7

Bevel Gears

"A good scientist is a person with original ideas. A good engineer is a person who makes a design that works with as few original ideas as possible. There are no prima donnas in engineering."

– Freeman Dyson

7.1. INTRODUCTION

Bevel gears are used to transmit power between two intersecting shafts. Bevel gears are commonly used in automotive differentials. The gears are formed by cutting teeth along the elements of frustum of a cone. That is, the pitch surface in the bevel gears are truncated cone, one of which rolls over the other, as shown in Fig.7.1. When teeth formed on the cones are straight, the gears are known as ***straight bevel*** and when inclined, they are known as ***spiral or helical bevel.***

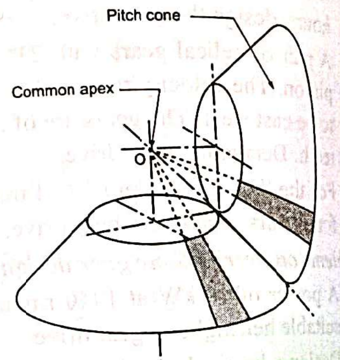


Fig. 7.1. *Bevel gear*

Bevel gears are mounted on intersecting shafts at any desired angle, although 90° shaft angle is most common. Bevel gears are not interchangeable. Because they are designed and manufactured in pairs.

The bevel gear teeth can be cast, milled, or generated. But the generated teeth is more accurate than cast and milled teeth.

7.2. TYPES OF BEVEL GEARS

7.2.1. Classification Based on the Teeth Shape

1. Straight Bevel Gears

If the teeth on the bevel gears are parallel to the lines generating the pitch cones, then they are called straight bevel gears. As shown in Fig.7.2, the teeth are straight, radial to the point of intersection of the shaft axes and vary in cross-section throughout their length. Usually, they are used to connect shafts at right angles which run at low speeds.

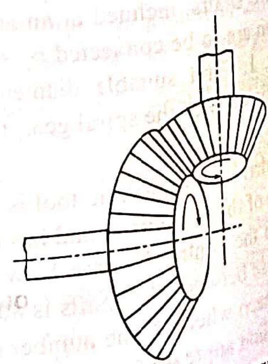


Fig. 7.2. *Straight bevel gear*

2. Spiral Bevel Gears

When the teeth of a bevel gear are inclined at an angle to the face of the bevel, they are known as spiral bevel gears, as shown in Fig.7.3. To reduce the noise, helical teeth is used on these bevel gears. They are smoother in action and quieter than straight tooth bevels as there is gradual load application and low impact stresses.

However, an axial thrust exists in the spiral bevel gears, so it requires stronger bearings and supporting assemblies. These gears are used for the drive to the differential of an automobile.

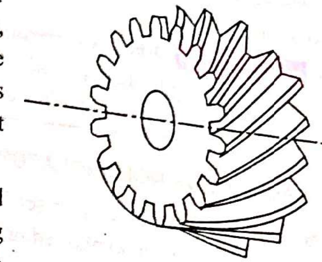


Fig. 7.3. *Spiral bevel gear*

3. Zerol Bevel Gears

Spiral bevel gears with curved teeth but with a zero degree spiral angle are known as zerol bevel gears. Refer Fig.7.4. Their tooth action and the end thrust are the same as that of straight bevel gears Zerol bevel gears are quicker in action than the straight bevel type as the teeth are curved.

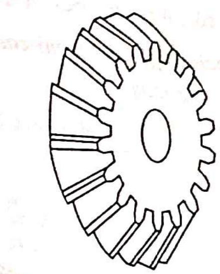


Fig. 7.4. *Zerol bevel gear*

4. Hypoid gears :

Hypoid gears are similar in appearance to spiral-bevel gears. They differ from spiral gears in that the axis of pinion is offset from the axis of the gear, as shown in Fig.7.5. The other difference is that their pitch surfaces are hyperboloids rather than cones.

In general, hypoid gears are most desirable for those applications involving large speed reduction ratios. They operate more smoothly and quietly than spiral bevel gears.

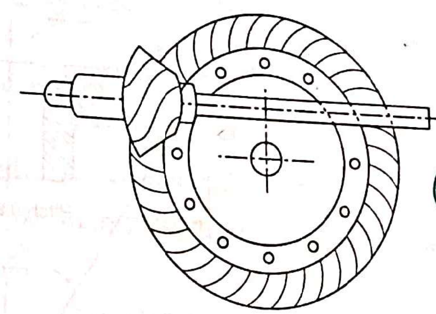


Fig. 7.5. *Hypoid gears*

7.2.2. Classification Based on Pitch Angle

1. **Crown gear :** A bevel gear having a pitch angle of 90° and a plane for its pitch surface is known as a ***crown gear.***

2. ***Internal bevel gear :*** When the pitch angle of a bevel gear exceeds 90°, it is called an ***internal bevel*** gear. Because of the manufacturing difficulties, the internal bevel gears are rarely used.

3. ***Mitre gears :*** When two meshing bevel gears have a shaft angle of 90° and have the same number of teeth, they are called ***mitre gears.*** In other words, mitre gears have a speed ratio of 1. Each of the two gears has a 45° pitch angle.

7.3. BEVEL GEAR NOMENCLATURE

The geometry of a bevel gear set is shown in Fig.7.6. The various terms used in the study of bevel gears have been explained below.

1. ***Pitch cone :*** It is the cone containing the pitch elements of the teeth.

2. ***Cone centre :*** It is the point where the axes of two mating gears intersect each other. In other words, it is the apex of the pitch cone.

3. ***Pitch angle (or semi-cone angle) (δ) :*** It is the angle made by the pitch line of a gear with the gear axis.

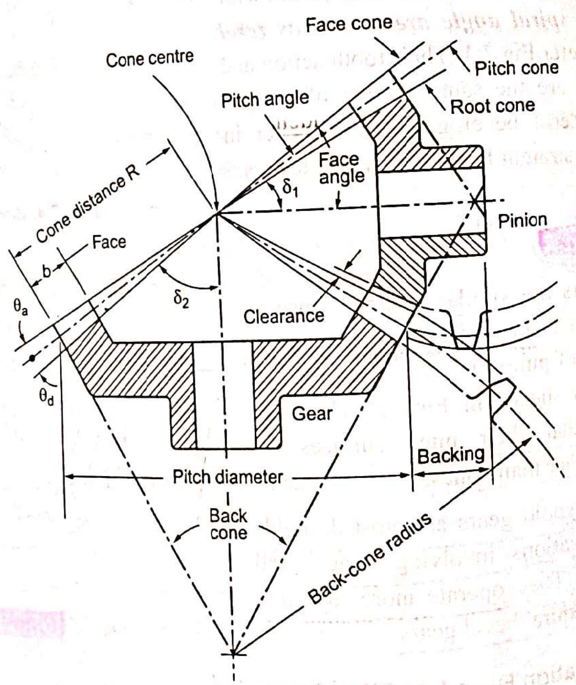


Fig. 7.6. Bevel gear nomenclature

4. ***Cone distance (or pitch cone radius) (R) :*** It is the length of the pitch cone element. Mathematically, cone distance (R) is given by

$$R = \frac{\text{Pitch radius}}{\sin \delta} = \frac{(d_1/2)}{\sin \delta_1} = \frac{(d_2/2)}{\sin \delta_2}$$

or

$$R = \frac{m_t \times z_1}{2 \sin \delta_1} = \frac{m_t \times z_2}{2 \cdot \sin \delta_2} \qquad [\because m_t = d/z] \qquad \dots (7.1)$$

where d_1 and d_2 = Pitch circle diameters of pinion and gear respectively,

z_1 and z_2 = Number of teeth of pinion and gear respectively,

δ_1 and δ_2 = Pitch angles of pinion and gear respectively, and

m_t = Transverse module

Cone distance can also be given by

$$R = \left[\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2\right]^{\frac{1}{2}} = \frac{1}{2}\sqrt{d_1^2 + d_2^2} \qquad \dots (7.2)$$

Note For right angle gears, $R = 0.5\, m_t \sqrt{z_1^2 + z_2^2} = 0.5\, m_t \times z_1 \sqrt{i^2 + 1}$

where i = Gear ratio = z_1/z_2. ... (7.3)

5. ***Addendum angle (θ_a) :*** It is the angle subtended by the addendum of the tooth at the cone centre. Mathematically,

$$\text{Addendum angle, } \theta_a = \tan^{-1}\left(\frac{\text{addendum}}{\text{cone distance}}\right) = \tan^{-1}\left(\frac{h_a}{R}\right) \qquad \dots (7.4)$$

6. ***Dedendum angle (θ_d) :*** It is the angle subtended by the dedendum of the tooth at the cone centre. Mathematically,

$$\text{Dedendum angle, } \theta_d = \tan^{-1}\left(\frac{\text{dedendum}}{\text{cone distance}}\right) = \tan^{-1}\left(\frac{h_f}{R}\right) \qquad \dots (7.5)$$

7. ***Tip (or face) angle :*** It is the angle subtended by the face of the tooth at the cone centre. Mathematically,

$$\text{Tip angle} = \text{Pitch angle} + \text{Addendum angle} = \delta + \theta_a \qquad \dots (7.6)$$

8. ***Root angle :*** It is the angle subtended by the root of the tooth at the cone centre. Mathematically,

$$\text{Root angle} = \text{Pitch angle} - \text{Dedendum angle} = \delta - \theta_f \qquad \dots (7.7)$$

9. ***Back (or normal) cone :*** It is an imaginary cone, perpendicular to the pitch cone at the end of the tooth.

10. ***Back cone distance (or back cone radius) :*** It is the length of the back cone.

11. ***Backing (B) :*** It is the distance of the pitch point from the back of the boss, parallel to the axis of the gear.

12. ***Mounting height :*** It is the distance of the back of the boss from the cone centre.

13. ***Pitch diameter :*** It is the diameter of the largest pitch circle.

14. ***Outside or addendum cone diameter :*** It is the maximum diameter of the teeth of the gear. It is equal to the diameter of the blank from which the gear can be cut. Mathematically,

$$\text{Outside diameter} = \text{Pitch diameter} + 2 \times h_a \times \cos\delta \quad \text{... (7.8)}$$

where h_a = Addendum, and

δ = Pitch angle.

15. ***Inside or dedendum cone diameter :*** It is given by

$$\text{Inside diameter} = \text{Pitch diameter} - 2 \times h_f \times \cos\delta \quad \text{... (7.9)}$$

where h_f = Dedendum

Angle relations for different bevel gears :

1. Acute angle bevel gears : If the shaft angle $\theta < 90°$, then the bevel gears are known as acute angle bevel gears. The pitch angles are given by

$$\tan\delta_1 = \frac{\sin\theta}{(z_2/z_1) + \cos\theta}$$

and

$$\tan\delta_2 = \frac{\sin\theta}{(z_1/z_2) + \cos\theta} \quad \text{... (7.10)}$$

where θ = Shaft angles = $\delta_1 + \delta_2$, and

δ_1 and δ_2 = Pitch angles of pinion and gear respectively.

2. Right angle bevel gears : If shaft angle $\theta = 90°$, then the bevel gears are known as right angle bevel gears. The pitch angles are given by

$$\tan\delta_2 = z_1/z_1 = i; \quad \text{and} \quad \delta_1 = 90° - \delta_2 \quad \text{... (7.11)}$$

3. Obtuse angle bevel gears : If shaft angle $\theta > 90°$, then the bevel gears are known as obtuse angle bevel gears. The pitch angles are given by

$$\tan\delta_1 = \frac{\sin(180° - \theta)}{(z_2/z_1) - \cos(180° - \theta)}$$

and

$$\tan\delta_2 = \frac{\sin(180° - \theta)}{(z_1/z_2) - \cos(180° - \theta)} \quad \text{... (7.12)}$$

7.4. VIRTUAL OR FORMATIVE OR EQUIVALENT NUMBER OF TEETH FOR BEVEL GEARS

In order to simplify the design calculation and analysis, bevel gears are replaced by equivalent spur gears. An imaginary spur gear considered in a plane perpendicular to the tooth at the larger end, is known as ***virtual or formative or equivalent spur gear.*** The virtual spur gear has pitch circle radius equal to the pitch cone radius 'R' of the bevel gear.

The number of teeth z_v on this imaginary spur gear is called ***virtual or formative or equivalent number of teeth.*** It is given by

$$z_v = \frac{z}{\cos\delta} \quad \text{... (7.13)}$$

where z = Actual number of teeth on the bevel gear.

Note The virtual number of teeth is used for selecting the cutters and in all design calculations of bevel gears.

7.5. PROPORTIONS FOR BEVEL GEARS

The proportions for the bevel gears are given below :

1. Addendum, $h_a = 1\ m_t$
2. Dedendum, $h_f = 1.2\ m_t$
3. Clearance, $c = 0.2\ m_t$
4. Working depth, $h_w = 2\ m_t$
5. Thickness of tooth $= 1.5708\ m_t$

where m_t = Transverse module.

7.6. BASIC DIMENSIONS OF BEVEL GEARS

The basic dimensions of straight bevel gears are listed in Table 7.1.

