UNIT – V

CAMS, CLUTCHES AND BRAKES

1. What is the significance of pressure angle in cam design? (Nov /Dec 2007)
Pressure Angle: It is the angle between line of action of cam follower motion and the normal to the pitch curve (cam profile). The pressure angle represents the steepness of cam profile. The pressure angle is limited to 30^o for smooth cam-follower action. Importance of Pressure angle in cam design. they are;Increasing pressure angle increases the side thrust and this increase the forces exerted on cam and follower.If the pressure angle is too large jamming of follower takes place.Reducing the pressure angle increases the cam size.

2. What are the effects of temperature rise in clutches? (Nov /Dec 2007)
Why is it necessary to dissipate the heat generated during clutch operation?

During operation of a clutch, most of the work done against frictional forces opposing the motion is liberated as heat at the interface. It has been found that at the actual point of contact, the temperature as high as 1000°C is reached for a very short duration (i.e. for 0.0001 second). Due to this, the temperature of the contact surfaces will increase and may destroy the clutch. In order to save the friction plates and lining materials from melting by the heat produced during operation, the generated heat should be dissipated. The heat of friction should be rapidly dissipated and tendency to grab should be at a minimum.

Distortion of the shape of the plates. Surface cracks in solid metallic plates caused by thermal stresses. In non- metallic and semi metallic clutch plates, the high temperature can cause excessive wear. High temperature existing at the rubbing interface may cause the individual plates to be welded together.

3. Differentiate between self energizing brakes and self locking brakes. (Nov / Dec 2009)
The moment of frictional force (μ. RN.a) adds to the moment of force (P.l). In other words, the frictional force helps to apply the brake. Such type of brakes are said to be self energizing brakes. When the frictional force is great enough to apply the brake with no external force, then the brake is said to be self-locking brake.

4. Under what condition of a clutch, uniform rate of wear assumption is more valid? Clutches are usually designed on the basis of uniform wear. Why? (May / June 2009)

It may be noted that when the friction surface is new, there is a uniform pressure distribution over the entire contact surface. This pressure will wear most rapidly where the sliding velocity is maximum and this will reduce the pressure between the friction surfaces. This wearing-in process continues until the product p.V is constant over the entire surface. After this, the wear will be uniform.

The uniform pressure theory gives a higher friction torque than the uniform wear theory. Therefore in case of friction clutches, uniform wear should be considered, unless otherwise stated.

5. List at least four characteristics of the materials used for the brake linings. (May / June 2009)

The material used for the brake lining should have the following characteristics :

1. It should have high coefficient of friction with minimum fading. In other words, the coefficient of friction should remain constant over the entire surface with change in temperature. **2.** It should have low wear rate. **3.** It should have high heat resistance. **4.** It should have high heat dissipation capacity. **5.** It should have low coefficient of thermal expansion. **6.** It should have adequate mechanical strength. **7.** It should not be affected by moisture and oil.

6. What are the materials used for lining of friction surfaces? List at least four characteristics of the materials used for the friction linings of clutches. (Nov / Dec 2008)

In dry friction clutches, the friction plates are made of steel or Cast Iron on a lining of some asbestos base friction materials.

In wet friction clutches, the friction plate are made of steels with subsequent hardening with cermet as lining elements.

Characteristics:

1. It should have a high and uniform coefficient of friction. **2.** It should not be affected by moisture and oil. **3**. It should have the ability to withstand high temperatures caused by slippage. **4.** It should have high heat conductivity. **5.** It should have high resistance to wear and scoring.

7. Define base circle, pitch circle and jerk with respect to cam

Base Circle: It is the smallest circle drawn to the cam profile from the centre of rotation of a radial cam. The size of the cam drive depends on the size of the base circle.

Pitch Circle: It is a circle with its centre as the centre of cam axis and radius such that it passes through the pitch point.

Jerk or Pulse: It is used to define the instantaneous time rate of change of acceleration. To minimize vibrations, its value should be as low as possible.

8. What is the advantage of conical clutch over plate clutch? Give its Limitations. (Nov / Dec 2007)

The cone clutch uses two conical surfaces to transmit torque by friction. The cone clutch transfers a higher torque than plate or disk clutches of the same size due to the wedging action and increased surface area.

Alternately, they may be made in smaller size or require less actuating force compared with plate clutch. The disadvantage is that if the cone angle (α) is very low, it is difficult to disengage them.

9. What is the axial force required at the engagement and disengagement of cone clutch? (May / June 2013)

Axial force required for engaging the clutch,

$W_e = W + \mu.W_n \cos\alpha = W_n. \sin\alpha + \mu W_n \cos\alpha = W_n (\sin\alpha + \mu \cos\alpha)$

It has been found experimentally that the term (μ Wn.cos α) is only 25 percent effective.

$\therefore W_e = W_n \sin\alpha + 0.25 \mu W_n \cos\alpha = W_n (\sin\alpha + 0.25 \mu \cos\alpha)$

Axial force required to disengage the clutch is given by,$W_d = W_n (\mu \cos\alpha - \sin\alpha)$

10. When do we use multiple disk clutches? (Apr/May 2010)

It is used when large amount of torque is to be transmitted. In a multi plate clutch, the number of frictional linings and the metal plates are increased which Increases the capacity of the clutch to transmit torque.

11. Why in automobiles, braking action when travelling in reverse is not as effective as when moving forward? (Apr/May 2004)

- When an automobile moves forward the braking force acts in the opposite direction to the direction of motion of the vehicle.
- Whereas in reverse travelling the braking force is in the same direction to the direction of motion of vehicle so it requires more braking force to apply brake.

