requirements are satisfied.

9.6

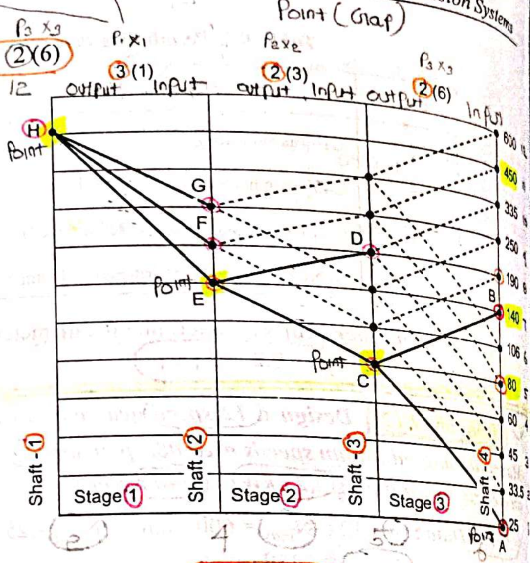


Fig. 9.21. Ray diagram for 12 speed gear box

① $\frac{1}{4} \leq i \leq 2$
② $\frac{N_{max}}{N_{min}} \leq 8$
③ $N_{max} \geq N_{input} > N_{min}$

Design Gear box rule

3. Kinematic arrangement : The kinematic arrangement for the given 12 speed gear box is constructed, as shown in Fig.9.22.

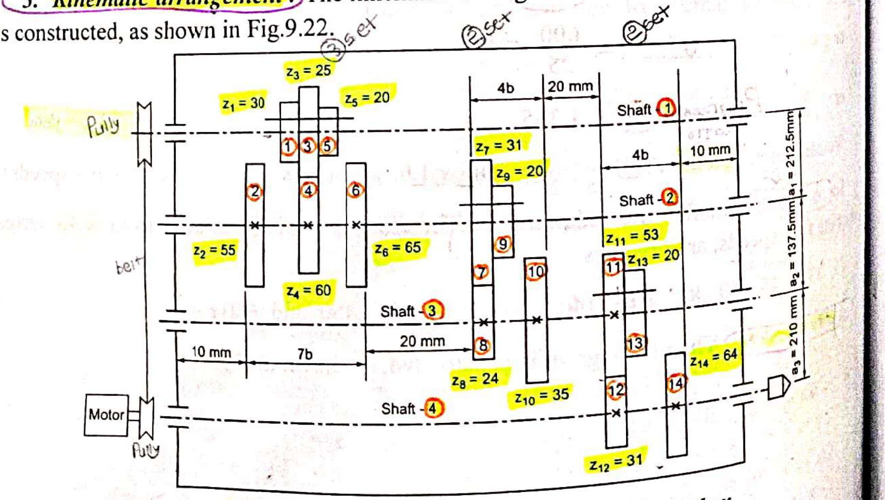


Fig. 9.22. Kinematic arrangement for 12 speed gear box

9.7

Str. formula 96 Page

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4. Calculation of number of teeth on all gears : The number of teeth on all gears are calculated as below, following the procedure used in Example 9.6.

Stage ③ :

First pair : Consider the ray that gives, maximum reduction *i.e.*, from 80 r.p.m. to 25 r.p.m. The corresponding gears are 13 and 14 on shaft 4.

We know that, $z_{min} \geq 17$. Therefore assume $z_{13} = 20$ (driver). ⟶ Page 9.9

Rules:- Min no of teeth in smallest drive should be greater than or equal to 17
$\boxed{teeth \geq 17}$ (or) $17 \leq$ teeth

$\therefore \quad \dfrac{z_{13}}{z_{14}} = \dfrac{N_{14}}{N_{13}}$ or $\dfrac{20}{z_{14}} = \dfrac{25}{80}$; $\therefore \boxed{z_{14} = 64}$

Second pair : Consider the other ray that gives speed increase from 80 r.p.m. to 140 r.p.m. The corresponding gears are 11 and 12.

$\therefore \quad \dfrac{z_{11}}{z_{12}} = \dfrac{N_{12}}{N_{11}} = \dfrac{140}{80}$ or $z_{11} = 1.75\ z_{12}$ ... (i)

We also know that the sum of number of teeth of mating gears should be equal.

$\therefore \quad z_{11} + z_{12} = z_{13} + z_{14} = 20 + 64 = 84$ ... (ii)

$z_{11} + z_{12} = 84$
$1.75 z_{12} + 1 z_{12} = 84$
$2.75 z_{12} = 84$
$z_{12} = \frac{84}{2.75} = 30.5 \approx 31$

On solving equations (i) and (ii), we get

$\boxed{z_{12} = 30.5 \approx 31}$ and $\boxed{z_{11} = 84 - 31 = 53}$

Stage ② :

Fist pair : Consider the ray that gives maximum reduction from 140 r.p.m. to 80 r.p.m. The corresponding gears are 9 and 10. Assume $z_9 = 20$ (driver).