Table 7.1. Basic dimensions of bevel gears (from data book, page no. 8.38)

S.No.	Nomenclature	Symbol	Units	Formula
1.	Transverse module	m_t	mm	$m_t = m_m + \frac{b \sin\delta}{z}$
2.	Mean module	m_m	mm	$m_m = m_t - \frac{b \sin\delta}{z}$
3.	Normal module	m_n	mm	$m_n = m_t \times \cos\beta_m$
4.	Cone distance	R	mm	$R = \frac{m_t z_1}{2\sin\delta_1} = \frac{m_t z_2}{2\sin\delta_2} = 0.5\, m_t \sqrt{z_1^2 + z_2^2}$
5.	Reference or pitch diameter	d	mm	$d_1 = m_t \times z_1;\ d_2 = m_t \times z_2$
6.	Tip (or face) diameter	d_a	mm	$d_{a1} = m_t(z_1 + 2\cos\delta_1)$ $d_{a2} = m_t(z_2 + 2\cos\delta_2)$
7.	Face width	b	mm	$b \approx 0.3$ R or 10 m_t, whichever is smaller
8.	Number of teeth on crown wheel	z_{cw}	–	$z_{cw} = \frac{2R}{m_t}$

S.No.	Nomenclature	Symbol	Units	Formula
9.	Minimum number of teeth on pinion to avoid undercutting	z_u	–	$z_u = \frac{2 X \cos\delta}{\sin^2\phi}$; (X = 1, for uncorrected gears)
		ϕ	degrees	$\phi = 20°$ usually
10.	Pressure angle	β_m	degrees	β_2 = 30 to 35° ; 35° preferred
11.	Mean spiral angle	δ	degrees	$\tan\delta_2 = i$; $\delta_1 = 90° - \delta_2$
12.	Reference or pitch angle	θ_a	degrees	$\tan\theta_{a1} = \tan\theta_{a2} = \frac{m_t \times f_0}{R}$
13.	Addendum angle			
14.	Dedendum angle	θ_f	degrees	$\tan\theta_{f1} = \tan\theta_{f2} = \frac{m_t(f_0 + c)}{R}$
15.	Height factor	f_0	–	$f_0 = 1$
16.	Clearance	c	–	$c = 0.2$
17.	Tip angle	δ_a	degrees	$\delta_a = \delta + \theta_a$
18.	Root angle	δ_f	degrees	$\delta_f = \delta - \theta_f$
19.	Tooth height	h	mm	$h = h_a + h_f$
20.	Working depth	h_w	mm	$h_w = 2\, m_t$
21.	Addendum	h_a	mm	$h_a = m_t$
22.	Dedendum	h_f	mm	$h_f = 1.1236\, m_t$
23.	Virtual number of teeth	z_v	–	$z_v = \frac{z}{\cos\delta}$; $z_{v1min} = 18$ for $\phi = 20°$

Example 7.1 ***A pair of straight bevel gears consists of a 30 teeth pinion meshing with a 48 teeth gear. The gears are mounted on shafts, which are intersecting at right angle. The module at the large end of the tooth is 4 mm. Calculate :***

(i) the pitch circle diameters of the pinion and the gear;

(ii) the pitch angles for the pinion and gear; and

(iii) the cone distance.

Given Data : $z_1 = 30$; $z_2 = 48$; $m_t = 4$ mm ; $\theta = 90°$.

To find : (i) d_1 and d_2 ; (ii) δ_1 and δ_2 ; and (iii) R.

☺ ***Solution :*** $i = \frac{z_2}{z_1} = \frac{48}{30} = 1.6$

*(i) **Pitch circle diameters of the pinion and the gear (i.e., d_1 and d_2) :***

We know that, $d_1 = m_t \times z_1 = 4 \times 30 = $ **120 mm ;**

and $d_2 = m_t \times z_2 = 4 \times 48 = $ **192 mm Ans.**

*(ii) **Pitch angles for the pinion and gear (i.e., δ_1 and δ_2) :***

We know that, $\tan\delta_2 = i = 1.6$ or $\delta_2 = \tan^{-1}(1.6) = $ **58° Ans.**

Given that, $\theta = \delta_1 + \delta_2 = 90°$

$\therefore$ $\delta_1 = 90° - \delta_2 = 90° - 58° = $ **32° Ans.**

*(iii) **Cone distance (R) :***

We know that $R = 0.5\, m_t \sqrt{z_1^2 + z_2^2}$

$= 0.5 \times 4 \sqrt{30^2 + 48^2} = $ **113.21 mm Ans.**

7.7. FORCE ANALYSIS ON BEVEL GEARS

In force analysis of bevel gears, it is assumed that the resultant tooth force between two meshing gears is concentrated at the midpoint along the face width of the tooth. The forces acting at the centre of the tooth are shown in Fig.7.7.

The components of the resultant force are :

1. Tangential or useful component (F_t), and
2. Separating force (F_s) : It is resolved into two components. They are

   (i) Axial force (F_a), and

   (ii) Radial force (F_r).

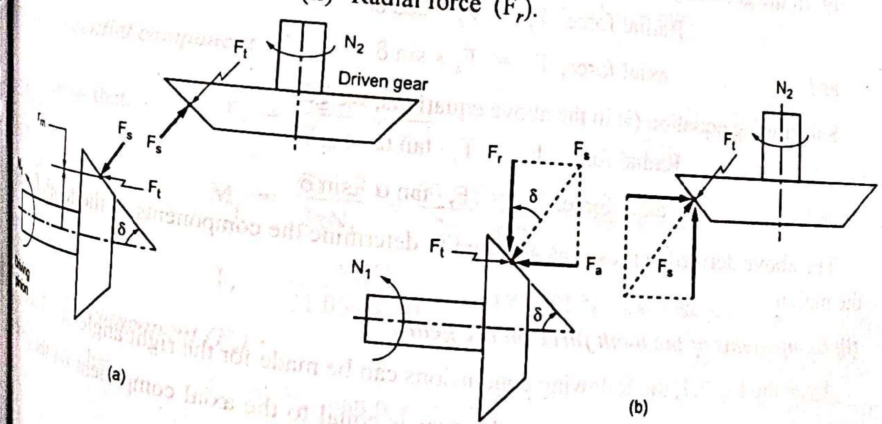


Fig. 7.7. Gear tooth forces

(i) Components of the tooth force on the pinion :

To find F_t : The tangential force can be determined using the familiar relationship

$$F_t = \frac{2\,M_t}{d_{1av}} = \frac{M_t}{r_m} \quad \ldots (7.14)$$

where M_t = Transmitted torque = $\frac{60 \times P}{2\pi N}$,

P = Power transmitted,

N = Speed of the gear,

d_{1av} = Average diameter of the pinion, at midpoint along the face width

$$= z_1 \cdot m_{av} = z_1\left(m_t - \frac{b \cdot \sin\delta}{z_1}\right), \text{ and}$$

$$r_m = \left(\frac{d_1}{2} - \frac{b \sin\delta_1}{2}\right)$$

= Mean radius of the pinion at midpoint along the face width

To find F_s : The analysis is similar to that of spur gears and the separating force can be determined using the relation

$$\text{Separating force, } F_s = F_t \times \tan\alpha \quad \ldots (i)$$

where α = Pressure angle

To find F_r and F_a : The separating force is further resolved into radial and axial forces as shown in Fig.7.7(b).

From the geometry of the Fig.7.7(b), we can write

$$\text{Radial force, } F_r = F_s \times \cos\delta \quad \ldots (ii)$$

$$\text{and} \quad \text{axial force, } F_a = F_s \times \sin\delta \quad \ldots (iii)$$

Substituting equation (i) in the above equations, we get

$$\text{Radial force, } F_r = F_t \cdot \tan\alpha \cdot \cos\delta \quad \ldots (7.15)$$

$$\text{and} \quad \text{axial force, } F_a = F_t \cdot \tan\alpha \cdot \sin\delta \quad \ldots (7.16)$$

The above derived expressions are used to determine the components of the tooth force on the pinion.

(ii) Components of the tooth force on the gear :

From the Fig.7.7, the following conclusions can be made for the right angle bevel gears :

✓ The radial component on the gear is equal to the axial component on the pinion but in opposite direction.

$$\therefore \ (F_r)_{gear} = -(F_a)_{pinion}$$

✓ **Similarly, the axial component on the gear is equal to the radial component on the pinion, but in opposite direction.**

$$\therefore \ (F_a)_{gear} = -(F_r)_{pinion}$$

Note The three forces F_t, F_r, and F_a are perpendicular to each other and can be used to determine the bearing loads by using the methods of statics.

Example 7.2 ***A pair of straight bevel gears has a velocity ratio of 2 : 1. The pitch circle diameter of the pinion is 80 mm at the large end of the tooth. A 5 kW power is supplied to the pinion, which rotates at 800 r.p.m. The face width is 40 mm and the pressure angle is 20°. Calculate the tangential, radial and axial components of the resultant tooth force acting on the pinion.***

Given Data : $i = 2$; $d_1 = 80$ mm ; P = 5 kW ; $N_1 = 800$ r.p.m. ; $b = 40$ mm ; $\alpha = 20°$.

To find : F_t, F_r and F_a on pinion.

☺ *Solution :* In order to calculate the force components, first let us find pitch angles (δ_1 and δ_2), and the mean radius of pinion.

We know that, $\tan\delta_2 = i = 2$, for shaft angle, $\theta = 90°$.

or $\delta_2 = \tan^{-1}(2) = 63.43°$

and $\delta_1 = 90° - \delta_2 = 90° - 63.43° = 26.57°$

The mean radius of the pinion at midpoint along the face width is given by,

$$r_m = \frac{d_1}{2} - \frac{b \cdot \sin\delta_1}{2}$$

$$= \frac{80}{2} - \frac{40 \times \sin 26.57°}{2} = 31.054 \text{ mm}$$

(i) Tangential component (F_t) :

We know that, $F_t = \frac{2\,M_t}{d_{1av}} = \frac{M_t}{r_m}$

where $M_t = \frac{60 \times P}{2\pi N_1} = \frac{60 \times 5 \times 10^3}{2\pi \times 800} = 59.68$ N-m

$$\therefore \ F_t = \frac{59.68}{31.054 \times 10^{-3}} = \mathbf{1921.91\ N} \quad \textbf{Ans.}$$

(ii) Radial component (F_r) :

We know that, $F_r = F_t \times \tan\alpha \times \cos\delta_1$

$$= 1921.91 \times \tan 20° \times \cos 26.57° = \mathbf{625.64\ N} \quad \textbf{Ans.}$$

(iii) Axial component (F_a) :

We know that, $F_a = F_t \times \tan\alpha \times \sin\delta_1$

$= 1921.91 \times \tan 20° \times \sin 26.57° = $ **312.89 N Ans.**

Example 7.3 ***For the data of above example, calculate the tangential, radial and axial components of the resultant tooth force acting on the gear wheel.***

Given Data : Refer Example 7.2 ; $N_2 = \frac{N_1}{i} = \frac{800}{2} = 400$ r.p.m.

To find : F_t, F_r and F_a on gear.

☺ *Solution :* $\delta_1 = 26.57°$ and $\delta_2 = 63.43°$ from Example 7.2.

(i) Tangential component (F_t) : We know that the tangential force on the pinion and gear are equal in magnitude and opposite in direction. Therefore,

$(F_t)_{gear} = (F_t)_{pinion} = $ **1921.91 N Ans.**

Note The above result can be verified as below.

$$(F_t)_{gear} = \frac{2\,(M_t)_{gear}}{d_{2av}} = \frac{2\,(M_t)_{gear}}{r_{m2}}$$

$$\text{where } (M_t)_{gear} = \frac{60 \times P}{2\pi N_2} = \frac{60 \times 5 \times 10^3}{2\pi \times 400} = 119.366 \text{ N-m}$$

r_{m2} = Mean radius of the gear at the midpoint

$$= \frac{d_2}{2} - \frac{b \cdot \sin\delta_2}{2} \qquad [\because d_2 = i \times d_1 = 2 \times 80 = 160 \text{ mm}]$$

$$= \frac{160}{2} - \frac{40 \times \sin 63.43°}{2} = 62.11 \text{ mm}$$

$$\therefore \quad (F_t)_{gear} = \frac{119.366}{62.11 \times 10^{-3}} = 1921.85 \text{ N, which is the same value.}$$

(ii) Radial component (F_r) :

We know that, $(F_r)_{gear} = F_t \times \tan\alpha \times \cos\delta_2$

$= 1921.91 \times \tan 20° \times \cos 63.43 = $ **312.88 N Ans.**

Note It can be seen that, $(F_r)_{gear} = (F_a)_{pinion}$.

(iii) Axial component (F_a) :

We know that, $(F_a)_{gear} = F_t \times \tan\alpha \times \sin\delta_2$

$= 1921.91 \times \tan 20° \times \sin 63.43 = $ **625.64 N Ans.**

Note It can be seen that, $(F_a)_{gear} = (F_r)_{pinion}$.

Example 7.4 ***A pair of bevel gears is to be used to transmit 10 kW from a pinion rotating at 420 r.p.m. to a gear mounted on a shaft which intersects the pinion shaft at an angle of 70°. Assuming that the pinion is to have an outside pitch diameter of 180 mm, a pressure angle of 20°, a face width of 45 mm, and the gear shaft is to rotate at 140 r.p.m., determine (i) the pitch angle for the gears; (ii) the forces on the pinion and gear; and (iii) the torque produced about the shaft axis.***

Given Data : P = 10 kW; $N_1 = 420$ r.p.m; $\theta = 70°$; $d_1 = 180$ mm; $\alpha = 20°$; $b = 45$ mm; $N_2 = 140$ r.p.m.

To find : (i) δ_1 and δ_2; (ii) F_t, F_r and F_a on pinion and gear; and (iii) $(M_t)_{gear}$.

☺ *Solution :* $i = \frac{N_1}{N_2} = \frac{420}{140} = 3 = \frac{z_2}{z_1}$

(i) Pitch angles for the gears (i.e., δ_1 and δ_2) :

We know that, $\tan\delta_1 = \frac{\sin\theta}{(z_2/z_1) + \cos\theta}$

$$= \frac{\sin 70°}{3 + \cos 70°} = 0.281 \qquad \ldots [\because \theta = 70°;\ \frac{z_2}{z_1} = i = 3]$$

or $\delta_1 = \tan^{-1}(0.281) = $ **15.7° Ans.**

We also know that, $\theta = \delta_1 + \delta_2$

or $\delta_2 = \theta - \delta_1 = 70° - 15.7° = $ **54.3° Ans.**

(ii) Forces on the pinion and the gear : Let us first find the mean radius of the pinion at midpoint.