12. Distinguish between Wet and Dry operations of clutches. (Apr/May 2004)

- When a clutch operates in the absence of a lubricant then that clutch is known as Dry clutch. In dry clutch the torque capacity is high but the heat dissipating capacity is low.
- When a clutch operates in the presence of a lubricant then that clutch is known as wet clutch. In Wet clutch the torque capacity is low but the heat dissipating capacity is high.

13. What is fade? (Nov/Dec 2004)

When the brake is applied continuously over a period of time the brake becomes overheated and the coefficient of friction drops. This results in sudden fall of efficiency of the brake. This phenomenon is known as Fade or Fading.

14. If a multi disc clutch has 8 discs in driving shaft and 9 discs in driven shaft then how many number of contact surfaces it will have? (May/June 2006)

$n = n_1 + n_2 - 1 = 8 + 9 - 1 = 16$

15. What are the factors upon which the torque capacity of a clutch depends? (Nov /Dec 2011)

Torque capacity of a clutch depends on (i) number of pair of contact surfaces (ii) coefficient of friction (iii) Axial thrust exerted by the spring and (iv) Mean radius of friction surface.

PART -B

UNIT – V - CAMS, CLUTCHES AND BRAKES

CLUTCH: A clutch is a machine member used to connect a driving shaft to a driven shaft so that the driven shaft may be started or stopped at will, without stopping the driving shaft. The use of a clutch is mostly found in automobiles. A little consideration will show that in order to change gears or to stop the vehicle, it is required that the driven shaft should stop, but the engine should continue to run. It is, therefore, necessary that the driven shaft should be disengaged from the driving shaft. The engagement and disengagement of the shafts is obtained by means of a clutch which is operated by a lever

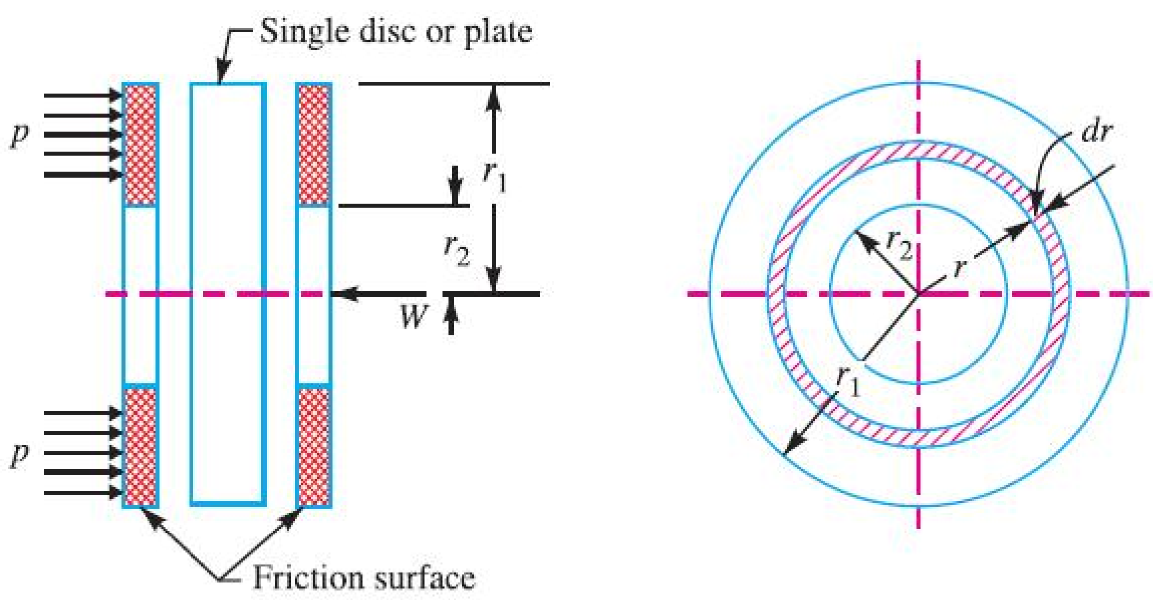
Types of Clutches: 1. Positive clutches, and 2. Friction clutches.

Types of Friction Clutches:

1. Disc or plate clutches (single disc or multiple disc clutch), **2.** Cone clutches, and **3.** Centrifugal clutches.

Note : The disc and cone clutches are known as **axial friction clutches,** while the centrifugal clutch is called **radial friction clutch.**

SINGLE PLATE CLUTCH:



I Considering Uniform Pressure:

$$p = \frac{W}{\pi\left[(r_1)^2 - (r_2)^2\right]}$$

$$T = \int_{r_2}^{r_1} 2\pi\,\mu.p.r^2.dr = 2\pi\mu.p\left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi\,\mu.p\left[\frac{(r_1)^3 - (r_2)^3}{3}\right] = 2\pi\,\mu \times \frac{W}{\pi\,[(r_1)^2 - (r_2)^2]}\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

... (Substituting the value of p)

$$= \frac{2}{3}\mu.\,W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \mu\,.W.R$$

$$R = \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$ = Mean radius of the friction surface.

II Considering uniform Wear:

$p.r = C$ (a constant) or $p = C/r$

$$C = \frac{W}{2\pi\,(r_1 - r_2)}$$

$$T = \int_{r_2}^{r_1} 2\pi\,\mu\,C.r.dr = 2\pi\,\mu\,C\left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= 2\pi\mu\,.C\left[\frac{(r_1)^2 - (r_2)^2}{2}\right] = \pi\,\mu.C\,[(r_1)^2 - (r_2)^2]$$

$$= \pi\mu \times \frac{W}{2\pi\,(r_1 - r_2)}\,[(r_1)^2 - (r_2)^2] = \frac{1}{2}\times\mu.W\,(r_1 + r_2) = \mu.W.R$$

$$R = \frac{r_1 + r_2}{2}$$ = Mean radius of the friction surface.