$\therefore \quad \dfrac{z_9}{z_{10}} = \dfrac{N_{10}}{N_9}$ or $\dfrac{20}{z_{10}} = \dfrac{80}{140}$; $\therefore \boxed{z_{10} = 35}$

Second pair : Consider the other ray that gives speed increase from 140 r.p.m. to 190 r.p.m. The corresponding gears are 7 and 8.

$\therefore \quad \dfrac{z_7}{z_8} = \dfrac{N_8}{N_7} = \dfrac{190}{140}$ or $z_7 = 1.357\ z_8$ ... (iii)

and $\quad z_7 + z_8 = z_9 + z_{10} = 20 + 35 = 55$ ... (iv)

$z_7 + z_8 = 55$
$1.357 z_8 + 1 z_8 = 55$
$2.357 z_8 = 55$
$z_8 = \frac{55}{2.357} = 23.4 \approx 24$

On solving equations (iii) and (iv), we get

$\boxed{z_8 = 23.3 \approx 24}$ and $\boxed{z_7 = 55 - 24 = 31}$

Stage ① :

First pair : Consider the ray that gives maximum reduction from 450 r.p.m. to 140 r.p.m. The corresponding gears are 5 and 6. Assume $z_5 = 20$ (driver).

$\therefore \quad \dfrac{z_5}{z_6} = \dfrac{N_6}{N_5}$ or $\dfrac{20}{z_6} = \dfrac{140}{450}$; $\therefore \boxed{z_6 = 64.28 \approx 65}$

Second pair : Consider the ray that gives speed reduction from 450 r.p.m. to 190 r.p.m. The corresponding gears are 3 and 4.

$$\frac{z_3}{z_4} = \frac{N_4}{N_3} = \frac{190}{450} \quad \text{or} \quad z_3 = 0.422\, z_4 \qquad \dots (v)$$

∴ $z_3 + z_4 = z_5 + z_6 = 20 + 65 = 85$ ... (vi)

and

On solving the equations (v) and (vi), we get

$$z_4 = 59.77 \approx 60 \quad \text{and} \quad z_3 = 85 - 60 = 25$$

Third pair : Consider the ray that gives speed reduction from 450 r.p.m. to 250 r.p.m. The corresponding gears are 1 and 2.

$$\frac{z_1}{z_2} = \frac{N_2}{N_1} = \frac{250}{450} \quad \text{or} \quad z_1 = 0.555\, z_2 \qquad \dots (vii)$$

∴ $z_1 + z_2 = z_3 + z_4 = 60 + 25 = 85$ ... (viii)

and

On solving the equations (vii) and (viii), we get

$$z_2 = 54.66 \approx 55 \quad \text{and} \quad z_1 = 85 - 55 = 30$$

5. *Material selection* : C 45

6. *Calculation of module* :

✓ ***To find torque* :** In this case, the lowest speed 25 r.p.m. is obtained by meshing gears 13 and 14. Therefore torque at 25 r.p.m. is given by

$$T_{14} = \frac{P \times 60}{2\,\pi\,N} = \frac{2.25 \times 10^3 \times 60}{2\,\pi \times 25} = 859.44 \text{ N-m}$$

✓ ***To find tangential force on gear 14* :**

$$\frac{\cancel{T}}{r} = \frac{2\cancel{T}}{zm} \Rightarrow r = \frac{zm}{2}$$

$$F_{t_{14}} = \frac{T}{r} = \frac{2 \times T_{14}}{z_{14} \times m} \qquad \left[\because T = F_t \times r \text{ and } r = \frac{z \times m}{2}\right]$$

$$F_t = \frac{2 \times 859.44 \times 10^3}{64 \times m} = \frac{26857.5}{m} \qquad \dots \text{[where module } (m) \text{ is in mm]}$$

✓ We know that, module, $m = \sqrt{F_t / \psi_m \times M}$

where $\psi_m = b/m = 10$, and

M = Material constant = 30, for C45, from Table 9.4.

$$\therefore \quad m = \sqrt{\frac{(26857.5 / m)}{10 \times 30}} = \sqrt{89.525 / m} \quad \text{or} \quad m^2 = 89.525/m$$

$$\sqrt[3]{89.525}$$

or module, $m = 4.47$ mm

From Table 5.8, the nearest higher standard module is **5 mm.**

7. *Calculation of centre distance* :

✓ Centre distance in stage 1,
$$a_1 = \left(\frac{z_1 + z_2}{2}\right) m = \left(\frac{30 + 55}{2}\right) \times 5 = \mathbf{212.5\ mm}$$