We know that, $r_{m1} = \frac{d_1}{2} - \frac{b \cdot \sin\delta_1}{2}$

$$= \frac{180}{2} - \frac{45 \times \sin 15.7°}{2} = 83.91 \text{ mm}$$

Torque transmitted on the pinion, $M_{t1} = \frac{60 \times P}{2\pi N_1} = \frac{60 \times 10 \times 10^3}{2\pi \times 420} = 227.36$ N-m

(a) Forces on the pinion :

(i) Tangential force : $F_{t1} = \frac{M_{t1}}{r_{m1}} = \frac{227.36}{83.91 \times 10^{-3}} = $ **2709.62 N Ans.**

(ii) Radial force : $F_{r1} = F_{t1} \times \tan\alpha \times \cos\delta_1$

$= 2709.62 \times \tan 20° \times \cos 15.7° = $ **949.43 N Ans.**

(iii) *Axial force :* $F_{a1} = F_{t1} \times \tan\alpha \times \sin\delta_1$

$= 2709.62 \times \tan 20° \times \sin 15.7° =$ **266.87 N Ans.**

(b) Forces on the gear :

(i) *Tangential force :* $F_{t2} = F_{t1} =$ **2709.62 N Ans.**

(ii) *Radial force :* $F_{r2} = F_{t2} \times \tan\alpha \times \cos\delta_2$

$= 2709.62 \times \tan 20° \times \cos 54.3° =$ **575.5 N Ans.**

(iii) *Axial force :* $F_{a2} = F_{t2} \times \tan\alpha \times \sin\delta_2$

$= 2709.62 \times \tan 20° \times \sin 54.3° =$ **800.89 N Ans.**

(iii) Torque produced about the shaft axis (i.e., torque on the gear shaft) :

We know that, $M_{t2} = i \times M_{t1}$ $\quad \left[\because M_{t2} = \frac{60 \times P}{2\pi N_2} = \frac{N_1}{N_2}\left(\frac{60 \times P}{2\pi N_1}\right) = i \times M_{t1}\right]$

$= 3 \times 227.36 =$ **682.08 N-m Ans.**

Example 7.5 ***A pair of bevel gears transmitting 15 kW at 600 r.p.m. as shown in Fig.7.8(a). The pressure angle is 20°. Determine the components of the resultant gear tooth force and draw a free body diagram of forces acting on the pinion and the gear.***

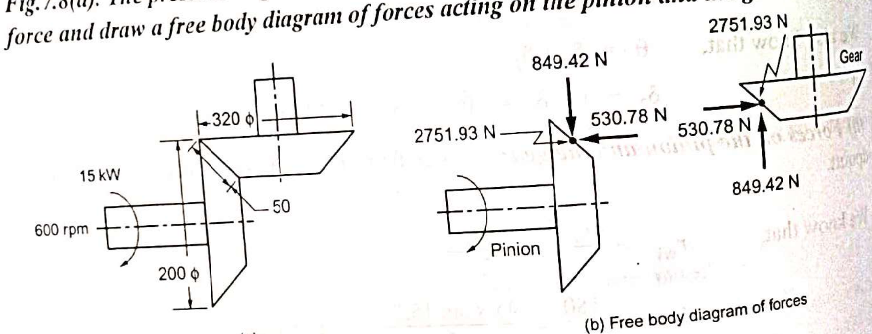


Fig. 7.8.

Given Data : $P = 15$ kW; $N_1 = 600$ r.p.m.; $\alpha = 20°$; $d_1 = 200$ mm; $d_2 = 320$ mm; $b = 50$ mm.

To find : 1. Components of the resultant gear tooth force, and

2. Draw a free body diagram acting on the pinion and gear.

☺ ***Solution :*** $i = \frac{N_1}{N_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1} = \frac{320}{200} = 1.6$

We know that, $\tan\delta_2 = i = 1.6$, for shaft angle $\theta = 90°$.

or $\delta_2 = \tan^{-1}(1.6) = 58°$

Then, $\delta_1 = 90° - \delta_1 = 90° - 58° = 32°$

Torque transmitted on pinion, $M_{t1} = \frac{60 \times P}{2\pi N_1} = \frac{60 \times 15 \times 10^3}{2\pi \times 600} = 238.73$ N-m

Mean radius of the pinion, $r_{m1} = \left[\frac{d_1}{2} - \frac{b \cdot \sin\delta_1}{2}\right]$

$= \left[\frac{200}{2} - \frac{50 \times \sin 32°}{2}\right] = 86.75$ mm

1. (i) Force components acting on the pinion :

✓ Tangential force : $F_{t1} = \frac{M_{t1}}{r_{m1}} = \frac{238.73}{86.75 \times 10^{-3}} =$ **2751.93 N Ans.**

✓ Radial force : $F_{r1} = F_{t1} \times \tan\alpha \times \cos\delta_1$

$= 2751.93 \times \tan 20° \times \cos 32° =$ **849.42 N Ans.**

✓ Axial force : $F_{a1} = F_{t1} \times \tan\alpha \times \sin\delta_1$

$= 2751.93 \times \tan 20° \times \sin 32° =$ **530.78 N Ans.**

(ii) Force components acting on the gear :

✓ Tangential force : $F_{t2} = F_{t1} = 2751.93$ N

✓ Radial force : $F_{r2} = F_{t2} \times \tan\alpha \times \cos\delta_2$

$= 2751.93 \times \tan 20° \times \cos 58° =$ **530.78 N Ans.**

✓ Axial force : $F_{a2} = F_{t2} \times \tan\alpha \times \sin\delta_2$

$= 2751.93 \times \tan 20° \times \sin 58° =$ **849.42 N Ans.**

2. Free body diagram of the forces :

The free body diagram of forces acting on the pinion and the gear are drawn, as shown in Fig.7.8(b).

Example 7.6 ***A bevel pinion shown in Fig.7.9 rotates at 600 r.p.m. in the direction shown and transmits 3.75 kW. The mounting distances, the location of all bearings, and the average pitch radii of the pinion and gear are shown in Fig.7.9. For simplicity, the teeth have been replaced by the pitch cones. Bearings A and C should take the radial and thrust loads, while bearings B and D can only take radial loads. Find the bearing forces (or reactions at the bearings) on the gearshaft.***

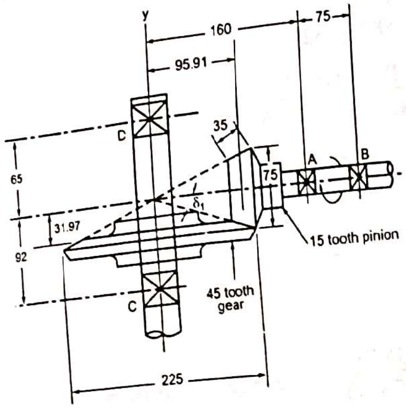


Fig. 7.9.

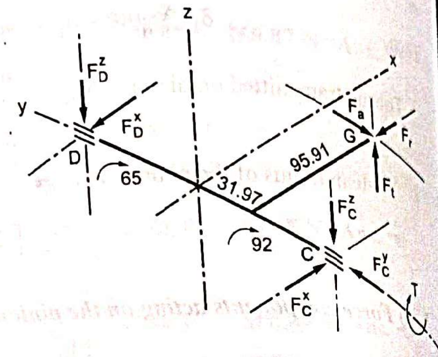


Fig. 7.10. Forces acting on gear shaft

Given Data : $N_1 = 600$ r.p.m ; $P = 3.75$ kW ; $d_1 = 75$ mm; $b = 35$ mm; $z_1 = 15$; $z_2 = 45$; $d_2 = 225$ mm.

To find : Bearing forces on the gearshaft.

☺ ***Solution :*** Gear ratio, $i = \dfrac{N_1}{N_2} = \dfrac{z_2}{z_1} = \dfrac{d_2}{d_1} = 3$

We know that, $\tan \delta_2 = i$, for right angle bevel gears

or $\delta_2 = \tan^{-1}(i) = \tan^{-1}(3) = 71.56°$

Then, $\delta_1 = 90° - \delta_2 = 90° - 71.56° = 18.43°$

Then, mean radius of the pinion, $r_m = \dfrac{d_1}{2} - \dfrac{b \cdot \sin \delta_1}{2}$

$$= \frac{75}{2} - \frac{35 \times \sin 18.43°}{2} = 31.97 \text{ mm}$$

Torque transmitted to the pinion, $M_{t1} = \dfrac{60 \times P}{2\pi N_1} = \dfrac{60 \times 3.75 \times 10^3}{2\pi \times 600} = 59.68$ N-m

Forces acting on the pinion :

✓ Tangential force : $F_{t1} = \dfrac{M_{t1}}{r_{m1}} = \dfrac{59.68}{31.97 \times 10^{-3}} = 1866.75$ N

✓ Radial force : $F_{r1} = F_{t1} \times \tan\alpha \times \cos\delta_1$

$$= 1866.75 \times \tan 20° \times \cos 18.43° = 644.59 \text{ N}$$

✓ Axial force : $F_{a1} = F_{t1} \times \tan\alpha \times \sin\delta_1$

$$= 1866.75 \times \tan 20° \times \sin 18.43° = 218.8 \text{ N}$$

Forces acting on the gear :

✓ Tangential force : $F_{t2} = F_{t1} = 1866.75$ N

✓ Radial force : $F_{r2} = F_{a1} = 218.8$ N

✓ Axial force : $F_{a2} = F_{r1} = 644.59$ N

Forces acting on the gearshaft : The forces acting on the gearshaft is shown in Fig.7.10.

Forces in the xy plane :

For $\Sigma x = 0$, *i.e.*, considering equilibrium of forces, we get

$$F_D^x + F_{r2} = F_C^x \quad \text{... (i)}$$

Taking moments of forces about bearing D, we get

$$F_C^x (92 + 65) - F_{r2} \times (r_m)_{pinion} - F_{a2} \times (r_m)_{gear} = 0$$

$$[\because (r_m)_{gear} = i \times (r_m)_{pinion} = 3 \times 31.97 = 95.91 \text{ mm}]$$

or $F_C^x (92 + 65) - 218.8 - 8 \times (31.97 + 65) - 644.59 \times 95.91 = 0$

or $F_C^x = \mathbf{528.91\ N}$ **Ans.**

Now from equation (i), we have

$F_D^x + 218.8 = 528.91$ N or $F_D^x = \mathbf{310.11\ N}$ **Ans.**

Forces in the xz plane : For $\Sigma y = 0$, $F_C^y = F_{a2} = \mathbf{644.59\ N}$ **Ans.**

For $\Sigma z = 0$, *i.e.*, considering equilibrium of forces, we get

$$F_D^z + F_C^z = F_{t2} \quad \text{... (ii)}$$

Taking moments of forces about bearing D, we get

$$F_C^z \times (92 + 65) = F_{t2} \times 95.91$$

or $F_C^z = \dfrac{1866.75 \times 95.91}{(92 + 65)} = \mathbf{1140.38\ N}$ **Ans.**

Now from equation (ii), we have

$F_D^z + 1140.38 = 1866.75$ or $F_D^z = \mathbf{726.37\ N}$ **Ans.**

Resultant bearing force :

Resultant bearing force at bearing C, $F_C = \sqrt{(F_C^x)^2 + (F_C^y)^2 + (F_C^z)^2}$

$$= \sqrt{(528.91)^2 + (644.59)^2 + (1140.38)^2}$$

$$= \mathbf{1412.69\ N} \quad \textbf{Ans.}$$

and resultant bearing force at bearing D,

$$F_D = \sqrt{(F_D^x)^2 + (F_D^y)^2 + (F_D^z)^2}$$

$$= \sqrt{(310.11)^2 + 0 + (726.37)^2} = \textbf{789.8 N} \quad \textbf{Ans.}$$

Example 7.7 ***For the data of Example 7.6, determine the bearing forces on the pinion shaft i.e., reactions at the bearing A and B. Assume that the bearing A can take radial as well as thrust load, while the bearing B can only take radial load.***

Given Data : Refer Example 7.6.

To find : Bearing forces on the pinion shaft (F_A an F_B)

☺ ***Solution :*** Refer Fig.7.9.

The forces acting on the pinion shaft is shown in Fig.7.11.

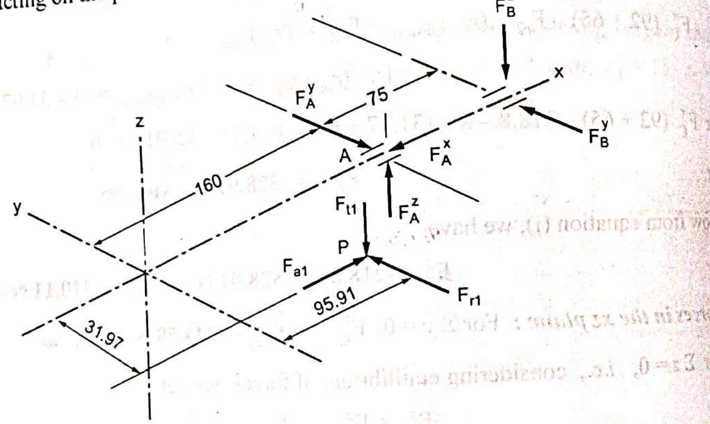


Fig. 7.11.

Forces in the xy plane : For $\Sigma y = 0$, *i.e.*, considering equilibrium of forces, we get

$$F_A^y = F_{r1} + F_B^y \quad \text{... (i)}$$

Taking moments of forces about bearing B, we get

$$F_{a1} \times 31.97 - F_{r1} \times [(160 - 95.91) + 75] + F_A^y \times 75 = 0$$

or $$218.8 \times 31.97 - 644.59 \times 139.09 + F_A^y \times 75 = 0$$

or $$F_A^y = \textbf{1102.15 N} \quad \textbf{Ans.}$$

Now from equation (i), we have

$$F_B^y = 1102.15 - 644.59 = \textbf{457.56 N} \quad \textbf{Ans.}$$

For $\Sigma x = 0$, $$F_A^x = F_{a1} = \textbf{218.8 N} \quad \textbf{Ans.}$$

Forces in the xz plane : For $\Sigma z = 0$, *i.e.*, considering equilibrium of forces, we get

$$F_A^z = F_{t1} + F_B^z$$

Taking moments of forces about bearing B, we get

$$F_{t1} \times [(160 - 95.91) + 75] - F_A^z \times 75 = 0 \quad \text{... (ii)}$$

or $$1866.75 \times 139.09 = F_A^z \times 75$$

or $$F_A^z = \textbf{3461.95 N} \quad \textbf{Ans.}$$

Now from equation (ii), we have

$$F_B^z = F_A^z - F_{t1} = 3461.95 - 1866.75 = \textbf{1595.2 N} \quad \textbf{Ans.}$$

Resultant bearing force :

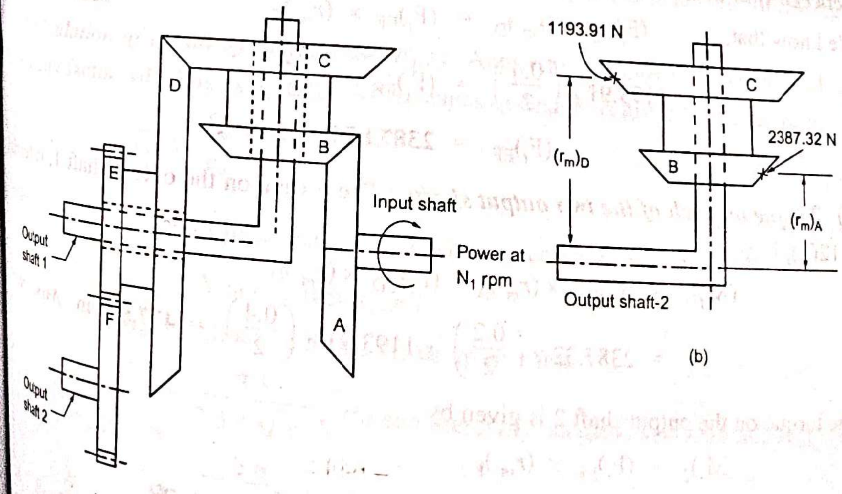
Resultant bearing force at bearing A, $F_A = \sqrt{(218.8)^2 + (1102.15)^2 + (3461.95)^2}$

$$= \textbf{3639.74 N} \quad \textbf{Ans.}$$

and resultant bearing force at bearing B, $F_B = \sqrt{(457.56)^2 + (1595.2)^2}$

$$= \textbf{1659.52 N} \quad \textbf{Ans.}$$

Example 7.8 ***A differential planetary gear train is shown in Fig.7.12(a). The input shaft receives 7.5 kW power at 300 r.p.m. The pitch circle diameters of bevel gears A, B, C and D at the midpoint along the face width are 200, 100, 200 and 400 mm respectively. The pitch circle diameters of spur gears E and F are 200 and 300 mm respectively. The gears rotate at constant speed. Determine the various forces acting on various gears and the torque on each of the two output shafts.***



(a) Differential planetary gear system

Fig. 7.12.