Notes : 1. In general, total frictional torque acting on the friction surfaces (or on the clutch) is given by

$$T = n.\mu.W.R$$

where n = Number of pairs of friction (or contact) surfaces, and

R = Mean radius of friction surface

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \quad \text{... (For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \quad \text{... (For uniform wear)}$$

2. For a single disc or plate clutch, normally both sides of the disc are effective. Therefore a single disc clutch has two pairs of surfaces in contact (*i.e.* $n = 2$).

3. Since the intensity of pressure is maximum at the inner radius (r_2) of the friction or contact surface, therefore equation (*ii*) may be written as

$$p_{max} \times r_2 = C \quad \text{or} \quad p_{max} = C / r_2$$

4. Since the intensity of pressure is minimum at the outer radius (r_1) of the friction or contact surface, therefore equation (*ii*) may be written as

$$p_{min} \times r_1 = C \quad \text{or} \quad p_{min} = C / r_1$$

5. The average pressure (p_{av}) on the friction or contact surface is given by

$$p_{av} = \frac{\text{Total force on friction surface}}{\text{Cross-sectional area of friction surface}} = \frac{W}{\pi[(r_1)^2 - (r_2)^2]}$$

<u>Pbm1:</u> A plate clutch having a single driving plate with contact surfaces on each side is required to transmit 110 kW at 1250 r.p.m. The outer diameter of the contact surfaces is to be 300 mm. The coefficient of friction is 0.4. (a) Assuming a uniform pressure of 0.17 N/mm^2; determine the inner diameter of the friction surfaces. (b) Assuming the same dimensions and the same total axial thrust, determine the maximum torque that can be transmitted and the maximum intensity of pressure when uniform wear conditions have been reached. (Nov / Dec 2008)

<u>Given Data :</u> P = 110 kW = 110 × 103W ; N = 1250 r.p.m. ; d1 = 300 mm or r1 = 150 mm ; μ = 0.4 ; p = 0.17 N/mm^2

(a) Inner diameter of the friction surfaces

Let d_2 = Inner diameter of the contact or friction surfaces, and

r_2 = Inner radius of the contact or friction surfaces.

We know that the torque transmitted by the clutch,

$$T = \frac{P \times 60}{2\pi N} = \frac{110 \times 10^3 \times 60}{2\pi \times 1250} = 840 \text{ N-m}$$

$$= 840 \times 10^3 \text{ N-mm}$$

Axial thrust with which the contact surfaces are held together,

$$W = \text{Pressure} \times \text{Area} = p \times \pi[(r_1)^2 - (r_2)^2]$$

$$= 0.17 \times \pi[(150)^2 - (r_2)^2] = 0.534[(150)^2 - (r_2)^2]$$

and mean radius of the contact surface for uniform pressure conditions,

$$R = \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \frac{2}{3}\left[\frac{(150)^3 - (r_2)^3}{(150)^2 - (r_2)^2}\right]$$

$\therefore$ Torque transmitted by the clutch (T),

$$840 \times 10^3 = n.\mu.W.R$$

$$= 2 \times 0.4 \times 0.534\ [(150)^2 - (r_2)^2] \times \frac{2}{3}\left[\frac{(150)^3 - (r_2)^3}{(150)^2 - (r_2)^2}\right] \qquad ...(\because n = 2)$$

$$= 0.285\ [(150)^3 - (r_2)^3]$$

$$(150)^3 - (r_2)^3 = 840 \times 10^3 / 0.285 = 2.95 \times 10^6$$

$$\therefore \quad (r_2)^3 = (150)^3 - 2.95 \times 10^6 = 0.425 \times 10^6 \text{ or } r_2 = 75 \text{ mm}$$

$$d_2 = 2r_2 = 2 \times 75 = 150 \text{ mm}$$ **Ans.**

(b) Maximum torque transmitted

We know that the axial thrust,

$$W = 0.534\ [(150)^2 - (r_2)^2] \qquad \text{... [From equation (i)]}$$

$$= 0.534\ [(150)^2 - (75)^2] = 9011 \text{ N}$$

and mean radius of the contact surfaces for uniform wear conditions,

$$R = \frac{r_1 + r_2}{2} = \frac{150 + 75}{2} = 112.5 \text{ mm}$$

$\therefore$ Maximum torque transmitted,

$$T = n.\mu.W.R = 2 \times 0.4 \times 9011 \times 112.5 = 811 \times 10^3 \text{ N-mm}$$

$$= 811 \text{ N-m}$$ **Ans.**

Maximum intensity of pressure

For uniform wear conditions, $p.r = C$ (a constant). Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$$p_{max} \times r_2 = C \quad \text{or} \quad C = p_{max} \times 75 \text{ N/mm}$$

We know that the axial thrust (W),

$$9011 = 2\pi C (r_1 - r_2) = 2\pi \times p_{max} \times 75\ (150 - 75) = 35\ 347\ p_{max}$$

$$\therefore \quad p_{max} = 9011 / 35\ 347 = 0.255 \text{ N/mm}^2$$ **Ans.**

Pbm 2: A single dry plate clutch is to be designed to transmit 7.5 kW at 900 r.p.m. Find :1. Diameter of the shaft, 2. Mean radius and face width of the friction lining assuming the ratio of the mean radius to the face width as 4, 3. Outer and inner radii of the clutch plate, and 4. Dimensions of the spring, assuming that the number of springs are 6 and spring index = 6. The allowable shear stress for the spring wire may be taken as 420 MPa. (May June 2012)

Given Data: P = 7.5 kW = 7500 W; N = 900 r.p.m. ; r / b = 4 ; No. of springs = 6 ; C = D/d = 6 ; τ = 420 MPa = 420 N/mm^2

1. Diameter of the shaft

Let d_s = Diameter of the shaft, and

τ_1 = Shear stress for the shaft material. It may be assumed as 40 N/mm^2.