✓ Centre distance in stage 2,
$$a_2 = \left(\frac{z_7 + z_8}{2}\right) m = \left(\frac{31 + 24}{2}\right) \times 5 = \mathbf{137.5\ mm}$$

✓ Centre distance in stage 3,
$$a_3 = \left(\frac{z_{11} + z_{12}}{2}\right) m = \left(\frac{53 + 31}{2}\right) 5 = \mathbf{210\ mm}$$

8. *Calculation of face width* : $b = \psi \times m = 10 \times 5 =$ **50 mm**

9. *Calculation of length of shaft* (*i.e., distance between the bearings*) : L

Length of shaft, $L = 25 + 10 + 7\,b + 20 + 4\,b + 20 + 4\,b + 10 + 25 = 110 + 15\,b$

$L = 110 + (15 \times 50) =$ **860 mm**

10. *Design of shafts* :

(i) *Design of spindle i.e., output shaft* :

✓ ***To find maximum bending moment (M)* :**

$$M = \frac{F_n \times L}{4}$$

where F_n = Normal load on gear $= \dfrac{F_t}{\cos \alpha} = \dfrac{(26857.5 / m)}{\cos \alpha}$

$$= \frac{(26857.5 / 5)}{\cos 20°} = 5716.23 \text{ N}$$

∴ Maximum bending moment, $M = \dfrac{5716.23 \times 860}{4} = 12.29 \times 10^5$ N-mm

✓ ***To find the equivalent torque (T_{eq})* :**

$$T_{eq} = \sqrt{M^2 + T_{14}^2} = \sqrt{(12.29 \times 10^5)^2 + (859.44 \times 10^3)^2}$$

$$T_{eq} = 1.5 \times 10^6 \text{ N-mm}$$

✓ ***Diameter of the spindle*** is given by (d_s) :

$$d_s = \left[\frac{16 \times T_{eq}}{\pi\,[\tau]}\right]^{\frac{1}{3}} \quad \text{where } \tau = 30 \text{ N/mm}^2, \text{ from Table 9.5.}$$

927

$$d_s = \left[\frac{16 \times 1.5 \times 10^6}{\pi \times 30}\right]^{\frac{1}{3}} = 63.38 \text{ mm}$$

✓ Rounded off value of the diameter, using R 40 series, is **67 mm.**

(ii) Design of other shafts :

(a) Diameter of shaft 1 :

H Input speed = 450 r.p.m.

$$\text{Torque} = \frac{P \times 60}{2\pi N} = \frac{2.25 \times 10^3 \times 60}{2\pi \times 450} = 47.746 \text{ N-m}$$

We know that,

$$T = 0.2\, d_s^3 [\tau]$$

$$47.746 \times 10^3 = 0.2 \cdot d_{s1}^3 \; 30$$ — Table 9.27 Page

or d_{s1} = 19.96 mm ≈ **20 mm** (R 40 series)

(b) Diameter of shaft 2 :

E Minimum speed = 140 r.p.m.

$$\text{Torque} = \frac{P \times 60}{2\pi N} = \frac{2.25 \times 10^3 \times 60}{2\pi \times 140} = 153.47 \text{ N-m}$$

We know that,

$$T = 0.2\, d_s^3 [\tau]$$

$$153.47 \times 10^3 = 0.2 \times d_{s2}^3 \times 30$$

or d_{s2} = 29.46 mm ≈ **30 mm** (R 40 series)

(c) Diameter of shaft 3 :

C Minimum speed = 80 r.p.m.

$$\text{Torque} = \frac{P \times 60}{2\pi N} = \frac{2.25 \times 10^3 \times 60}{2\pi \times 80} = 268.57 \text{ N-m}$$

$$T = 0.2 \cdot d_{s3}^3 \cdot (\tau)$$

$$\therefore \; 268.57 \times 10^3 = 0.2 \times d_{s3}^3 \times 30$$

or d_{s3} = **35.5 mm**

Sem

16 Mark

Example 9.13 ***Design a gear drive to give 18 speeds for a spindle of a milling machine. The drive is from an electric motor of 3.75 kW at 1440 r.p.m. Maximum and minimum speeds of the spindle are to be around 650 and 35 r.p.m. respectively.***

Given Data : $n = 18$; $P = 3.75$ kW ; N_{motor} = 1440 r.p.m. ; N_{max} = 650 r.p.m. ;

N_{min} = 35 r.p.m.

To find : Design the gear box.