Given Data : P = 7.5 kW ; N_1 = 300 r.p.m ; $(d_m)_A$ = 200 mm ; $(d_m)_B$ = 100 mm ; $(d_m)_C$ = 200 mm ; $(d_m)_D$ = 400 mm ; d_E = 200 mm ; d_F = 300 mm.

To find : (i) Tangential forces acting on various gears, and

(ii) Torque on each of the two output shafts.

☺ ***Solution :*** The torque acting on the input shaft is given by

$$(M_t)_1 = \frac{60 \times P}{2\pi N_1} = \frac{60 \times 7.5 \times 10^3}{2\pi \times 300} = 238.73 \text{ N-m}$$

(i) Tangential forces acting on various gears :

Between gears A and B :

$$(F_t)_{AB} = (F_t)_{BA} = \frac{(M_t)_1}{(r_m)_A} = \frac{2 \times (M_t)_1}{(d_m)_A}$$

$$= \frac{2 \times 238.73}{200 \times 10^{-3}} = \mathbf{2387.32\ N} \quad \textbf{Ans.}$$

Between gears C and D : We know that, for constant speed,

$$(F_t)_{AB} \times (r_m)_B = (F_t)_{CD} \times (r_m)_C$$

or $$2387.82 \times \left(\frac{0.100}{2}\right) = (F_t)_{CD} \times \left(\frac{0.200}{2}\right)$$

∴ $$(F_t)_{CD} = \mathbf{1193.91\ N} \quad \textbf{Ans.}$$

Between spur gears E and F :

We know that, $$(F_t)_{CD} \times (r_m)_D = (F_t)_{EF} \times (r_m)_E$$

or $$1193.91 \times \left(\frac{0.4}{2}\right) = (F_t)_{EF} \times \left(\frac{0.2}{2}\right)$$

∴ $$(F_t)_{EF} = \mathbf{2387.82\ N} \quad \textbf{Ans.}$$

(ii) Torque on each of the two output shafts : The torque on the output shaft 1, referring Fig.7.12(b), is given by

$$(M_t)_1 = (F_t)_{BA} \times (r_m)_A + (F_t)_{CD} \times (r_m)_D$$

$$= 2387.32 \times \left(\frac{0.2}{2}\right) + 1193.91 \times \left(\frac{0.4}{2}\right) = \mathbf{477.5\ N\text{-}m} \quad \textbf{Ans.}$$

The torque on the output shaft 2 is given by

$$(M_t)_2 = (F_t)_{EF} \times (r_m)_F$$

$$= 2387.82 \times \left(\frac{0.3}{2}\right) = \mathbf{358.17\ N\text{-}m} \quad \textbf{Ans.}$$

DESIGN OF BEVEL GEARS

I. BEVEL GEAR DESIGN USING LEWIS AND BUCKINGHAM'S EQUATION (Helical Gear Design Recommended by AGMA)

7.8. BEAM STRENGTH OF BEVEL GEARS (OR LEWIS EQUATION)

Since the tooth thickness varies along its length, a modified Lewis equation of beam strength is used for bevel gears. It is given by

$$\text{Beam strength, } F_s = \pi \times m_t \times b \times [\sigma_b] \times y' \left(\frac{R-b}{R}\right) \quad \text{... (7.17)}$$

where m_t = Transverse module,

b = Face width = 10 m_t or 0.3 R,

$[\sigma_b]$ = Permissible or allowable static stress, from Table 5.4,

y' = Lewis form factor based on virtual number of teeth, and

R = Cone distance = $0.5\, m_t \sqrt{z_1^2 + z_2^2}$

Note The factor $\left(\frac{R-b}{R}\right)$ may be called as ***bevel factor.***

7.9. DYNAMIC LOAD ON BEVEL GEAR TOOTH (Effective Load on Gear Tooth)

As discussed in Section 6.10, in order to account for dynamic loads, the following two methods are used.

1. ***Calculation of initial dynamic load (F_D) :*** Approximate value of dynamic load, using the velocity factor, which is used in the initial stages of design, is given by the relation

$$F_d = \frac{F_t}{C_v} \quad \text{... (7.18)}$$

where F_t = Tangential load considering service factor = $\frac{P}{v} \times K_0$,

C_v = Velocity factor, and

K_0 = Shock / service factor, from Table 5.6.

$= \frac{3.5}{3.5 + \sqrt{v}}$, for commercially cut gears and $v \leq 5$ m/s

$= \frac{5.6}{5.6 + \sqrt{v}}$, for generated teeth.

2. ***Buckingham's equation for dynamic load :*** Buckingham's equation, used for accurate estimation of dynamic load, is given by

Dynamic load, $F_d = F_t + \dfrac{21\,v\,(bc + F_t)}{21\,v + \sqrt{bc + F_t}}$ ... (7.19)

where F_t = Tangential load neglecting service factor $= \dfrac{P}{v}$,

v = Pitch line velocity,

b = Face width, and

c = Deformation or dynamic factor, from Tables 5.7(a) and (b).

7.10. WEAR STRENGTH OF BEVEL GEARS (Wear Tooth Load)

The modified wear strength equation to suit bevel gears is given by

Wear load, $F_w = \dfrac{0.75\, d_1 \times b \times Q' \times K_w}{\cos \delta_1}$ ... (7.20)

where d_1 = Pitch circle diameter of the big end of the pinion,

b = Face width,

Q' = Ratio factor, based on virtual number of teeth

$= \dfrac{2 \times z_{v2}}{z_{v1} + z_{v2}}$, for external gears

$= \dfrac{2 \times z_{v2}}{z_{v2} - z_{v1}}$, for internal gears

K_w = Load stress factor, from Table 5.9 (or)

$= \left[\dfrac{f_{es}^2 \times \sin \alpha}{1.4}\right]\left[\dfrac{1}{E_p} + \dfrac{1}{E_g}\right]$

where f_{es} = Surface endurance limit, from Table 5.9,

α = Pressure angle, and

E_p and E_g = Young's modulus of pinion and gear respe...

δ_1 = Pitch cone angle of the pinion.

Note: In the wear load formula, use d_1 and δ_1, irrespective of whether pinion or gear is ...

7.11. DESIGN PROCEDURE

(1) Lewis & Buckingham eq

The design procedure for bevel gears are the same as for spur gears.

1. Select the materials.
2. Calculate z_1 and z_2. If not given, assume $z_1 \geq 17$.

3. Calculate the pitch angles (*i.e.*, δ_1 and δ_2) and the virtual number of teeth (*i.e.*, z_{v1} and z_{v2}) using the following relations.

   ✓ *Pitch angles :* $\tan \delta_2 = i$ and $\delta_1 = 90° - \delta_2$, for right angle bevel gears

   ✓ $z_{v1} = \dfrac{z_1}{\cos \delta_1}$ and $z_{v2} = \dfrac{z_2}{\cos \delta_2}$

4. Calculate the tangential load on tooth using the relation $F_t = \dfrac{P}{v} \times K_0$
5. Calculate the preliminary value of dynamic load using the relation $F_d = \dfrac{F_t}{c_v}$.
6. Calculate the beam strength F_s in terms of transverse module using the relation

   $$F_s = \pi \times m_t \times b \times [\sigma_b] \times y' \times \left(\frac{R - b}{R}\right)$$

   Initially assume $b = 10\, m_t$.

7. Calculate the transverse module m_t by equating F_s and F_d.
8. Calculate the values of b, d_1 and v using the following relations :

   ✓ Face width : $b = 10\, m_t$

   ✓ Pitch circle diameter : $d_1 = m_t \times z_1$

   ✓ Pitch line velocity : $v = \dfrac{\pi\, d_1\, N_1}{60}$

9. Recalculate the beam strength using the relation

   $$F_s = \pi \times m_t \times b \times [\sigma_b] \times y' \times \left(\frac{R - b}{R}\right)$$

10. Calculate the dynamic load more accurately using Buckingham's equation,

    $$F_d = F_t + \frac{21\, v\, (bc + F_t)}{21\, v + \sqrt{bc + F_t}}$$

11. Check for beam strength (or tooth breakage). If $F_d \leq F_s$, the gear tooth has adequate beam strength and will not fail by breakage. Thus the design is satisfactory.
12. Calculate the maximum wear load using the relation $F_w = \dfrac{0.75 \times d_1 \times b \times Q' \times K_w}{\cos \delta_1}$
13. Check for wear strength. If $F_d \leq F_w$, the gear tooth has adequate wear capacity and will not wear out. Thus the design is safe and satisfactory.
14. Calculate the basic dimensions of pinion and gear using the Table 7.1.

Example 7.9 *Design a pair of bevel gears to transmit 10 kW at a pinion speed of 1440 r.p.m. Required transmission ratio is 4. Material for gears is 15 Ni 2Cr 1 Mo 15/Steel. The tooth profiles of the gears are of 20° composite form.*

Given Data : P = 10 kW; N_1 = 1440 r.p.m. ; i = 4 ; $\alpha = 20°$.

To find : Design the pair of bevel gears.

Solution : Since the same material is used for both pinion and gear, the pinion is weaker than the gear. Therefore, ***we have to design only pinion.***

1. ***Material for gears :*** 15 Ni 2 Cr 1 Mo 15 ... (Given)

2. ***Calculation of z_1 and z_2 :*** Assume $z_1 = 20$, then $z_2 = i \times z_1 = 4 \times 20 = 80$

3. ***Calculation of pitch angles and virtual number of teeth :***

✓ Pitch angles : We know that, $\tan \delta_2 = i = 6$ or $\delta_2 = \tan^{-1}(4) = 75.96°$

Then, $\delta_1 = 90° - \delta_2 = 90° - 75.96° = 14.04°$

✓ To find z_{v1} and z_{v2} :

$$z_{v1} = \frac{z_1}{\cos \delta_1} = \frac{20}{\cos 14.04°} = 20.61 \approx 21, \text{ and}$$

$$z_{v2} = \frac{z_2}{\cos \delta_2} = \frac{80}{\cos 75.96°} = 329.76 \approx 330.$$

4. ***Calculation of tangential load on tooth (F_t) :***

We know that, $F_t = \frac{P}{v} \times K_0$

where
$$v = \frac{\pi d_1 N_1}{60} = \frac{\pi N_1}{60}\left(\frac{m_t \times z_1}{1000}\right) \ldots [\because d_1 = m_t \times z_1 \text{ and } m_t \text{ is in mm}]$$

$$= \frac{\pi \times 1440}{60}\left(\frac{m_t \times 20}{1000}\right) = 1.508\, m_t \text{ m/s}$$

K_0 = 1.25, assuming medium shock, from Table 5.6.

$$\therefore \quad F_t = \frac{10 \times 10^3}{1.508\, m_t} \times 1.25 = \frac{8289.32}{m_t}$$

5. ***Calculation of initial dynamic load (F_d) :***

We know that, $F_d = \frac{F_t}{c_v}$

where $c_v = \frac{5.6}{5.6 + \sqrt{v}}$, for generated teeth

$$= \frac{5.6}{5.6 + \sqrt{5}} = 0.714, \text{ assuming } v = 5 \text{ m/s}$$

$$\therefore \quad F_d = \left(\frac{8289.32}{m_t}\right) \times \frac{1}{0.714} = \frac{11599.23}{m_t}$$

6. ***Calculation of beam strength (F_s) :***

We know that, $F_s = \pi \times m_t \times b \times [\sigma_b] \times y' \times \left(\frac{R - b}{R}\right)$

where $b = 10\, m_t$

$[\sigma_b]$ = 450 N/mm², for alloy steel, from Table 5.4. ... (initially assumed)

y' = Form factor based on virtual number of teeth

$$= 0.154 - \frac{0.912}{z_{v1}}, \text{ for } 20° \text{ full depth}$$

$$= 0.154 - \frac{0.912}{21} = 0.1106$$

$$R = \text{Cone distance} = 0.5\, m_t \sqrt{z_1^2 + z_2^2}$$

$$= 0.5 \times m_t \sqrt{20^2 + 80^2} = 41.23\, m_t$$

$$\therefore \quad F_s = \pi \times m_t \times 10\, m_t \times 450 \times 0.1106 \times \left(\frac{41.23\, m_t - 10\, m_t}{41.23\, m_t}\right)$$

$$= 1184.38\, m_t^2$$

7. ***Calculation of transverse module (m_t) :***

We know that, $F_s \geq F_d$

$$1184.38\, m_t^2 \geq \frac{11599.23}{m_t}$$

or $m_t \geq 2.14$

From Table 5.8, the nearest higher standard transverse module is 3 mm.

8. ***Calculation of b, d_1 and v :***

✓ Face width : $b = 10\, m_t = 10 \times 3 = 30$ mm

✓ Pitch circle diameter : $d_1 = m_t \times z_1 = 3 \times 20 = 60$ mm

✓ Pitch line velocity : $v = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 60 \times 10^{-3} \times 1440}{60} = 4.52$ m/s

9. ***Recalculation of beam strength :*** We know that,

$$F_s = 1184.38\, m_t^2 = 1184.38 \times 3^2 = 10659 \text{ N}$$

10. *Calculation of accurate dynamic load* (F_d) : We know that,

$$F_d = F_t + \frac{21\,v\,(bc + F_t)}{21\,v + \sqrt{bc + F_t}}$$

where $F_t = \frac{P}{v} = \frac{10 \times 10^3}{4.52} = 2212.4$ N

c = Deformation factor, from Tables 5.7(a) and (b)

= $11860\,e$, for steel and steel and 20° FD, from Table 5.7(a), and

e = 0.0125, for precision gears and m_t upto 4, from Table 5.7(b).