We know that the torque transmitted,

$$T = \frac{P \times 60}{2\pi N} = \frac{7500 \times 60}{2\pi \times 900} = 79.6 \text{ N-m} = 79\ 600 \text{ N-mm} \qquad ...(i)$$

We also know that the torque transmitted (T),

$$79\ 600 = \frac{\pi}{16} \times \tau_1\ (d_s)^3 = \frac{\pi}{16} \times 40\ (d_s)^3 = 7.855\ (d_s)^3$$

$$\therefore \quad (d_s)^3 = 79\ 600 / 7.855 = 10\ 134 \text{ or } d_s = 21.6 \text{ say } 25 \text{ mm}$$ **Ans.**

2. *Mean radius and face width of the friction lining*

Let R = Mean radius of the friction lining, and

b = Face width of the friction lining = $R/4$

We know that the area of the friction faces,

$$A = 2\pi\, R.b$$

∴ Normal or the axial force acting on the friction faces,

$$W = A \times p = 2\pi\, R.b.p$$

and torque transmitted, $T = \mu\ W.R.n = \mu\ (2\pi\ Rb.p)\ R.n$

$$= \mu\left(2\,\pi R \times \frac{R}{4} \times p\right) R.n = \frac{\pi}{2} \times \mu.R^3.p.n \qquad ...(ii)$$

Assuming the intensity of pressure (p) as 0.07 N/mm^2 and coefficient of friction (μ) as 0.25, we have from equations (*i*) and (*ii*),

$$79\ 600 = \frac{\pi}{2} \times 0.25 \times R^3 \times 0.07 \times 2 = 0.055\ R^3$$

... ($\because$ $n = 2$, for both sides of plate effective)

∴ $R^3 = 79\ 600 / 0.055 = 1.45 \times 10^6$ or $R = 113.2$ say 114 mm **Ans.**

and $b = R / 4 = 114 / 4 = 28.5$ mm **Ans.**

3. *Outer and inner radii of the clutch plate*

Let r_1 and r_2 = Outer and inner radii of the clutch plate respectively.

Since the face width (or radial width) of the plate is equal to the difference of the outer and inner radii, therefore,

$$b = r_1 - r_2 \quad \text{or} \quad r_1 - r_2 = 28.5 \text{ mm} \qquad ...(iii)$$

We know that for uniform wear, mean radius of the clutch plate,

$$R = \frac{r_1 + r_2}{2} \quad \text{or} \quad r_1 + r_2 = 2R = 2 \times 114 = 228 \text{ mm} \qquad ...(iv)$$

From equations (*iii*), and (*iv*), we find that

r_1 = 128.25 mm and r_2 = 99.75 mm **Ans.**

4. *Dimensions of the spring*

Let D = Mean diameter of the spring, and

d = Diameter of the spring wire.

We know that the axial force on the friction faces,

$$W = 2\pi\ R.b.p = 2\pi \times 114 \times 28.5 \times 0.07 = 1429.2 \text{ N}$$

In order to allow for adjustment and for maximum engine torque, the spring is designed for an overload of 25%.

∴ Total load on the springs

$$= 1.25\ W = 1.25 \times 1429.2 = 1786.5 \text{ N}$$

Since there are 6 springs, therefore maximum load on each spring,

$$W_s = 1786.5 / 6 = 297.75 \text{ N}$$

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

We also know that maximum shear stress induced in the wire (τ),

$$420 = K \times \frac{8 W_s C}{\pi d^2} = 1.2525 \times \frac{8 \times 297.75 \times 6}{\pi d^2} = \frac{5697}{d^2}$$

$$\therefore \quad d^2 = 5697 / 420 = 13.56 \text{ or } d = 3.68 \text{ mm}$$

We shall take a standard wire of size *SWG* 8 having diameter (d) = 4.064 mm **Ans.**

and mean diameter of the spring,

$$D = C.d = 6 \times 4.064 = 24.384 \text{ say } 24.4 \text{ mm} \quad \textbf{Ans.}$$

Let us assume that the spring has 4 active turns (*i.e.* $n = 4$). Therefore compression of the spring,

$$\delta = \frac{8 W_s.C^3.n}{G.d} = \frac{8 \times 297.75 \times 6^3 \times 4}{84 \times 10^3 \times 4.064} = 6.03 \text{ mm}$$

... (Taking $G = 84 \times 10^3$ N/mm^2)

Assuming squared and ground ends, total number of turns,

$$n' = n + 2 = 4 + 2 = 6$$

We know that free length of the spring,

$$L_F = n'.d + \delta + 0.15\,\delta$$

$$= 6 \times 4.064 + 6.03 + 0.15 \times 6.03 = 31.32 \text{ mm} \quad \textbf{Ans.}$$

$$\text{pitch of the coils} = \frac{L_F}{n' - 1} = \frac{31.32}{6 - 1} = 6.264 \text{ mm} \quad \textbf{Ans.}$$

Pbm 3: A multiple disc clutch, steel on bronze, is to transmit 4.5 kW at 750 r.p.m. The inner radius of the contact is 40 mm and outer radius of the contact is 70 mm. The clutch operates in oil with an expected coefficient of 0.1. The average allowable pressure is 0.35 N/mm^2. Find : 1. the total number of steel and bronze discs; 2. the actual axial force required; 3. the actual average pressure; and 4. the actual maximum pressure. (Nov/Dec 2007)

Given Data : P = 4.5 kW = 4500 W; N = 750 r.p.m. ; r_2 = 40 mm ; r_1 = 70 mm ; μ = 0.1 ; p_{av} = 0.35 N/mm^2.