☺ Solution :

1. Selection of spindle speeds :

2

$$\frac{650}{35} = 18.57$$

$$\phi^{17} = 18.57$$

$$\phi = (18.57)^{1/17}$$

$$\phi = 1.1875$$

We know that,

$$\phi^{n-1} = \frac{N_{max}}{N_{min}}$$

or

$$\phi^{18-1} = \frac{650}{35} \quad \text{or} \quad \phi = 1.1875$$

35

$35 \times (1.1875)^1 \rightarrow 41.6$

$35 \times (1.1875)^2 \rightarrow 49.4$

$35 \times (1.1875)^3 \rightarrow 58.6$

4 → 69.6

5 → 82.6

6 → 98.1

7 → 116.5

8 → 138

9 → 164

10 → 195

11 → 231

12 → 275

13 → 326

14 → 388

15 → 460

16 → 547

17 → 650

650

$(1.06)^3 = 1.191$

We can write, $1.06 \times (\underline{1.06 \times 1.06}) = 1.191$

7.20

So $\phi = 1.06$ satisfies the requirement. Therefore the spindle speeds from R 40 series, skipping 2 speeds, are given by … (skip 2 speeds)

35.5, 42.5, 50, 60, 71, 85, 100, 118, 140, 170, 200, 236, 280, 335, 400, 475, 560 and 670 r.p.m.

2. Ray diagram : The ray diagram is constructed, as shown in Fig.9.23.

Structural formula : 2 (1) 3 (2) 3 (6)

Stage 3 :

$$\frac{N_{min}}{N_{input}} = \frac{35.5}{140} = 0.253 > \frac{1}{4} \quad \text{and}$$

$$\frac{N_{max}}{N_{input}} = \frac{280}{140} = 2$$

Stage 2 :

$$\frac{N_{min}}{N_{input}} = \frac{140}{236} = 0.59 > \frac{1}{4} \quad \text{and}$$

$$\frac{N_{max}}{N_{input}} = \frac{280}{236} = 1.186$$

Stage 1 :

$$\frac{N_{min}}{N_{input}} = \frac{236}{475} = 0.497 > \frac{1}{4} \quad \text{and}$$

$$\frac{N_{max}}{N_{input}} = \frac{280}{475} = 0.59 < 2$$

$\therefore$ Ratio requirements are satisfied.

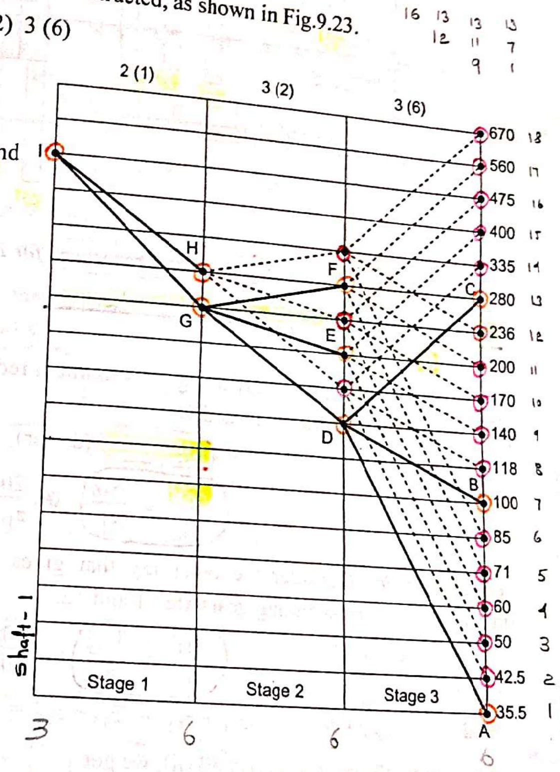

Fig. 9.23. Ray diagram for 18 speed gear box

3. Kinematic arrangement : The kinematic arrangement for the given 18 speed gear box is drawn, as shown in Fig.9.24.

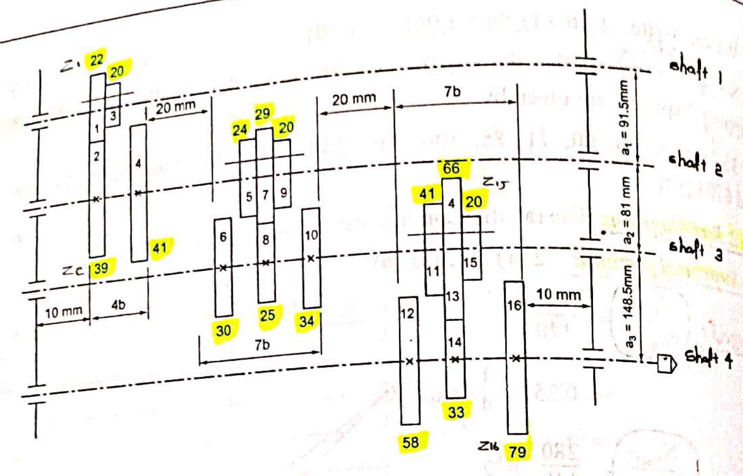

Fig. 9.24 Kinematic arrangement for 18 speed gear box

4. Calculation of number of teeth on all gears :

Stage 3 :

***First* pair :** Consider the ray that gives maximum reduction from 140 r.p.m. to 35.5 r.p.m. The corresponding gears are 15 and 16.