∴ $c = 11860 \times 0.0125 = 148.25$ N/mm

Then, $$F_d = 2212.4 + \frac{21 \times 4.52 \times 10^3\,(30 \times 148.25 + 2212.4)}{21 \times 4.52 \times 10^3 + \sqrt{30 \times 148.25 + 2212.4}}$$

$= 8866.58$ N

11. *Check for beam strength (or tooth breakage)* : We find $\mathbf{F_s > F_d}$. It means the gear tooth has adequate beam strength and will not fail by breakage. Thus ***the design is satisfactory.***

12. *Calculation of maximum wear load* (F_w) :

We know that, $$F_w = \frac{0.75 \times d_1 \times b \times Q' \times K_w}{\cos \delta_1}$$

where Q' = Ratio factor $= \frac{2 \times z_{v2}}{z_{v1} + z_{v2}} = \frac{2 \times 330}{21 + 330} = 1.88$

K_w = 2.553 N/mm², for steel gears hardened to 400 BHN, from Table 5.9.

∴ $$F_w = \frac{0.75 \times 60 \times 30 \times 1.88 \times 2.553}{\cos 14.04°} = 6679 \text{ N}$$

13. *Check for wear* : Since $F_w < F_d$, the design is unsatisfactory.

***Trial 2* :** Now we have to increase the transverse module to 5 mm. Repeating from step again, we get

$b = 10 \times m_t = 10 \times 5 = 50$ mm

$d_1 = m_t \times z_1 = 5 \times 20 = 100$ mm

$v_1 = \frac{\pi \times 0.1 \times 1440}{60} = 7.54$ m/s

$$F_s = \pi \times m_t \times b \times [\sigma_b] \times y' \times \left(\frac{R - b}{R}\right)$$

$$= \pi \times 5 \times 50 \times 450 \times 0.1106 \times \left(\frac{41.23 \times 5 - 10 \times 5}{41.23 \times 5}\right) = 29608.5\text{N}$$

$F_t = \frac{P}{v} = \frac{10 \times 10^3}{7.54} = 1326.26$ N

$$F_d = 1326.26 + \frac{21 \times 7.54 \times 10^3\,(50 \times 148.25 + 1326.26)}{21 \times 7.54 \times 10^3 + \sqrt{50 \times 148.25 + 1326.26}}$$

$= 10059.9$ N

We find $\mathbf{F_s > F_d}$, so ***the design is safe against beam strength.***

$$F_w = \frac{0.75 \times 100 \times 50 \times 1.88 \times 2.553}{\cos 14.04°} = 18552.89 \text{ N}$$

Now we find $\mathbf{F_w > F_d}$. It means the gear tooth has adequate wear capacity and will not wear out. Thus ***the design is safe against wear failure also.***

14. *Calculation of basic dimensions of pinion and gear* : Refer Table 7.1.

✓ Transverse module : $\mathbf{m_t = 5}$ **mm**

✓ Number of teeth : $\mathbf{z_1 = 20}$; and $\mathbf{z_2 = 80}$

✓ Pitch circle diameter : $\mathbf{d_1 = 100}$ **mm** ; and $d_2 = m_t \times z_2 = 5 \times 80 = \mathbf{400}$ **mm**

✓ Cone distance : $R = 0.5\,m_t \sqrt{z_1^2 + z_2^2} = 0.5 \times 5 \sqrt{20^2 + 80^2}$

$= \mathbf{206.15}$ **mm**

✓ Face width : $b = 10\,m_t = 10 \times 5 = \mathbf{50}$ **mm**

✓ Pitch angles : $\mathbf{\delta_1 = 14.04°}$; and $\mathbf{\delta_2 = 75.96°}$

✓ Tip diameter : $d_{a1} = m_t\,(z_1 + 2\cos\delta_1)$

$= 5\,(20 + 2 \times \cos 14.04°) = \mathbf{109.7}$ **mm** ; and

$d_{a2} = m_t\,(z_2 + 2\cos\delta_2)$

$= 5\,(80 + 2 \times \cos 75.96°) = \mathbf{402.43}$ **mm.**

✓ Height factor : $\mathbf{f_0 = 1}$

✓ Clearance : $\mathbf{c = 0.2}$

✓ Addendum angle : $\tan\theta_{a1} = \tan\theta_{a2} = \frac{m_t \times f_0}{R}$

$= \frac{5 \times 1}{206.15} = 0.02425$

or $\mathbf{\theta_{a1} = \theta_{a2} = 1.4°}$

✓ Dedendum angle : $\tan\theta_{f1} = \tan\theta_{f2} = \frac{m_t\,(f_0 + c)}{R} = \frac{5\,(1 + 0.2)}{206.15} = 0.0291$

or $\mathbf{\theta_{f1} = \theta_{f2} = .67°}$

✓ Tip angle :

$$\delta_{a1} = \delta_1 + \theta_{a1} = 14.04° + 1.4° = \mathbf{15.44°}; \text{ and}$$

$$\delta_{a2} = \delta_2 + \theta_{a2} = 75.96° + 1.4° = \mathbf{77.36°}$$

✓ Root angle :

$$\delta_{f1} = \delta_1 - \theta_{f1} = 14.04° - 1.67° = \mathbf{12.37°}; \text{ and}$$

$$\delta_{f2} = \delta_2 - \theta_{f2} = 75.96° - 1.67° = \mathbf{74.29°}$$

✓ Virtual number of teeth : $z_{v1} = \mathbf{21}$; and $z_{v2} = \mathbf{330}$

(3) **Example 7.10** ***A pair of 20° full depth involute teeth bevel gears connect two shafts at right angles having a velocity ratio 3.2 :1. The gear is made of cast steel with an allowable static stress as 72 N/mm² and the pinion is made of steel having a static stress of 100 N/mm². The pinion transmits 40 kW at 840 r.p.m. Find the module, face width and pitch diameter from the stand point of beam strength and check the design from the stand point of wear.***

Given Data : $\alpha = 20°$; $\theta = 90°$; $i = 3.2$; $[\sigma_{b2}] = 72$ N/mm² ; $[\sigma_{b1}] = 100$ N/mm²; P = 40 kW ; $N_1 = 840$ r.p.m.

To find : Module, face width and pitch diameter of the gears.

☺ *Solution :* Since the materials of pinion and gear are different, we have to evaluate $[\sigma_{b1}]y_1$ and $[\sigma_{b2}]y_2$ to find out the weaker element.

Assume $z_1 = 20$, then $z_2 = i \times z_1 = 3.2 \times 20 = 64$.

The pitch angles are given by

$$\tan\delta_2 = i = 3.2 \text{ or } \delta_2 = \tan^{-1}(3.2) = 72.64°$$

Then,

$$\delta_1 = 90° - \delta_2 = 90° - 72.64° = 17.36°$$

The virtual number of teeth on gears are given by

$$z_{v1} = \frac{z_1}{\cos\delta_1} = \frac{20}{\cos 17.36°} \approx 21$$

and

$$z_{v2} = \frac{z}{\cos\delta_2} = \frac{64}{\cos 72.64°} \approx 215$$

Then form factors based on virtual number of teeth are given by

$$y_1' = 0.154 - \frac{0.912}{z_{v1}} = 0.154 - \frac{0.912}{21} = 0.1106$$

and

$$y_2' = 0.154 - \frac{0.912}{z_{v2}} = 0.154 - \frac{0.912}{215} = 0.1497$$

For pinion : $[\sigma_{b1}] \times y_1' = 100 \times 0.1106 = 11.06$ N/mm²

For gear : $[\sigma_{b2}] \times y_2' = 72 \times 0.1497 = 10.78$ N/mm²

We find, $[\sigma_{b2}]y_2' < [\sigma_{b1}]y_1'$. It means, the gear is weaker than the pinion. Thus we have to ***design the gear only.***

1. ***Material :*** Pinion – steel, and Gear – cast steel.

2. $z_1 = 20$; and $z_2 = 64$.

3. $\delta_1 = 17.36°$; $\delta_2 = 72.64°$, $z_{v1} = 21$; and $z_{v2} = 215$.

4. ***Calculation of tangential load :***

We know that,

$$F_t = \frac{P}{v} \times K_0$$

where

$$v = \frac{\pi d_1 N_1}{60} = \frac{\pi \times N_1}{60} \times \left(\frac{m_t \times z_1}{1000}\right)$$

$$= \frac{\pi \times 840}{60} \times \left(\frac{m_t \times 20}{1000}\right) = 0.879\, m_t$$

$K_0 = 1.25$, assuming medium shock, from Table 5.6.

$$\therefore \quad F_t = \frac{40 \times 10^3}{0.879\, m_t} \times 1.25 = \frac{56841}{m_t}$$

5. ***Calculation of initial dynamic load :***

We know that,

$$F_d = \frac{F_t}{c_v}$$

where

$$c_v = \frac{5.6}{5.6 + \sqrt{v}}, \text{ for generated teeth}$$

$$= \frac{5.6}{5.6 + \sqrt{5}} = 0.715, \text{ assuming } v = 5 \text{ m/s}$$

$$\therefore \quad F_d = \frac{56841}{m_t} \times \frac{1}{0.715} = \frac{79497.9}{m_t}$$

6. ***Calculation of beam strength :***

We know that,

$$F_s = \pi \times m_t \times b \times [\sigma_{b2}] \times y_2' \times \left(\frac{R - b}{R}\right)$$

where

$b = 10\, m_t$ ... *(initially assumed)*

$[\sigma_{b2}] = 72$ N/mm², ... *(given)*

$y_2' = 0.1497$, ... *(already calculated)*

$$R = 0.5\, m_t \sqrt{z_1^2 + z_2^2}$$

$= 0.5\, m_t \sqrt{20^2 + 64^2} = 33.53\, m_t$

$F_s = \pi \times m_t \times 10\, m_t \times 72 \times 0.1497 \times \left(\frac{33.53\, m_t - 10\, m_t}{33.53\, m_t}\right)$

$\therefore \quad = 237.62\, m_t^2$

7. Calculation of transverse module (m_t) :

We know that, $F_s \geq F_d$

$237.62\, m_t^2 \geq \frac{79497.9}{m_t}$

or $m_t \geq 6.94$ mm

From Table 5.8, the nearest higher value of transverse module is 7 mm.

8. Calculation of b, d_1 and v :

✓ Face width : $b = 10\, m_t = 10 \times 7 = 70$ mm

✓ Pitch circle diameter : $d_1 = m_t \times z_1 = 7 \times 20 = 140$ mm

✓ Pitch line velocity : $v_2 = v_1 = \frac{\pi\, d_1\, N_1}{60} = \frac{\pi \times 0.140 \times 840}{60} = 6.16$ m/s

9. Recalculation of beam strength :

$F_s = 237.62 \times m_t^2 = 237.62 \times 7^2 = 11643.38$ N

10. Calculation of accurate dynamic load (F_d) :

We know that, $F_d = F_t + \frac{21\, v\,(bc + F_t)}{21\, v + \sqrt{bc + F_t}}$

where $F_t = \frac{P}{v} = \frac{40 \times 10^3}{6.16} = 6493.5$ N

$c = 11860 \times 0.017 = 201.62$ N/mm, from Tables 5.7(a) and (b).

$\therefore \quad F_d = 6493.5 + \frac{21 \times 6.16 \times 10^3\,(70 \times 201.62 + 6493.5)}{21 \times 6.16 \times 10^3 + \sqrt{70 \times 201.62 + 6493.5}}$

$= 27077.55$ N

11. Check for beam strength : Since $F_d >> F_s$. The design is not satisfactory.

Trial 2 : Increase the transverse module to 14 m. Repeating from Step 8 again, we get

$b = 10 \times m_t = 10 \times 14 = 140$ mm

$d_1 = m_t \times z_1 = 14 \times 20 = 280$ mm

$v_1 = v_2 = \frac{\pi\, d_1\, N_1}{60} = \frac{\pi \times 0.28 \times 840}{60} = 12.315$ m/s

$F_s = 237.62 \times m_t^2 = 237.62 \times 14^2 = 46573.52$ N

$F_t = \frac{P}{v} = \frac{40 \times 10^3}{12.315} = 3248$ N

$c \approx 11860 \times e = 11860 \times 0.025 \approx 296.5$ N/mm

$F_d = 3248 + \frac{21 \times 12.315 \times 10^3\,(140 \times 296.5 + 3248)}{21 \times 12.315 \times 10^3 \sqrt{140 \times 296.5 + 3248}}$

$= 47969.4$ N

We find $F_s > F_d$. Now ***the design is safe and satisfactory against the tooth failure.***

12. Calculation of wear load (F_w) :

We know that, $F_w = \frac{0.75 \times d_1 \times b \times Q' \times K_w}{\cos \delta_1}$

where $Q' = \text{Ratio factor} = \frac{2 \times z_{v2}}{z_{v1} \pm z_{v2}} = \frac{2 \times 215}{21 + 215} = 1.822$, and

$K_w = 0.919$ N/mm², for steel gears hardened to 250 BHN, from Table 5.9.

$\therefore \quad F_w = \frac{0.75 \times 280 \times 140 \times 1.822 \times 0.919}{\cos 17.36°} = 51578.25$ N

13. Check for wear : We find $F_w > F_d$. It means the gear tooth has adequate wear capacity and will not wear out. Thus ***the design is safe against wear failure also.***

14. Basic dimensions of pinion and gear : Refer Table 7.1.

✓ Module : m_t = **14 mm**

✓ Face width : $b = 10 \times m_t = 10 \times 14 =$ **140 mm**

✓ Pitch diameter : $d_1 = m_t \times z_1 = 14 \times 20 =$ **280 mm;** and

$d_2 = m_t \times z_2 = 14 \times 64 =$ **896 mm.**

II. BEVEL GEAR DESIGN BASED ON GEAR LIFE

(Bevel Gear Design using Basic Equations)

BEVEL GEAR

7.12. DESIGN FORMULAS FOR BEVEL GEAR DESIGN

(i) Design torque (or Design load) $[M_t]$:

$[M_t] = M_t \times K \times K_d$ ... (7.21)

where M_t = Transmitted torque $= \frac{60 \times P}{2\pi N}$,

K = Load concentration factor, from Table 7.2, and

K_d = Dynamic load factor, from Table 5.12.