1. *Total number of steel and bronze discs*

Let n = Number of pairs of contact surfaces.

We know that the torque transmitted by the clutch,

$$T = \frac{P \times 60}{2\pi N} = \frac{4500 \times 60}{2\pi \times 750} = 57.3 \text{ N-m} = 57\,300 \text{ N-mm}$$

For uniform wear, mean radius of the contact surfaces,

$$R = \frac{r_1 + r_2}{2} = \frac{70 + 40}{2} = 55 \text{ mm}$$

and average axial force required,

$$W = p_{av} \times \pi [(r_1)^2 - (r_2)^2] = 0.35 \times \pi [(70)^2 - (40)^2] = 3630 \text{ N}$$

We also know that the torque transmitted (T),

$$57\,300 = n.\mu.W.R = n \times 0.1 \times 3630 \times 55 = 19\,965\ n$$

$$\therefore \quad n = 57\,300 / 19\,965 = 2.87$$

Since the number of pairs of contact surfaces must be even, therefore we shall use 4 pairs of contact surfaces with 3 steel discs and 2 bronze discs (because the number of pairs of contact surfaces is one less than the total number of discs). **Ans.**

2. Actual axial force required

Let W' = Actual axial force required.

Since the actual number of pairs of contact surfaces is 4, therefore actual torque developed by the clutch for one pair of contact surface,

$$T' = \frac{T}{n} = \frac{57\,300}{4} = 14\,325 \text{ N-mm}$$

We know that torque developed for one pair of contact surface (T'),

$$14\,325 = \mu.W'.R = 0.1 \times W' \times 55 = 5.5\ W'$$

$$\therefore \quad W' = 14\,325 / 5.5 = 2604.5 \text{ N}$$ **Ans.**

3. Actual average pressure

We know that the actual average pressure,

$$p'_{av} = \frac{W'}{\pi[(r_1)^2 - (r_2)^2]} = \frac{2604.5}{\pi\,[(70)^2 - (40)^2]} = 0.25 \text{ N/mm}^2$$ **Ans.**

4. Actual maximum pressure

Let p_{max} = Actual maximum pressure.

For uniform wear, $p.r = C$. Since the intensity of pressure is maximum at the inner radius, therefore,

$$p_{max} \times r_2 = C \quad \text{or} \quad C = 40\ p_{max} \text{ N/mm}$$

We know that the actual axial force (W'),

$$2604.5 = 2\pi C\,(r_1 - r_2) = 2\pi \times 40\ p_{max}\,(70 - 40) = 7541\ p_{max}$$

$$\therefore \quad p_{max} = 2604.5 / 7541 = 0.345 \text{ N/mm}^2$$ **Ans.**

Pbm 4: **A multi-disc clutch has three discs on the driving shaft and two on the driven shaft. The inside diameter of the contact surface is 120 mm. The maximum pressure between the surface is limited to 0.1 N/mm^2. Design the clutch for transmitting 25 kW at 1575 r.p.m. Assume uniform wear condition and coefficient of friction as 0.3. (May / June 2009)**

Given Data: $n_1 = 3$; $n_2 = 2$; $d_2 = 120$ mm or $r_2 = 60$ mm ; $p_{max} = 0.1$ N/mm^2 ; P = 25 kW = 25 × 103 W ; N = 1575 r.p.m. ; μ = 0.3

$$T = \frac{P \times 60}{2\,\pi\,N} = \frac{25 \times 10^3 \times 60}{2\,\pi \times 1575} = 151.6 \text{ N-m} = 151\,600 \text{ N-mm}$$

For uniform wear, we know that $p.r = C$. Since the intensity of pressure is maximum at the inner radius (r_2), therefore,

$$p_{max} \times r_2 = C \qquad \text{or} \qquad C = 0.1 \times 60 = 6 \text{ N/mm}$$

We know that the axial force on each friction surface,

$$W = 2\pi C\,(r_1 - r_2) = 2\pi \times 6(r_1 - 60) = 37.7\,(r_1 - 60) \qquad \ldots(i)$$

For uniform wear, mean radius of the contact surface,

$$R = \frac{r_1 + r_2}{2} = \frac{r_1 + 60}{2} = 0.5\,r_1 + 30$$

We know that number of pairs of contact surfaces,

$$n = n_1 + n_2 - 1 = 3 + 2 - 1 = 4$$

$\therefore$ Torque transmitted (T),

$$151\,600 = n.\mu.W.R = 4 \times 0.3 \times 37.7\,(r_1 - 60)\,(0.5\,r_1 + 30)$$

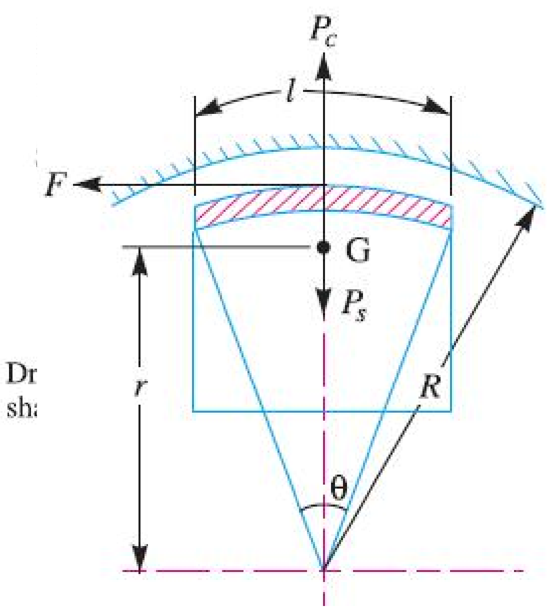
... [Substituting the value of W from equation (i)]