Assume $z_{15} = 20$ (driver)

$$\therefore \quad \boxed{\frac{z_{15}}{z_{16}} = \frac{N_{16}}{N_{15}}} \text{ or } \frac{20}{z_{16}} = \frac{35.5}{140} ; \quad \therefore \boxed{z_{16} = 78.87 \approx 79}$$

***Second* pair :** Consider the other ray that gives speed reduction from 140 r.p.m. to 100 r.p.m. The corresponding gears are 11 and 12.

z15 = 20
z16 = 79
z15+z16 = 99
z11 = 0.714 z12

= 99
+1z12 = 99
= 99
= 99/1.714

$$\therefore \quad \boxed{\frac{z_{11}}{z_{12}} = \frac{N_{12}}{N_{11}}} = \frac{100}{140} \text{ or } z_{11} = 0.714\, z_{12} \qquad \text{... (i)}$$

and $$z_{11} + z_{12} = z_{15} + z_{16} = 20 + 79 = 99 \qquad \text{... (ii)}$$

On solving the equations (i) and (ii), we get

$$\boxed{z_{12} = 57.75 \approx 58} \text{ and } \boxed{z_{11} = 99 - 58 = 41}$$

***Third* pair :** Consider the other ray that gives speed increase from 140 r.p.m. to 280 r.p.m. The corresponding gears are 13 and 14.

$$\therefore \quad \boxed{\frac{z_{13}}{z_{14}} = \frac{N_{14}}{N_{13}}} = \frac{280}{140} \text{ or } z_{13} = 2 \times z_{14} \qquad \text{... (iii)}$$

and $$z_{13} + z_{14} = z_{15} + z_{16} = 20 + 79 = 99 \qquad \text{... (iv)}$$

On solving the equations (iii) and (iv), we get

$$\boxed{z_{14} = 33} \text{ and } \boxed{z_{13} = 99 - 33 = 66}$$

Stage 2 :

***First* pair :** Consider the ray that gives maximum reduction from 236 r.p.m. to 140 r.p.m. The corresponding gears are 9 and 10.

Assume $z_9 = 20$ (driver)

$$\therefore \quad \boxed{\frac{z_9}{z_{10}} = \frac{N_{10}}{N_9}} \text{ or } \frac{20}{z_{10}} = \frac{140}{236} ; \quad \therefore \boxed{z_{10} = 33.71 \approx 34}$$

***Second* pair :** Consider the other ray that gives speed reduction from 236 r.p.m. to 200 r.p.m. The corresponding gears are 5 and 6.

$$\therefore \quad \boxed{\frac{z_5}{z_6} = \frac{N_6}{N_5}} = \frac{200}{236} \text{ or } z_5 = 0.847\, z_6 \qquad \text{... (v)}$$

and $$z_5 + z_6 = z_9 + z_{10} = 20 + 34 = 54 \qquad \text{... (vi)}$$

On solving the equations (v) and (vi), we get

$$\boxed{z_6 = 29.24 \approx 30} \text{ and } \boxed{z_5 = 54 - 30 = 24}$$

***Third* pair :** Consider the other ray that gives the speed increase from 236 r.p.m. to 280 r.p.m. The corresponding gears are 7 and 8.

$$\therefore \quad \boxed{\frac{z_7}{z_8} = \frac{N_8}{N_7}} = \frac{280}{236} \text{ or } z_7 = 1.186\, z_8 \qquad \text{... (vii)}$$

and $$z_7 + z_8 = z_9 + z_{10} = 20 + 34 = 54 \qquad \text{... (viii)}$$

On solving the equations (vii) and (viii), we get

$$\boxed{z_8 = 24.69 \approx 25} \text{ and } \boxed{z_7 = 54 - 25 = 29}$$

Stage 1 :

***First* pair :** Consider the ray that gives maximum speed reduction from 475 r.p.m. to 236 r.p.m. The corresponding gears are 3 and 4.

Assume $z_3 = 20$ (driver)

$$\therefore \quad \boxed{\frac{z_3}{z_4} = \frac{N_4}{N_3}} \text{ or } \frac{20}{z_4} = \frac{236}{475} ; \quad \therefore \boxed{z_4 = 40.25 \approx 41}$$

***Second* pair :** Consider the other ray that gives speed reduction from 475 r.p.m. to 280 r.p.m. The corresponding gears are 1 and 2.