Table 7.2. **Load concentration factor, K for bevel gears (from data book, page no. 8.15)**

Surface hardness of gears, HB	b / d_{1av} ratio		
	≤ 1	1 to 1.6	1.6 to 1.8
> 350 for both the gears	1.6	–	–
≤ 350 for both the gears or atleast for wheel	1.1	1.2	1.3

d_{1av} – Average pitch diameter of bevel pinion $= m_t z_1 [(R - 0.5\,b) / R]$ mm

(ii) Induced bending stress (σ_b) :

$$\sigma_b = \frac{R\sqrt{i^2+1}\;[M_t]}{(R-0.5\,b)^2 \cdot b \cdot m_t \cdot y_v} \quad \ldots (7.22)$$

where R = Cone distance,

i = Gear ratio $= \frac{N_1}{N_2} = \frac{z_2}{z_1}$,

b = Face width,

m_t = Transverse module, and

y_v = Form factor based on virtual number of teeth, from Table 5.13.

(iii) Design bending stress $[\sigma_b]$:

$$[\sigma_b] = \frac{1.4\,K_{bl}}{n \cdot K_\sigma} \times \sigma_{-1}, \text{ for rotation in one direction} \quad \ldots (7.23(a))$$

$$= \frac{K_{bl}}{n \cdot K_\sigma} \times \sigma_{-1}, \text{ for rotation in both directions} \quad \ldots (7.23(b))$$

where K_{bl} = Life factor for bending, from Table 5.14,

K_σ = Stress concentration factor for fillet, from Table 5.15,

σ_{-1} = Endurance limit in reversed bending, from Table 5.16, and

n = Factor of safety, from Table 5.17.

(iv) Induced contact stress (σ_c) :

$$\sigma_c = \frac{0.72}{(R-0.5\,b)}\left[\frac{\sqrt{(i^2 \pm 1)^3}}{ib} \times E_{eq} \times [M_t]\right]^{\frac{1}{2}}$$

where E_{eq} = Equivalent Young's modulus, from Table 5.20.

(v) Design contact stress $[\sigma_c]$:

$$[\sigma_c] = C_B \times HB \times K_{cl} \quad \text{or} \quad [\sigma_c] = C_R \times HRC \times K_{cl} \quad \ldots (7.25)$$

where C_B and C_R = Coefficients depending on the surface hardness, from Table 5.18,

HB = Brinell hardness number,

HBC = Rockwell hardness number, and

K_{cl} = Life factor for surface strength, from Table 5.19.

(vi) Cone distance (R) :

$$R \geq \psi_y \sqrt{i^2+1}\;\sqrt[3]{\left(\frac{0.72}{(\psi_y - 0.5)[\sigma_c]}\right)^2 \frac{E_{eq}\,[M_t]}{i}} \quad \ldots (7.26)$$

where ψ_y = Ratio of cone distance to face width $= \frac{R}{b}$

Take ψ_y = 3 for initial calculations.

(vii) Transverse module (m_t) : The transverse module (m_t) can be found by using the following cone distance equation

$$R = 0.5\,m_t\sqrt{z_1^2 + z_2^2} \quad \ldots (7.27)$$

or

$$m_t = \frac{R}{0.5\sqrt{z_1^2+z_2^2}} \quad \ldots (7.28)$$

7.13. DESIGN PROCEDURE

1. Calculate ***gear ratio*** and the pitch angles.
2. Select the suitable combination of ***materials for pinion and wheel***, consulting Table 5.3.
3. If not given, assume ***gear life*** (say 20,000 hrs).
4. ***Calculate the initial design torque*** $[M_t]$. Use $[M_t] = M_t \times K \times K_d$
   Initially assume $K \cdot K_d = 1.3$.
5. ***Calculation of*** E_{eq}, $[\sigma_b]$ ***and*** $[\sigma_c]$:
   - ✓ Calculate the equivalent Young's modulus, E_{eq} consulting Table 5.20.
   - ✓ To find $[\sigma_b]$: Calculate the design bending stress $[\sigma_b]$ using the equation 7.23.
   - ✓ To find $[\sigma_c]$: Calculate the design contact stress $[\sigma_c]$ using the equation 7.25.
6. ***Calculate the cone distance (R)*** using the equation 7.26.

7. ***Selection of number of teeth :***

✓ If not given, assume $z_1 \geq 17$. Then $z_2 = i \times z_1$.

✓ Virtual number of teeth : $z_{v1} = \frac{z_1}{\cos \delta_1}$; and $z_{v2} = \frac{z_2}{\cos \delta_2}$

8. ***Calculate the transverse module (m_t)*** using the equation 7.28. Then, choose the nearest higher standard transverse module from Table 5.8.

9. ***Revise the cone distance (R)*** using the relation $R = 0.5\, m_t \sqrt{z_1^2 + z_2^2}$.

10. ***Calculation of b, m_{av}, d_{1av}, v and ψ_y :***

✓ Calculate face width (b) : $b = \frac{R}{\psi_y}$

✓ Calculate average module (m_{av}) : $m_{av} = m_t - \frac{b \sin \delta_1}{z_1}$

✓ Calculate average pcd (d_{1av}) : $d_{1av} = m_{av} \cdot z_1$

✓ Calculate pitch line velocity (v) : $v = \frac{\pi \times d_{1av} \times N_1}{60}$

✓ Calculate ψ_y : Use $\psi_y = \frac{b}{d_{1av}}$

11. Select the suitable ***quality of gear,*** consulting Table 5.22.

12. ***Revision of design torque [M_t] :***

✓ Revise K, using ψ_y and Table 5.11.

✓ Revise K_d, using Table 5.12.

✓ Revise [M_t], using the revised values of K and K_d. Use $[M_t] = M_t \times K \times K_d$

13. ***Check for bending :***

✓ Calculate the induced bending stress using the equation 7.22.

✓ Compare the induced bending stress with the design bending stress.

✓ If $\sigma_b \leq [\sigma_b]$, then the design is satisfactory.

14. If $\sigma_b > [\sigma_b]$, then the design is not satisfactory. Then increase the transverse module or face width, or change the gear material. The above procedure is repeated until the design is satisfactory. *i.e.,* $\sigma_b \leq [\sigma_b]$.

15. ***Check for wear strength :***

✓ Calculate the induced contact stress using the equation 7.24.

✓ Compare the induced contact stress with design contact stress.

✓ If $\sigma_c \leq [\sigma_c]$, then the design is safe and satisfactory.

16. ***Calculation of basic dimensions of the gear pair :*** Calculate all the basic dimensions of the pinion and gear using the equations listed in Table 7.1.

Note

✓ The above listed procedure is for the design of pinion.

✓ As discussed earlier, if the materials of the pinion and gear are same, then design only the pinion. If the materials of the pinion and gear are different, then design the pinion first and check for both pinion and gear.

✓ The induced bending stress in the gear (σ_{b2}) can be determined by using the relation

$$\sigma_{b1}\, y_{v1} = \sigma_{b2} \cdot y_{v2}$$

where σ_{b1} and σ_{b2} = Induced bending stresses of pinion and gear respectively, and

y_{v1} and y_{v2} = Form factors of pinion and gear respectively based on the virtual number of teeth.

✓ Since the contact area is same, the induced contact stress is same for both pinion and gear. *i.e.,* $\sigma_{c1} = \sigma_{c2}$.

Example 7.11 ***Design a cast iron bevel gear drive for a pillar drilling machine to transmit 1875 W at 800 r.p.m. to a spindle at 400 r.p.m. The gear is to work for 40 hours per week for 3 years. Pressure angle is 20°.***

Given Data : P = 1875 W ; N_1 = 800 r.p.m. ; N_2 = 400 r.p.m. ; $\alpha = 20°$.

To find : Design a bevel gear drive.

Solution : Since the materials of pinion and gear are same, we have to ***design only the pinion.***

1. ***Gear ratio :***

$$i = \frac{N_1}{N_2} = \frac{800}{400} = 2$$

Pitch angles : For right angle bevel gears, $\tan \delta_2 = i = 2$

or $\delta_2 = \tan^{-1}(2) = 63.43°$

and $\delta_1 = 90° - \delta_2 = 90° - 63.43° = 26.57°$

2. ***Material for pinion and gear :*** Cast iron, Grade 35 heat treated.

σ_u = 350 N/mm², from Table 5.3

3. ***Gear life*** in hours = (40 hrs / week) × (52 weeks / year × 3 years) = 6240 hours

Gear life in cycles, N = 6240 × 800 × 60 = 29.952 × 10⁷ cycles

4. ***Calculation of initial design torque [M_t] :***

We know that, $[M_t] = M_t \times K \times K_d$

where

$$M_t = \frac{60 \times P}{2\pi N_1} = \frac{60 \times 1875}{2\pi \times 800} = 22.38 \text{ N-m, and}$$

$K \cdot K_0 = 1.3$. ... (initially assumed)

$\therefore \quad [M_t] = 22.38 \times 1.3 = 29.095$ N-m

5. Calculation of E_{eq}, $[\sigma_b]$ and $[\sigma_c]$:

✓ *To find E_{eq}*: $E_{eq} = 1.4 \times 10^5$ N/mm² for cast iron, $\sigma_u > 280$ N/mm², from Table 5.20.

✓ *To find $[\sigma_b]$*: We know that the design bending stress,

$$[\sigma_b] = \frac{1.4\, K_{bl}}{n \cdot K_\sigma} \times \sigma_{-1}, \text{ for rotation in one direction}$$

where $K_{bl} = \sqrt[9]{\frac{10^7}{N}} = \sqrt[9]{\frac{10^7}{29.952 \times 10^7}} = 0.8852$, for C.I, from Table 5.14

$K_\sigma = 1.2$, for C.I. from Table 5.15.

$n = 2$, from Table 5.17, and

$\sigma_{-1} = 0.45\, \sigma_u$.

But $\sigma_u = 350$ N/mm², for C.I., from Table 5.3

$\therefore \quad \sigma_{-1} = 0.45 \times 350 = 157.5$ N/mm²

Then, $[\sigma_b] = \frac{1.4 \times 0.8852}{2 \times 1.2} \times 157.5 = 81.33$ N/mm²

✓ *To find $[\sigma_c]$*: We know that the design contact stress,

$$[\sigma_c] = C_B \times HB \times K_{cl}$$

where $C_B = 2.3$. from Table 5.18,

$HB = 200$ to 260, from Table 5.18, and

$$K_{cl} = \sqrt[6]{\frac{10^7}{N}} = \sqrt[6]{\frac{10^7}{29.952 \times 10^7}}$$

$= 0.833$, for C.I, from Table 5.19.

$\therefore \quad [\sigma_c] = 2.3 \times 260 \times 0.833 = 498.08$ N/mm²

6. Calculation of cone distance (R):

We know that,

$$R \geq \psi_y \sqrt{i^2 + 1}\, \sqrt[3]{\left[\frac{0.72}{(\psi_y - 0.5)[\sigma_c]}\right]^2 \times \frac{E_{eq}[M_t]}{i}}$$

where $\psi_y = R/b = 3$, initially assumed.

$$\therefore \quad R \geq 3\sqrt{2^2 + 1}\, \sqrt[3]{\left[\frac{0.72}{(3 - 0.5)\, 498.08}\right]^2 \times \frac{1.4 \times 10^5 \times 17.905 \times 10^3}{2}}$$

≥ 50.2

or $R = 51$ mm

7. Assume $z_1 = 20$; Then $z_2 = i \times z_1 = 2 \times 20 = 40$

Virtual number of teeth: $z_{v1} = \frac{z_1}{\cos \delta_1} = \frac{20}{\cos 26.57°} \approx 23$; and

$$z_{v2} = \frac{z_2}{\cos \delta_2} = \frac{40}{\cos 63.43°} \approx 90$$

8. Calculation of transverse module (m_t):

We know that,

$$m_t = \frac{R}{0.5\sqrt{z_1^2 + z_2^2}} = \frac{51}{0.5\sqrt{20^2 + 40^2}} = 2.28 \text{ mm}$$

From Table 5.8, the nearest higher standard transverse module is 2.5 mm

9. Revision of cone distance (R):

We know that,

$$R = 0.5\, m_t \sqrt{z_1^2 + z_2^2} = 0.5 \times 2.5\sqrt{20^2 + 40^2} = 55.9 \text{ mm}$$

10. Calculation of b, m_{av}, d_{1av}, v and ψ_y:

✓ Face width (b): $b = \frac{R}{\psi_y} = \frac{55.9}{3} = 18.63$ mm

✓ Average module (m_{av}): $m_{av} = m_t - \frac{b \sin \delta_1}{z_1} = 2.5 - \frac{18.63 \times \sin 26.57°}{20}$

$= 2.083$ mm

✓ Average pcd of pinion (d_{1av}): $d_{1av} = m_{av} \times z_1 = 2.083 \times 20 = 41.66$ m

✓ Pitch line velocity (v): $v = \frac{\pi \times d_{1av} \times N_1}{60} = \frac{\pi \times 41.66 \times 10^{-3} \times 800}{60}$

$= 1.745$ m/s

✓ *To find ψ_y*: $\psi_y = \frac{b}{d_{1av}} = \frac{18.63}{41.66} = 0.447$.

11. IS quality 6 bevel gear is assumed, from Table 5.22.

12. Revision of design torque $[M_t]$:

We know that, $[M_t] = M_t \times K \times K_d$

where $K = 1.1$, for $b/d_{1av} \le 1$, from Table 7.2, and

$K_d = 1.35$, for IS quality 6 and v upto 3 m/s, from Table 5.12.

$\therefore [M_t] = 22.38 \times 1.1 \times 1.35 = 33.24$ N-m

13. Check for bending : We know that the induced bending stress,

$$\sigma_b = \frac{R\sqrt{i^2+1}\ [M_t]}{(R-0.5\,b)^2 \times b \times m_t \times y_{v1}}$$

where $y_{v1} \approx 0.408$, for $z_{v1} = 23$, from Table 5.13

$$\therefore \sigma_b = \frac{55.9\sqrt{2^2+1} \times 33.24 \times 10^3}{(55.9 - 0.5 \times 18.63)^2 \times 18.63 \times 2.5 \times 0.408}$$

$= 100.75$ N/mm²

We find $\sigma_b > [\sigma_b]$. Thus the design is not satisfactory.