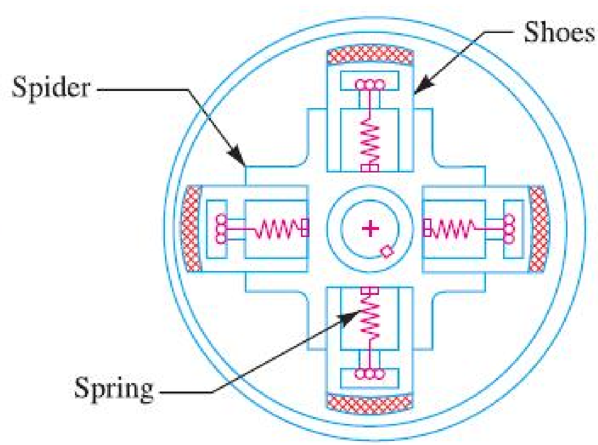
$$= 22.62\,(r_1)^2 - 81\,432$$

$$\therefore \qquad (r_1)^2 = \frac{151\,600 + 81\,432}{22.62} = 10\,302$$

or $\qquad r_1 = 101.5$ mm **Ans.**

CENTRIFUGAL CLUTCH:





P_c =Centrifugal Force

P_s = Spring Force

F= Frictional Force

r = Distance of centre of gravity of shoe from the centre of the spider.

R = Inside radius of the pulley rim.

l = contact length of each shoes.

θ = Angle subtended by the shoes.

<u>Pbm 5 :</u> A centrifugal clutch is to be designed to transmit 15 kW at 900 r.p.m. The shoes are four in number. The speed at which the engagement begins is 3/4th of the running speed. The inside radius of the pulley rim is 150 mm. The shoes are lined with Ferrodo for which the coefficient of friction may be taken as 0.25. Determine: 1. mass of the shoes, and 2. size of the shoes.

Given Data: *P* = 15 kW = 15 × 103 W ; *N* = 900 r.p.m. ; *n* = 4 ; *R* = 150 mm = 0.15 m ; μ = 0.25

1. *Mass of the shoes*

Let m = Mass of the shoes.

We know that the angular running speed,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 900}{60} = 94.26 \text{ rad/s}$$

Since the speed at which the engagement begins is 3/4 th of the running speed, therefore angular speed at which engagement begins is

$$\omega_1 = \frac{3}{4}\omega = \frac{3}{4} \times 94.26 = 70.7 \text{ rad/s}$$

Assuming that the centre of gravity of the shoe lies at a distance of 120 mm (30 mm less than R) from the centre of the spider, *i.e.*

$$r = 120 \text{ mm} = 0.12 \text{ m}$$

We know that the centrifugal force acting on each shoe,

$$P_c = m.\omega^2.r = m\,(94.26)^2\,0.12 = 1066\,m \text{ N}$$

and the inward force on each shoe exerted by the spring *i.e.* the centrifugal force at the engagement speed, ω_1,

$$P_s = m(\omega_1)^2\,r = m\,(70.7)^2\,0.12 = 600\,m \text{ N}$$

We know that the torque transmitted at the running speed,

$$T = \frac{P \times 60}{2\pi N} = \frac{15 \times 10^3 \times 60}{2\pi \times 900} = 159 \text{ N-m}$$

We also know that the torque transmitted (T),

$$159 = \mu\,(P_c - P_s)\,R \times n = 0.25\,(1066\,m - 600\,m)\,0.15 \times 4 = 70\,m$$

$$\therefore \quad m = 159/70 = 2.27 \text{ kg} \quad \textbf{Ans.}$$

2. *Size of the shoes*

Let l = Contact length of shoes in mm, and

b = Width of the shoes in mm.

Assuming that the arc of contact of the shoes subtend an angle of $\theta = 60°$ or $\pi/3$ radians, at the centre of the spider, therefore

$$l = \theta.R = \frac{\pi}{3} \times 150 = 157 \text{ mm}$$

Area of contact of the shoes

$$A = l.b = 157\,b \text{ mm}^2$$

Assuming that the intensity of pressure (p) exerted on the shoes is 0.1 N/mm², therefore force with which the shoe presses against the rim

$$= A.p = 157b \times 0.1 = 15.7\, b\, \text{N} \quad ...(i)$$

We also know that the force with which the shoe presses against the rim

$$= P_c - P_s = 1066\, m - 600\, m = 466\, m$$

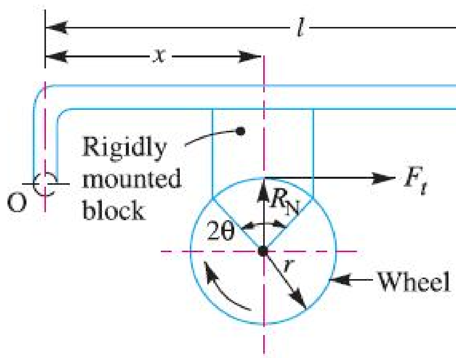
$$= 466 \times 2.27 = 1058 \text{ N} \quad ...(ii)$$

From equations (*i*) and (*ii*), we find that

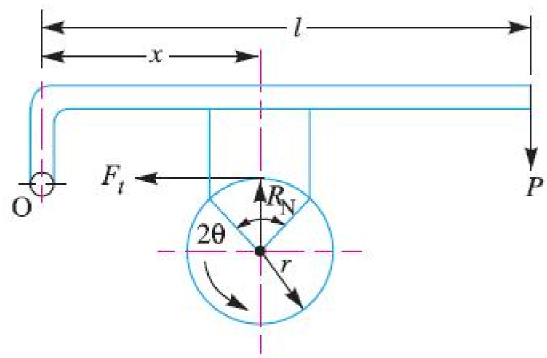
$$b = 1058 / 15.7 = 67.4 \text{ mm} \quad \textbf{Ans.}$$

BRAKE: A brake is a device by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of a machine. In the process of performing this function, the brake absorbs either kinetic energy of the moving member or potential energy given up by objects being lowered by hoists, elevators etc. The energy absorbed by brakes is dissipated in the form of heat. This heat is dissipated in the surrounding air (or water which is circulated through the passages in the brake drum) so that excessive heating of the brake lining does not take place.