$$\therefore \quad \boxed{\frac{z_1}{z_2} = \frac{N_2}{N_1}} = \frac{280}{475} \text{ or } z_1 = 0.59\, z_2 \qquad \text{... (ix)}$$

and $$z_1 + z_2 = z_3 + z_4 = 20 + 41 = 61 \qquad \text{... (x)}$$

On solving the equations (ix) and (x), we get

$$\boxed{z_2 = 38.37 \approx 39} \text{ and } \boxed{z_1 = 61 - 39 = 22}$$

5. Material selection : Take 40 Ni 2 Cr 1 Mo 28 (hardened and tempered)

6. Calculation of module :

✓ *To find torque :* The gear 16 has the lowest speed of 35.5 r.p.m.

$$T_{16} = \frac{P \times 60}{2\pi N} = \frac{3.75 \times 10^3 \times 60}{2\pi \times 35.5} = 1008.73 \text{ N-m}$$

✓ To find tangential force on gear 16 :

$$F_{t_{16}} = \frac{T}{r} = \frac{2 \times T_{16}}{z_{16} \times m}$$

$$F_t = \frac{2 \times 1008.73 \times 10^3}{79 \times m} = \frac{25537.43}{m}$$

✓ We know that, module, $m = \sqrt{F_t / \psi_m \times M}$

where $\psi_m = \frac{b}{m} = 10$, and

M = Material constant = 100, for 40 Ni 2 Cr 1 Mo 28, from Table 9.4

$$\therefore \quad m = \sqrt{\frac{(25537.43 / m)}{10 \times 100}} = \sqrt{25.537/m} \quad \text{or} \quad m^2 = 25.537/m$$

or module, m = 2.945 mm

From Table 5.8, the nearest higher standard module is **3 mm.**

✓ *Calculation of centre distance :*

✓ Centre distance in stage 1, $a_1 = \left(\frac{z_1 + z_2}{2}\right) m = \left(\frac{22 + 39}{2}\right) 3 = 91.5 \text{ mm}$

✓ Centre distance in stage 2, $a_2 = \left(\frac{z_5 + z_6}{2}\right) m = \left(\frac{24 + 30}{2}\right) 3 = 81 \text{ mm}$

✓ Centre distance in stage 3, $a_3 = \left(\frac{z_{11} + z_{12}}{2}\right) m = \left(\frac{41 + 58}{2}\right) 3 = 148.5 \text{ mm}$

8. Calculation of face width : $b = \psi \times m = 10 \times 3 = \mathbf{30 \text{ mm}}$

9. Calculation of length of shaft (i.e., distance between the bearings) : L

Length of shaft, $L = 25 + 10 + 4\,b + 20 + 7\,b + 20 + 7\,b + 10 + 25 = 110 + 18b$

$$L = 110 + 18 \times 30 = \mathbf{650 \text{ mm}}$$

10. Design of shafts :

(i) Design of spindle i.e., output shaft :

✓ M Maximum bending moment, $M = \frac{F_n \times L}{4}$

where F_n = Normal load on gear $= \frac{F_t}{\cos\alpha} = \frac{(25537.43 / m)}{\cos\alpha}$

$$F_n = \frac{(25537.43 / 3)}{\cos 20^\circ} = 9058.79 \text{ N}$$

$$\therefore \quad M = \frac{9058.79 \times 650}{4} = 1.472 \times 10^6 \text{ N-mm}$$

T_{eq} ✓ Equivalent torque, $T_{eq} = \sqrt{M^2 + T_{16}^2}$

$$= \sqrt{(1.472 \times 10^6)^2 + (1008.73 \times 10^3)^2}$$

$$T_{eq} = 1.784 \times 10^6 \text{ N-mm}$$

d_s ✓ Diameter of the spindle is given by $\quad \tau = \left(\frac{16\,T_{eq}}{\pi d_s^3}\right)$

$$d_s = \left[\frac{16 \times T_{eq}}{\pi [\tau]}\right]^{\frac{1}{3}} \quad \text{where } [\tau] = 55 \text{ N/mm}^2, \text{ from Table 9.5.}$$

$$d_s = \left[\frac{16 \times 1.784 \times 10^6}{\pi \times 55}\right]^{\frac{1}{3}} = 54.87 \text{ mm} \approx \mathbf{56 \text{ mm}} \text{ (R 40 series)}$$

(ii) Design of other shafts :

(a) Diameter of shaft 1 : Input speed = 475 r.p.m.

$$\text{Torque} = \frac{P \times 60}{2\pi N} = \frac{3.75 \times 10^3 \times 60}{2\pi \times 475} = 75.39 \text{ N-m}$$

We know that, $T = 0.2 \times d_{s1}^3 \times [\tau]$

or $75.39 \times 10^3 = 0.2 \times d_{s1}^3 \times 55$

$$d_{s1} = 18.99 \text{ mm} \approx \mathbf{19 \text{ mm}} \text{ (R 40 series)}$$

(b) Diameter of shaft 2 :

Minimum speed = 236 r.p.m.