Trial 2 : Now we will try with increased transverse module 3 mm. Repeating from Step again, we get

$$R = 0.5 \times m_t \times \sqrt{z_1^2 + z_2^2} = 0.5 \times 3 \times \sqrt{20^2 + 40^2} = 67.08 \text{ mm}$$

$$b = \frac{R}{\psi_y} = \frac{67.08}{3} = 22.36 \text{ mm}$$

$$m_{av} = m_t - \frac{b \sin \delta_1}{z_1} = 3 - \frac{22.36 \times \sin 26.57°}{20} = 2.5 \text{ mm}$$

$$d_{1av} = m_{av} \times z_1 = 2.5 \times 20 = 50 \text{ mm}$$

$$v = \frac{\pi \times d_{1av} \times N_1}{60} = \frac{\pi \times 50 \times 10^{-3} \times 800}{60} = 2.094 \text{ m/s}$$

$$\psi_y = \frac{b}{d_{1av}} = \frac{22.36}{50} = 0.447$$

IS quality 6 bevel gear is assumed.

$K = 1.1$, from Table 7.2

$K_d = 1.35$, from Table 5.12

$[M_t] = M_t \times K \times K_d = 22.38 \times 1.1 \times 1.35$

$= 33.24$ N-m

$$\sigma_b = \frac{67.08\sqrt{2^2+1} \times 33.24 \times 10^3}{(67.08 - 0.5 \times 22.36)^2 \times 22.36 \times 3 \times 0.408} = 58.3 \text{ N/mm}^2$$

Now we find $\sigma_b < [\sigma_b]$. Thus ***the design is satisfactory.***

14. Check for wear strength : We know that the induced contact stress,

$$\sigma_c = \frac{0.72}{(R - 0.5\,b)} \left[\frac{\sqrt{(i^2+1)^3}}{i \times b} \times E_{eq}\ [M_t]\right]^{\frac{1}{2}}$$

$$= \frac{0.72}{(67.08 - 0.5 \times 22.36)} \left[\frac{\sqrt{(2^2+1)^3}}{2 \times 22.36} \times 1.4 \times 10^5 \times 33.24 \times 10^3\right]^{\frac{1}{2}}$$

$= 439.33$ N/mm²

We find $\sigma_c < [\sigma_c]$. Thus ***the design is satisfactory.***

15. Calculation of basic dimensions of pinion and gear : Refer Table 7.1.

✓ Transverse module : $m_t = 3$ mm

✓ Number of teeth : $z_1 = 20$; and $z_2 = 40$.

✓ Pitch circle diameter : $d_1 = m_t \times z_1 = 3 \times 20 = 60$ mm ; and

$d_2 = m_t \times z_2 = 3 \times 40 = 120$ mm.

✓ Cone distance : $R = 67.08$ mm

✓ Face width : $b = 22.36$ mm

✓ Pitch angles : $\delta_1 = 26.57°$; and $\delta_2 = 63.43°$

✓ Tip diameter : $d_{a1} = m_t(z_1 + 2\cos\delta_1) = 3(20 + 2\cos 26.57°)$

$= 65.37$ mm ; and

$d_{a2} = m_t(z_2 + 2\cos\delta_2) = 3(40 + 2\cos 63.43°)$

$= 122.68$ mm

✓ Height factor : $f_0 = 1$

✓ Clearance : $c = 0.2$

✓ Addendum angle : $\tan\theta_{a1} = \tan\theta_{a2} = \frac{m_t \times f_0}{R} = \frac{3 \times 1}{67.08} = 0.0447$

or $\theta_{a1} = \theta_{a2} = 2.56°$

✓ Dedendum angle : $\tan\theta_{f1} = \tan\theta_{f2} = \frac{m_t(f_0 + c)}{R} = \frac{3(1 + 0.2)}{67.08}$

$= 0.05366$

or $\theta_{f1} = \theta_{f2} = 3.07°$

✓ Tip angle : $\delta_{a1} = \delta_1 + \theta_{a1} = 26.57° + 2.56° = 29.13°$; and

$\delta_{a2} = \delta_2 + \theta_{a2} = 63.43° + 2.56° = 65.99°$

✓ **Root angle :** $\delta_{f1} = \delta_1 - \theta_{f1} = 26.57° - 3.07° = \mathbf{23.5°}$; and

$\delta_{f2} = \delta_2 - \theta_{f2} = 63.43° - 3.07° = \mathbf{60.36°}$

✓ Virtual number of teeth : $z_{v1} = \mathbf{23}$; and $z_{v2} = \mathbf{90}$.

Example 7.12 *Design a straight bevel gear drive between two shafts at right angles to each other. Speed of the pinion shaft is 360 r.p.m. and the speed of the gear wheel shaft is 120 r.p.m. Pinion is of steel and wheel of cast iron. Each gear is expected to work 2 hours / day for 10 years. The drive transmits 9.37 kW.*

Given Data : $\theta = 90°$; $N_1 = 360$ r.p.m. ; $N_2 = 120$ r.p.m. ; $P = 9.37$ kW.

To find : Design the bevel gear drive.

Solution : Since the materials of pinion and gear are different, we have to design the pinion first and check the gear.

1. **Gear ratio :** $i = \dfrac{N_1}{N_2} = \dfrac{360}{120} = 3$

Pitch angles : $\tan\delta_2 = i = 3$ or $\delta_2 = \tan^{-1}(3) = 71.56°$

Then, $\delta_1 = 90° - \delta_2 = 90° - 71.56° = 18.44°$

2. **Material selection :** Pinion – C45 Steel, $\sigma_u = 700$ N/mm² and $\sigma_y = 360$ N/mm²

Gear – CI grade 35, $\sigma_u = 350$ N/mm², from Table 5.3.

3. **Gear life** in hours = (2 hours/day) × (365 days / year × 10 years) = 7300 hours

∴ Gear life in cycles, $N = 7300 \times 360 \times 60 = 15.768 \times 10^7$ cycles

4. **Calculation of initial design torque $[M_t]$:**

We know that, $[M_t] = M_t \times K \times K_d$

where $M_t = \dfrac{60 \times P}{2\pi N_1} = \dfrac{60 \times 9.37 \times 10^3}{2\pi \times 360} = 248.6$ N-m, and

$K \cdot K_d = 1.3$, initially assumed.

∴ $[M_t] = 248.6 \times 1.3 = 323.28$ N-m

5. **Calculation of E_{eq}, $[\sigma_b]$ and $[\sigma_c]$:**

✓ *To find* E_{eq} : $E_{eq} = 1.7 \times 10^5$ N/mm², from Table 5.20.

✓ *To find* $[\sigma_{b1}]$: We know that the design bending stress for pinion,

$[\sigma_{b1}] = \dfrac{1.4\, K_{bl}}{n \cdot K_\sigma} \times \sigma_{-1}$, for rotation in one direction

where $K_{bl} = 1$, for HB ≤ 350 and $N \geq 10^7$, from Table 5.14,

$K_\sigma = 1.5$, for steel pinion, from Table 5.15,

$n = 2.5$, steel hardened, from Table 5.17.

$\sigma_{-1} = 0.25(\sigma_u + \sigma_y) + 50$, for forged steel, from Table 5.16.

$= 0.25(700 + 360) + 50 = 315$ N/mm²

∴ $[\sigma_{b1}] = \dfrac{1.4 \times 1}{2.5 \times 1.5} \times 315 = 117.6$ N/mm²

✓ *To find* $[\sigma_{c1}]$: We know that the design contact stress for pinion,

$[\sigma_{c1}] = C_R \cdot HRC \times K_{cl}$

where $C_R = 23$, from Table 5.18,

$HRC = 40$ to 55, from Table 5.18, and

$K_{cl} = 1$, for steel pinion, HB ≤ 350 and $N \geq 10^7$, from Table 5.19.

∴ $[\sigma_{c1}] = 23 \times 50 \times 1 = 1150$ N/mm²

6. **Calculation of cone distance (R) :**

We know that, $R \geq \psi_y \sqrt{i^2+1}\ \sqrt[3]{\left[\dfrac{0.72}{(\psi_y - 0.5)[\sigma_c]}\right]^2 \times \dfrac{E_{eq}[M_t]}{i}}$

where $\psi_y = R/b = 3$, initially assumed.

∴ $R \geq 3\sqrt{3^2+1}\ \sqrt[3]{\left[\dfrac{0.72}{(3-0.5)\,1150}\right]^2 \times \dfrac{1.7 \times 10^5 \times 323.28 \times 10^3}{3}}$

≥ 99.36

or $R = 100$ mm.

7. Assume $z_1 = 20$; Then $z_2 = i \times z_1 = 3 \times 20 = 60$

Virtual number of teeth : $z_{v1} = \dfrac{z_1}{\cos\delta_1} = \dfrac{20}{\cos 18.44°} \approx 22$; and

$z_{v2} = \dfrac{z_2}{\cos\delta_2} = \dfrac{60}{\cos 71.56°} \approx 190.$

8. **Calculation of transverse module (m_t) :**

We know that,

$m_t = \dfrac{R}{0.5\sqrt{z_1^2 + z_2^2}}$

$= \dfrac{100}{0.5\sqrt{20^2 + 60^2}} = 3.162$ mm

From Table 5.8, the nearest higher standard transverse module is 4 mm.

9. *Revision of cone distance (R) :*

We know that, $R = 0.5\, m_t \sqrt{z_1^2 + z_2^2} = 0.5 \times 4 \sqrt{20^2 + 60^2} = 126.49$ mm

10. *Calculation of b, m_{av}, d_{1av}, v and ψ_y :*

✓ Face width (b) : $b = \dfrac{R}{\psi_y} = \dfrac{126.49}{3} = 42.16$ mm

✓ Average module (m_{av}) : $m_{av} = m_t - \dfrac{b \sin \delta_1}{z_1} = 4 - \dfrac{42.16 \times \sin 18.44°}{20}$

$= 3.333$

✓ Average pcd of pinion (d_{1av}) : $d_{1av} = m_{av} \times z_1 = 3.333 \times 20 = 66.66$ mm

✓ Pitch line velocity (v) : $v = \dfrac{\pi \times d_{1av} \times N_1}{60} = \dfrac{\pi \times 66.66 \times 10^{-3} \times 360}{60} = 1.256$ m/s

✓ $\psi_y = \dfrac{b}{d_{1av}} = \dfrac{42.16}{66.66} = 0.632$

11. *IS quality 6 bevel gear* is assumed, from Table 5.22.

12. *Revision of design torque [M_t] :*

We know that, $[M_t] = M_t \times K \times K_d$

where $K = 1.1$, from Table 7.2, and

$K_d = 1.35$, from Table 5.12.

$\therefore$ $[M_t] = 248.6 \times 1.1 \times 1.35 = 369.28$ N-m

13. *Check for bending of pinion :* We know that the induced bending stress,

$$\sigma_{b1} = \frac{R\sqrt{i^2+1}\;[M_t]}{(R - 0.5\, b)^2 \times b \times m_t \times y_{v1}}$$

where $y_{v1} = 0.402$, for $z_{v1} = 22$, from Table 5.13

$$\therefore \quad \sigma_b = \frac{126.49\sqrt{3^2+1} \times 369.28 \times 10^3}{(126.49 - 0.5 \times 42.16)^2 \times 42.16 \times 4 \times 0.402} = 196.09 \text{ N/mm}^2$$

We find $\sigma_{b1} > [\sigma_{b1}]$. Thus the design is unsatisfactory.

Trial 2 : Now we will try with increased transverse module 5 mm. Repeating from Step 1 again, we get

$R = 0.5 \times m_t \times \sqrt{z_1^2 + z_2^2} = 0.5 \times 5 \times \sqrt{20^2 + 60^2} = 158.11$ mm

$b = \dfrac{R}{\psi_y} = \dfrac{158.11}{3} = 52.7$ mm

$m_{av} = m_t - \dfrac{b \sin \delta_1}{z_1} = 5 - \dfrac{52.7 \times \sin 18.44}{20} = 4.166$ mm

$d_{1av} = m_{av} \times z_1 = 4.166 \times 20 = 83.33$ mm

$v = \dfrac{\pi \times d_{1av} \times N_1}{60} = \dfrac{\pi \times 83.33 \times 10^{-3} \times 360}{60} = 1.57$ m/s

$\psi_y = \dfrac{b}{d_{1av}} = \dfrac{52.7}{83.33} = 0.632$

IS quality 6 bevel gear is assumed.

$K = 1.1$; $K_d = 1.35$.

$M_t = 248.6 \times 1.1 \times 1.35 = 369.28$ N-m

$$\therefore \quad \sigma_{b1} = \frac{158.11\sqrt{3^2+1} \times 369.28 \times 10^3}{(158.11 - 0.5 \times 52.7)^2 \times 52.7 \times 5 \times 0.402} = 100.4 \text{ N/mm}^2$$

Now we find $\sigma_{b1} < [\sigma_{b1}]$, thus ***the design is satisfactory.***

14. *Check for wearing of pinion :* We know that the induced contact stress,

$$\sigma_{c1} = \left(\frac{0.72}{R - 0.5\, b}\right)\left[\frac{\sqrt{(i^2+1)^3}}{i\, b} \times E_{eq} \times [M_t]\right]^{\frac{1}{2}}$$

$$= \left[\frac{0.72}{158.11 - 0.5 \times 52.7}\right]\left[\frac{\sqrt{(3^2+1)^3}}{3 \times 52.7} \times 1.7 \times 10^5 \times 369.28 \times 10^3\right]^{\frac{1}{2}}$$

$= 612.33$ N/mm²

We find $\sigma_{c1} < [\sigma_{c1}]$. Thus ***the design is satisfactory for pinion.***

15. *Check for gear (i.e., wheel) : Gear material :* CI grade 30.

First we have to calculate $[\sigma_{b2}]$ and $[\sigma_{c2}]$.

Gear life of wheel, $N = \dfrac{N_{pinion}}{3} = \dfrac{15.768 \times 10^7}{3} = 5.256 \times 10^7$ cycles

✓ ***To find [σ_{b2}] :*** We know that the design bending stress for gear,

$$[\sigma_{b2}] = \frac{1.4 \times K_{bl}}{n \times K_\sigma} \times \sigma_{-1}$$

where $K_{bl} = \sqrt[9]{\dfrac{10^7}{N}} = \sqrt[9]{\dfrac{10^7}{5.256 \times 10^7}} = 0.832$, from Table 5.14,

$K_\sigma = 1.2$, from Table 5.15,

$n = 2$, from Table 5.17.