Single Block or Shoe Brake:



(*a*) Clockwise rotation of brake wheel.



(*b*) Anticlockwise rotation of brake wheel.

Case 1. When the line of action of tangential braking force (F_t) passes through the fulcrum O of the lever, and the brake wheel rotates clockwise as shown in Fig. 25.1 (*a*), then for equilibrium, taking moments about the fulcrum O, we have

$$R_N \times x = P \times l \qquad \text{or} \qquad R_N = \frac{P \times l}{x}$$

$$\therefore \quad \text{Braking torque,} \quad T_B = \mu.R_N.r = \mu \times \frac{P.l}{x} \times r = \frac{\mu.P.l.r}{x}$$

It may be noted that when the brake wheel rotates anticlockwise as shown in Fig. 25.1 (*b*), then the braking torque is same, *i.e.*

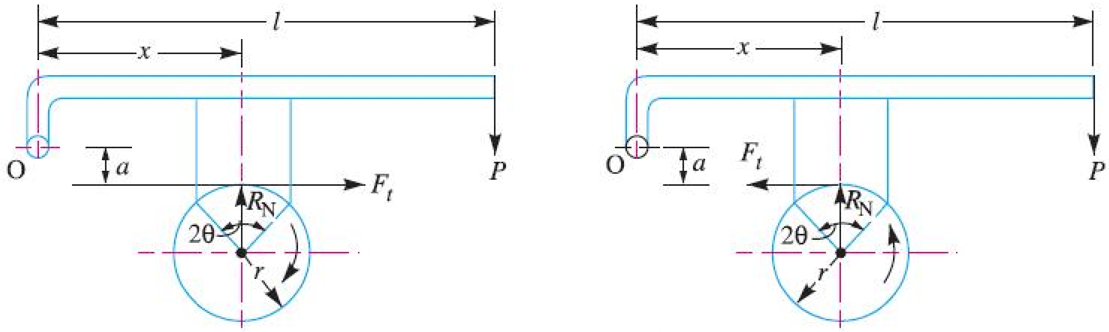
$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x}$$

Case 2. When the line of action of the tangential braking force (F_t) passes through a distance 'a' below the fulcrum O, and the brake wheel rotates clockwise as shown in Fig. 25.2 (*a*), then for equilibrium, taking moments about the fulcrum O,

$$R_N \times x + F_t \times a = P.l$$

$$\text{or} \quad R_N \times x + \mu\, R_N \times a = P.l \qquad \text{or} \qquad R_N = \frac{P.l}{x + \mu.a}$$

and braking torque, $$T_B = \mu\ R_N.r = \frac{\mu.P.l.r}{x + \mu.a}$$



(*a*) Clockwise rotation of brake wheel. (*b*) Anticlockwise rotation of brake wheel.

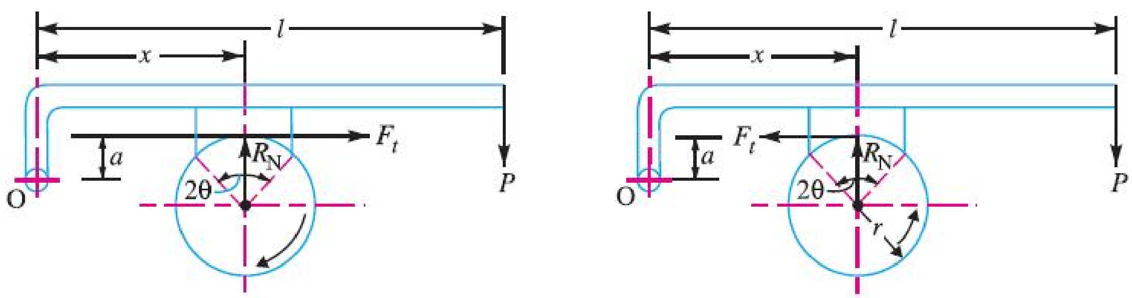
When the brake wheel rotates anticlockwise, as shown in Fig. 25.2 (*b*), then for equilibrium,

$$R_N.x = P.l + F_t.a = P.l + \mu.R_N.a \quad ...(i)$$

or $$R_N\ (x - \mu.a) = P.l \quad \text{or} \quad R_N = \frac{P.l}{x - \mu.a}$$

and braking torque, $$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x - \mu.a}$$

Case 3. When the line of action of the tangential braking force passes through a distance '*a*' above the fulcrum, and the brake wheel rotates clockwise as shown in Fig. 25.3 (*a*), then for equilibrium, taking moments about the fulcrum *O*, we have



(*a*) Clockwise rotation of brake wheel. (*b*) Anticlockwise rotation of brake wheel.

$$R_N.x = P.l + F_t.a = P.l + \mu.R_N.a \quad ...(ii)$$

or $$R_N\ (x - \mu.a) = P.l \quad \text{or} \quad R_N = \frac{P.l}{x - \mu.a}$$

and braking torque, $$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x - \mu.a}$$