$$\therefore \quad \text{Torque} = \frac{P \times 60}{2\pi N} = \frac{3.75 \times 10^3 \times 60}{2\pi \times 236} = 151.74 \text{ N-m}$$

or $151.74 \times 10^3 = 0.2 \times d_{s2}^3 \times 55 \qquad T = 0.2 \cdot d_{se}^3 \cdot [\tau]$

$$d_{s2} = 23.98 \text{ mm} \approx \mathbf{25 \text{ mm}} \text{ (R 40 series)}$$

(c) ***Diameter of shaft 3 :***

$$\text{Minimum speed} = 140 \text{ r.p.m.}$$

$$\text{Torque} = \frac{P \times 60}{2\pi N} = \frac{3.75 \times 10^3 \times 60}{2\pi \times 140} = 255.78 \text{ N-m}$$

$T = 0.2 \cdot d_{s3}^3 \cdot (\tau)$

$$255.78 \times 10^3 = 0.2 \times d_{s3}^3 \times 55$$

$$\therefore \quad d_{s3} = 28.54 \text{ mm} \approx \mathbf{30\ mm} \quad \text{(R 40 series)}$$

or

REVIEW AND SUMMARY

- ✓ *Gear boxes are used to obtain different spindle speeds in most of the machine tools.*
- ✓ *The speeds in machine tool gear boxes are in geometric progression.*
- ✓ *The concept of preferred numbers and their significance are presented in the beginning of this chapter.*
- ✓ ***Step ratio*** *(or series ratio or progression ratio) (ϕ) is given by*

$$\phi^{n-1} = \frac{N_{max}}{N_{min}} \quad \textit{where } n = \textit{Number of spindle speeds required.}$$

- ✓ ***Structural formula :*** $n = p_1(X_1) \cdot p_2(X_2) \cdot p_3(X_3)$

  where $X_1 = 1;\ X_2 = p_1;\ X_3 = p_1 \cdot p_2$
- ✓ *The kinematic layout shows the arrangement of gears in a gear box.*
- ✓ *Ray or speed diagram is the graphical representation of the structural formula.*
- ✓ *Ray diagram serves to determine the specific values of all the transmission ratios and speeds of all the shafts in the drive.*
- ✓ *The basic rules to be followed while designing the gear boxes are also discussed.*
- ✓ *If the engagement of two different sets of gears provide the same, then the gear box is known as* ***overlapping speed gear box.***
- ✓ *The step by step procedure for gear box design is presented at the end of this chapter.*

REVIEW QUESTIONS

1. What situations demand use of gear boxes ?
2. What are the requirements of a speed gear box ?
3. The speeds in machine tool gear boxes are in geometric progression. Why ?
4. What are preferred numbers ?
5. Explain briefly the term 'progression ratio'.
6. Write an engineering brief on :
   (i) Kinematic layout of gear box ; and (ii) Ray diagram.

7. Write the significance of structural formula.
8. Differentiate ray diagram and structural diagram.
9. List out the basic rules to be followed for optimum gear box design.
10. Write the step by step procedure to design the gear box.

PROBLEMS FOR PRACTICE

Problems on construction of ray diagram and kinematic layout :

1. A six speed gear box is to provide a speed range of 100 r.p.m. to 1000 r.p.m. Draw the speed diagram and the kinematic layout of the gear box.
2. The minimum and maximum speed of 6 speed gear box are to be 500 and 1600 r.p.m. Construct the speed diagram and the kinematic arrangement of the gear box.
3. A nine speed gear box is required to give output speeds ranging from 100 r.p.m. to 600 r.p.m. The input power is 4 kW at 1000 r.p.m. Draw the structural diagram and the kinematic arrangement of gears. Also calculate the percentage deviation of the obtainable speeds from the calculated ones.
4. Draw the kinematic arrangement and the speed diagram of the headstock gear box of a turret lathe having arrangement for 9 spindle speeds ranging from 50 r.p.m. to 1500 r.p.m. Calculate the number of teeth on each gear if the minimum number of teeth on a gear is 23.
5. A machine tool gear box is to have 9 speeds. The gear box is driven by an electric motor whose shaft rotational speed is 1400 r.p.m. The gear box is connected to the motor by a belt drive. The maximum and minimum speeds required at the gear box output are 1000 r.p.m. and 200 r.p.m. Suitable speed reduction can also be provided in the belt drive. What is the step ratio and what are the values of 9 speeds ? Sketch the arrangement. Obtain the number of teeth on each gear and also the actual output speeds.
6. A 12 speed gear box is to provide a minimum speed of 30 r.p.m. with a step ratio of 1.12. Using standard step ratios, find the number of teeth on all gears.
7. Draw the speed diagram of a 12 speed gear box to give speeds in the range of 63 to 2800 r.p.m. Consider any 4 possible different alternates and indicate the best with reasons.
8. A machine tool gear box is to provide 14 spindle speeds ranging from 20 to 400 r.p.m. Draw the kinematic arrangement and the ray diagram.
9. A 16 speed gear box is to furnish speeds in the range of 100 r.p.m. to 560 r.p.m. Sketch the kinematic layout and speed diagram. Use standard progression ratio.
10. A gear box is to give 18 speeds for a spindle of a milling machine. Maximum and minimum speeds of the spindle are to be around 16 to 800 r.p.m. respectively. Find the speed ratios which will give the desired speeds and draw the structural diagram and the kinematic arrangement of the drive.