$\sigma_{-1} = 0.45\, \sigma_u = 0.45 \times 350 = 157.5$ N/mm²

$\therefore \quad [\sigma_{b2}] = \frac{1.4 \times 0.832}{2 \times 1.2} \times 157.5 = 76.44\ N/mm^2$

✓ *To find* $[\sigma_{c2}]$: We know that the design contact stress for gear,

$$[\sigma_{c2}] = C_B \times HB \times K_{cl}$$

where $C_B = 2.3$, from Table 5.18,

$HB = 200$ to 260, from Table 5.18, and

$$K_{cl} = \sqrt[6]{\frac{10^7}{N}} = \sqrt[6]{\frac{10^7}{5.256 \times 10^7}} = 0.758$$

$\therefore \quad [\sigma_{c2}] = 2.3 \times 260 \times 0.758 = 453.284\ N/mm^2$

(a) ***Check for bending of gear*** : The induced bending stress for gear can be calculated using the relation

$$\sigma_{b1} \times y_{v1} = \sigma_{b2} \times y_{v2}$$

where $y_{v1} = 0.402$, for $z_{v1} = 22$, from Table 5.13, and

$y_{v2} \approx 0.520$, for $z_{v2} = 190$, from Table 5.13.

$\therefore \quad 100.4 \times 0.402 = \sigma_{b2} \times 0.520$

or $\quad \sigma_{b2} = 77.6\ N/mm^2$

We find σ_{b2} is almost equal to $[\sigma_{b2}]$. Thus the design is okay and it can be accepted.

(b) ***Check for wearing of gear*** : Since the contact area is same,

$$\sigma_{c2} = \sigma_{c1} = 612.33\ N/mm^2$$

We find $\sigma_{c2} > [\sigma_{c2}]$. It means the gear does not have adequate beam strength. In order to increase the wear strength of the gear, surface hardness may be raised to 360 BHN. Then we get

$$[\sigma_{b2}] = 2.3 \times 360 \times 0.758 = 627.62\ N/mm^2.$$

Now we find $\sigma_{b2} < [\sigma_{b2}]$, thus the ***design is safe and satisfactory.***

REVIEW AND SUMMARY

- ✓ *Bevel gears are used to transmit power between two intersecting shafts.*
- ✓ ***Types :*** *1. Straight bevel gears ; 2. Spiral bevel gears ; 3. Zerol bevel gears ; and 4. Hypoid gears.*
- ✓ ***Classification based on pitch angle :*** *1. Crown gear; 2. Internal bevel gear; and 3. Mitre gears.*
- ✓ *The bevel gear nomenclature and its kinematics are presented in the beginning of the chapter.*

- ✓ ***Virtual number of teeth :*** $z_v = \frac{z}{\cos\delta}$
- ✓ *Tooth proportions and basic dimensions of bevel gears are tabulated in Table 7.1.*
- ✓ ***Force analysis on bevel gears :***

Three components of the resultant force on the gear tooth are :

1. *Tangential component :* $F_t = \frac{2\,M_t}{d_{1m}} = \frac{M_t}{r_m}$

   where r_m = *Mean radius of the pinion* $= \frac{d_1}{2} - \frac{b \cdot \sin\delta_1}{2}$

2. *Radial component :* $F_r = F_t \times \tan\alpha \times \cos\delta$

3. *Axial component :* $F_a = F_t \times \tan\alpha \times \sin\delta$

- ✓ ***Two methods of designing a bevel gear :*** *1. Bevel gear design using Lewis and Buckingham's equations; and 2. Bevel gear design based on gear life.*
- ✓ *The step by step procedure for the above said two methods are presented with sufficient illustrative problems.*
- ✓ ***Lewis beam strength for bevel gears :***

Beam strength, $F_s = \pi \times m_t \times b \times [\sigma_b] \times y' \times \left(\frac{R-b}{R}\right)$

where R = *Cone distance* $= 0.5\, m_t \sqrt{z_1^2 + z_2^2}$

- ✓ ***Buckingham's equation for bevel gears :***

(i) *Dynamic load,* $F_d = F_t + \frac{21\,v\,(bc + F_t)}{21\,v + \sqrt{bc + F_t}}$

(ii) *Wear tooth load,* $F_w = \frac{0.75\, d_1 \times b \times Q' \times K_w}{\cos\delta_1}$

REVIEW QUESTIONS

1. Under what situation, bevel gears are used ?
2. How are the bevel gears classified ? Explain with sketches.
3. What do you mean by spiral bevel gear ? What is its advantage over straight bevel gear ?
4. What is a zerol bevel gear ?
5. What is a hypoid gear ? Why is it used in automobiles ?
6. Differentiate an angular gear and a mitre gear.
7. With neat sketch, explain the nomenclature of a straight bevel gear.

8. Differentiate the following terms with respect to bevel gears :
(i) Pitch angle; (ii) Tip angle; (iii) Root angle; (iv) Addendum angle; and
(v) Dedendum angle.
9. What virtual number of teeth of bevel gear refers to ?
10. What are the various forces acting on a bevel gear ? Deduce the expressions of it.
11. Write the expressions for beam strength, dynamic load, and limiting wear load for bevel gears and explain the various terms used in it.
12. Write the step by step procedure for the design of bevel gears.

PROBLEMS FOR PRACTICE

Problems on kinematics of bevel gears :

1. A 20° straight-tooth bevel pinion having 14 teeth and a transverse module of 4 mm drives a 32-tooth gear. The two shafts are at right angles and in the same plane. Find : (i) the cone distance; (ii) the pitch angles; (iii) the pitch diameter; and (iv) the face width.

[*Ans :* (i) 69.85 mm; (ii) 23.63°; 66.37°; (iii) 56 mm; 128 mm; (iv) 40 mm]

2. A pair of bevel gears consists of a 24-teeth pinion meshing with a 60 teeth gear. The gears are mounted on shafts, which are intersecting at right angles. The module at the large end of the tooth is 3.5 mm. Calculate : (i) the pitch diameters of the pinion and the gear; (ii) the pitch angles for the pinion and the gear, and (iii) the cone distance.

[*Ans :* (i) 84 mm ; 210 mm; (ii) 21.8° ; 68.2° ; (iii) 113.09 mm]

Problems on force analysis of bevel gears :

3. A pair of straight bevel gears has a speed ratio of 3. A 7.5 kW power is supplied to the pinion, which rotates at 1200 r.p.m. The pitch circle diameter of the pinion is 100 mm at the large end of the tooth. The face width is 45 mm and the pressure angle is 20°. Calculate the tangential, radial and axial components of the resultant tooth force acting on the pinion. [*Ans :* 1391.86 N; 480.58 N; 160.24 N]

4. For the data of previous problem, calculate the tangential, radial and axial components of the resultant tooth force acting on the gear wheel.

[*Ans :* 1391.86 N; 160.24 N; 480.58 N]

5. A pair of bevel gears is to be used to transmit 12 kW from a pinion rotating at 360 r.p.m. to a gear mounted on a shaft which intersects the pinion shaft at an angle of 70°. Assuming that the pinion is to have an outside pitch diameter of 200 mm, a pressure angle of 20°, a face width of 40 mm, and the gear shaft is to rotate at 120 r.p.m. determine (i) the pitch angle for the gears; (ii) the forces on the gears; and (iii) the torque produced about the shaft.

[*Ans :*(i) 15.7°; 54.3°; (ii) 3365 N; 1179 N; 331 N; 3365 N; 715 N; 995 N; (iii) 955 N-m]

6. A pair of bevel gears transmitting 7.5 kW at 300 r.p.m. is shown in Fig.7.13. The pressure angle is 20°. Determine the components of the resultant gear tooth force and draw a free body diagram of forces acting on the pinion and gear.

[*Ans :* 3789.4 N; 1103.38 N; 827.54 N]

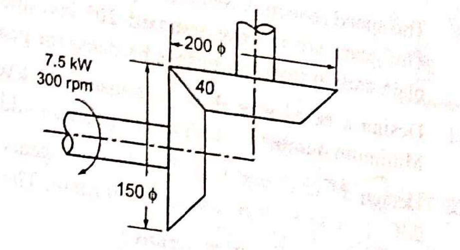


Fig. 7.13.

7. The dimensions of a pair of bevel gears are given in Fig.7.14. The gear G delivers 5 kW power at 500 r.p.m. to the output shaft. The bearings A and B are mounted on the output shaft in such a way that the bearing B can take radial as well as thrust load, while the bearing A can only take the radial load. Determine the reactions at the two bearings.

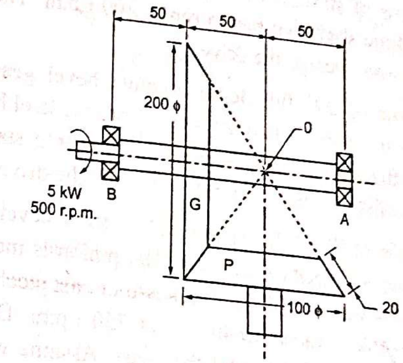


Fig. 7.14.

[*Ans :* $F_A^y = 145.25$ N; $F_A^z = 380.84$ N; $F_B^x = 341.4$ N; $F_B^y = 316$ N; $F_B^z = 667.91$ N]

8. A differential planetary gear train is shown in Fig.7.12(a). The input shaft receives 10 kW power at 500 r.p.m. The pitch circle diameters of bevel gears A, B, C and D at the midpoint along the face width are 250, 125, 250 and 500 mm respectively. The pitch circle diameters of spur gears E and F are 250 and 350 mm respectively. The gears rotate at constant speed. Draw a free body diagram of forces acting on various gears and calculate the torque on each of the two output shafts.

[*Ans :* $F^t_{AB} = 1527.89$ N; $F^t_{CD} = 763.95$ N; $F^t_{EF} = 1527.89$ N; $(M_{t1}) = 381.97$ N-m; $(M_{t2}) = 267.38$ N-m]

Problems on bevel gear design using Lewis and Buckingham's equations :

(i) ***Gears are of same material :***

9. Design a pair of cast iron straight bevel gears with the transmission ratio of 2.5. The input is from a 3 kW electric motor running at 960 r.p.m. Select suitable grade of material for the gears and determine the dimensions.

10. A pair of bevel gears have their shaft at right angles and intersecting each other. This gear pair is required to transmit 5 kW power from an IC engine to a casting machine.

The speed reduction rendered by the gear is 2.5. The IC engine rated speed is 900 r.p.m. The gears are to have standard 20° involute straight tooth system. Selecting suitable plain carbon steel and surface hardness for gears, design all the parameters.

11. Design a bevel gear drive to transmit 10 kW at 1440 r.p.m. Desired speed ratio is 3. Minimum number of teeth on the gears should be 20. Gear materials is C15 steel.

12. Design a pair of cast iron straight bevel gears to transmit 90 kW from a shaft running at 240 r.p.m. to another running at 80 r.p.m. The operation has to be continuous.

(ii) Gears are of different materials :

13. A pair of straight bevel gears is to transmit 3.5 kW at 1440 r.p.m. from a motor to a machine shaft that has to run at 200 r.p.m. The pinion is of cast steel and the gear is of cast iron. Design the drive.

14. A pair of 20° full depth involute bevel gears connect shafts at right angles having velocity ratio 3 : 1. Gear is made of cast steel having allowable static strength 70 N/mm² and the pinion is of steel with allowable static stress of 100 N/mm² and the pinion transmits 35 kW at 750 r.p.m. Design the drive.

15. A pair of 20° full depth involute teeth bevel gears connect two shafts at right angles having a velocity ratio of 3. The pinion is made of steel with allowable static stress of 100 N/mm² and the gear is made of cast steel with allowable static stress of 70 N/mm². The pinion transmits 40 kW at 750 r.p.m. Determine the module, face width and the pitch circle diameters of the gears. Assume width as one-third the length of pitch cone and tooth form factors as $0.154 - (0.912 / z_v)$ where z_v is the virtual number of teeth.

Problems on bevel gear design, based on gear life :

(i) Gears are of same material :

16. Design a bevel gear drive to transmit 7.5 kW at 1440 r.p.m. Gear ratio is 3; Pinion and gear are made of C45 steel; Life of gears 10000 hrs.

17. Design a bevel gear drive to transmit a power of 9 kW at 20 r.p.s. of the pinion. Gear ratio is to be 3. Material to be used is C20. $\sigma_u = 500$ N/mm²; $\sigma_y = 260$ N/mm²; Assume the expected gear life as 10000 hours.

(ii) Gears are of different materials :

18. Design a bevel gear drive to transmit 4 kW with the following specifications : Speed ratio = 3.5 ; driving shaft speed = 300 r.p.m; drive is non-reversible; material for pinion is steel; material for wheel is cast iron; and life 20000 hours.

19. Design a bevel gear drive to transmit 12 kW at 1400 r.p.m. for the following data : Gear ratio = 3; Material for pinion is steel; Material for wheel is cast iron; and life 10,000 hours.

8 Worm Gears

"First they ignore you, then they laugh at you, then they fight you, then you win."

– Mahatma Gandhi

8.1. INTRODUCTION

The worm gears are used to transmit power between two non-intersecting, non-parallel shafts. The angle between the non-intersecting shafts is usually, but not necessarily, a right angle. As discussed in the previous chapters, crossed helical and hypoid gears are also used to connect non-intersecting non-parallel shafts. But crossed-helical and hypoid gears are suitable only for low speed ratios (upto 8) and low power ratings. Whereas worm gears can be used for high speed ratios as high as 300 : 1.

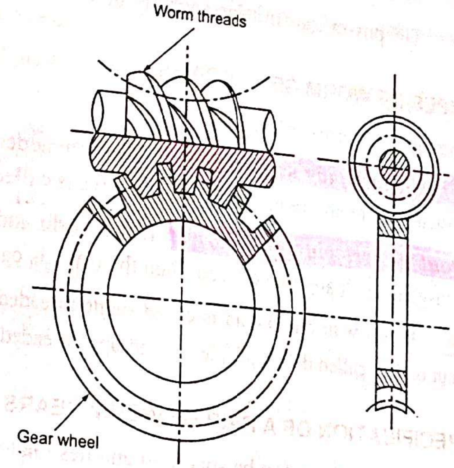


Fig. 8.1. Worm and Worm gears

The worm gear drive consists of a worm and a worm wheel, as shown in Fig.8.1. If a tooth of a helical gear makes complete revolutions on the pitch cylinder, the resulting gear is known as a ***worm.*** The mating gear is called ***worm gear*** or ***worm wheel.*** The worm in worm and worm gear drive is same as screw in screw and nut pair.

8.1.1. Applications of Worm Gear Drives

Worm gear drives are widely used as a speed reducer in materials handling equipment, machine tools and automobiles.

8.2. ADVANTAGES AND DISADVANTAGES OF WORM GEAR DRIVE

8.2.1. Advantages of Worm Gear Drives

- The worm gear drives can be used for speed ratios as high as 300 : 1.
- The operation is smooth and silent.