Problems on design of gear box :

11. Design a six speed gear box with a step ratio of 1.25. The input is from a motor running at 1440 r.p.m. The minimum speed is 112 r.p.m. Power of motor is 3 kW.

12. A six speed gear box is required to provide output speeds in the range of 125 to 400 r.p.m., with a step ratio of 1.25 and transmit a power of 5 kW at 710 r.p.m. Draw the speed diagram and kinematic diagram. Determine the number of teeth, module and face width of all gears, assuming the suitable materials for the gears. Determine the length of the gear box along the axis of the gear shaft.

13. Design a headstock gear box of a lathe having nine spindle speeds ranging from 30 to 1000 r.p.m. The power of the machine is 4.5 kW and the speed of the motor is 1440 r.p.m. Minimum number of teeth on the gear is to be 25. Sketch the layout of the gear box and calculate the number of teeth on the gears.

14. Design a 9 speed gear box to give output speeds between 280 and 1800 r.p.m. The input power is 5.5 kW at 1400 r.p.m. Draw the kinematic layout diagram and the speed diagram. Determine the number of teeth on all gears and the length of all the shafts.

15. Design a nine speed gear box with a minimum speed of 200 r.p.m. and speed ratio 1.5. The input is from a motor of 2 kW at 1400 r.p.m.

16. Design a 12 speed gear box for an all geared headstock of a lathe. Maximum and minimum speeds are 900 r.p.m. and 23 r.p.m. respectively. The drive is from an electric motor giving 2.2 kW at 1440 r.p.m.

17. The spindle of a pillar drill is to run at 12 different speeds in the range of 70 r.p.m. and 325 r.p.m. Design a three stage gear box with a standard step ratio. The gear box receives 4 kW from an electric motor running at 330 r.p.m. Sketch the layout of the gear box, indicating the number of teeth on each gear. Also sketch the speed diagram.

18. A gear box is to be designed for the following specifications;

Power to be transmitted	–	15 kW
Number of speeds	–	18
Minimum speed	–	16 r.p.m.
Step ratio	–	1.25
Motor speed	–	1400 r.p.m.
Arrangement scheme	–	2 × 3 × 3

Sketch the speed diagram and the kinematic arrangement.

19. Design the layout of a gear box for a milling machine having an output of speeds ranging from 180 to 2000 r.p.m. Power is supplied to the gear box by 6 kW induction motor at 1440 r.p.m. Choose standard step ratio and construct the speed diagram. Decide upon the various reduction ratios and number of teeth on each gear wheel. Sketch the arrangement of the gear box.

4th unit end ....

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10

Clutches

> *"One sees great things from the valley, only small things from the peak."*
>
> *– G.K. Chesterton*

2 Mark

10.1. INTRODUCTION

The clutch is a mechanical device which is used to connect or disconnect the source of power at the operator's will. The use of clutches is mostly found in automobile. In an automobile, clutch is a transmission device which is used to engage and disengage the power from the engine to the rest of the system.

For the above said reason, the clutch is located in between the engine and the transmission system. When the clutch is engaged to the engine, power is transmitted to the wheels. If it is disengaged, the power is not transmitted to the rest of the system even though engine is running and hence the vehicle stops. Therefore for coupling the engine smoothly to the power transmission during starting from rest and gear shifting, clutch is used.

10.2. FUNCTIONS OF THE CLUTCH

The important functions of a clutch are :

- ✓ To connect and disconnect the shafts at will.
- ✓ To start or stop a machine (or a rotating element) without starting and stopping the prime mover.
- ✓ To maintain constant speed, torque and power.
- ✓ To reduce shocks transmitted between machine shafts.
- ✓ For automatic disconnect, quick start and stop, gradual starts, and non-reversing and over running functions.

10.3. PRINCIPLES OF OPERATION OF THE CLUTCH

The clutch works on the principle of friction. When two friction surfaces are brought in contact with each other and pressed, they are united due to the friction between them. The friction between the two surfaces depends upon the area of the surfaces, pressure applied upon them and coefficient of friction of the surface materials. The principle of friction clutch is understood with the help of Fig.10.1. The two surfaces can be separated and brought into contact when required. One surface is considered as driving member and the other as driven