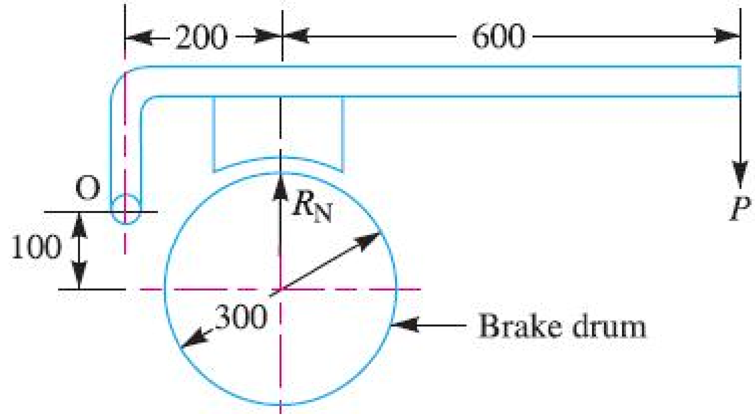
When the brake wheel rotates anticlockwise as shown in Fig. 25.3 (*b*), then for equilibrium, taking moments about the fulcrum *O*, we have

$$R_N \times x + F_t \times a = P.l$$

or $$R_N \times x + \mu.R_N \times a = P.l \quad \text{or} \quad R_N = \frac{P.l}{x + \mu.a}$$

and braking torque, $$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x + \mu.a}$$

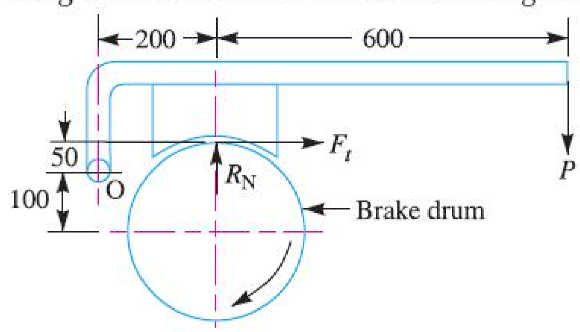
<u>Pbm 6 :</u> The block brake, as shown in Fig., provides a braking torque of 360 N-m. The diameter of the brake drum is 300 mm. The coefficient of friction is 0.3. Find : 1. The force (P) to be applied at the end of the lever for the clockwise and counter clockwise rotation of the brake drum; and 2. The location of the pivot or fulcrum to make the brake self locking for the clockwise rotation of the brake drum. (May / June 2007)

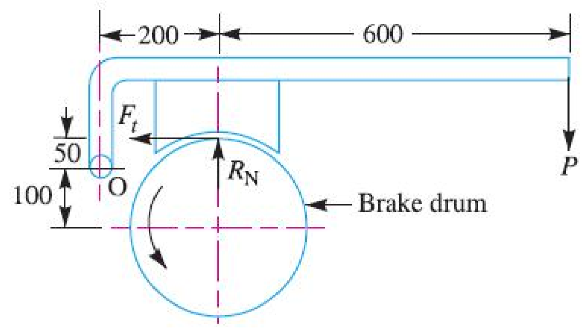


<u>Given Data:</u> T_B = 360 N-m = 360 × 103 N-mm ; d = 300 mm or r = 150 mm = 0.15 m ; μ = 0.3

1. Force (P) for the clockwise and counter clockwise rotation of the brake drum

For the clockwise rotation of the brake drum, the frictional force or the tangential force (F_t) acting at the contact surfaces is shown in Fig. 25.8.





We know that braking torque (T_B),

$$360 = F_t \times r = F_t \times 0.15 \quad \text{or} \quad F_t = 360 / 0.15 = 2400 \text{ N}$$

and normal force, $\quad R_N = F_t / \mu = 2400 / 0.3 = 8000$ N

Now taking moments about the fulcrum *O*, we have

$$P(600 + 200) + F_t \times 50 = R_N \times 200$$

$$P \times 800 + 2400 \times 50 = 8000 \times 200$$

$$P \times 800 = 8000 \times 200 - 2400 \times 50 = 1480 \times 10^3$$

$$\therefore \quad P = 1480 \times 10^3 / 800 = 1850 \text{ N} \quad \textbf{Ans.}$$

For the counter clockwise rotation of the drum, the frictional force or the tangential force (F_t) acting at the contact surfaces is shown in Fig. 25.9.

Taking moments about the fulcrum O, we have

$$P(600 + 200) = F_t \times 50 + R_N \times 200$$

$$P \times 800 = 2400 \times 50 + 8000 \times 200 = 1720 \times 10^3$$

$$\therefore \quad P = 1720 \times 10^3 / 800 = 2150 \text{ N} \quad \textbf{Ans.}$$

2. Location of the pivot or fulcrum to make the brake self-locking

The clockwise rotation of the brake drum is shown in Fig. 25.8. Let x be the distance of the pivot or fulcrum O from the line of action of the tangential force (F_t). Taking moments about the fulcrum O, we have

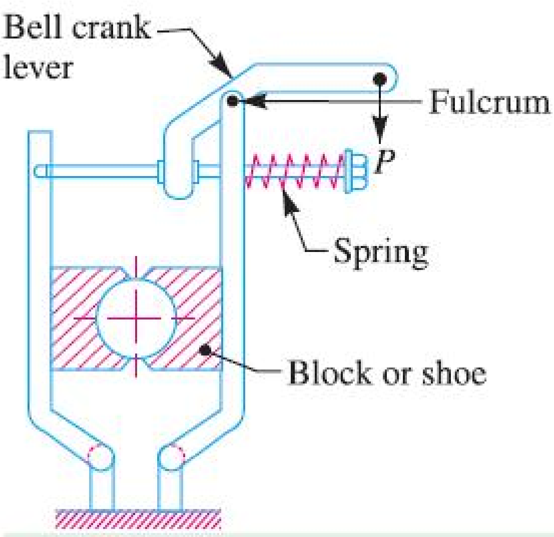
$$P(600 + 200) + F_t \times x - R_N \times 200 = 0$$

In order to make the brake self-locking, $F_t \times x$ must be equal to $R_N \times 200$ so that the force P is zero.

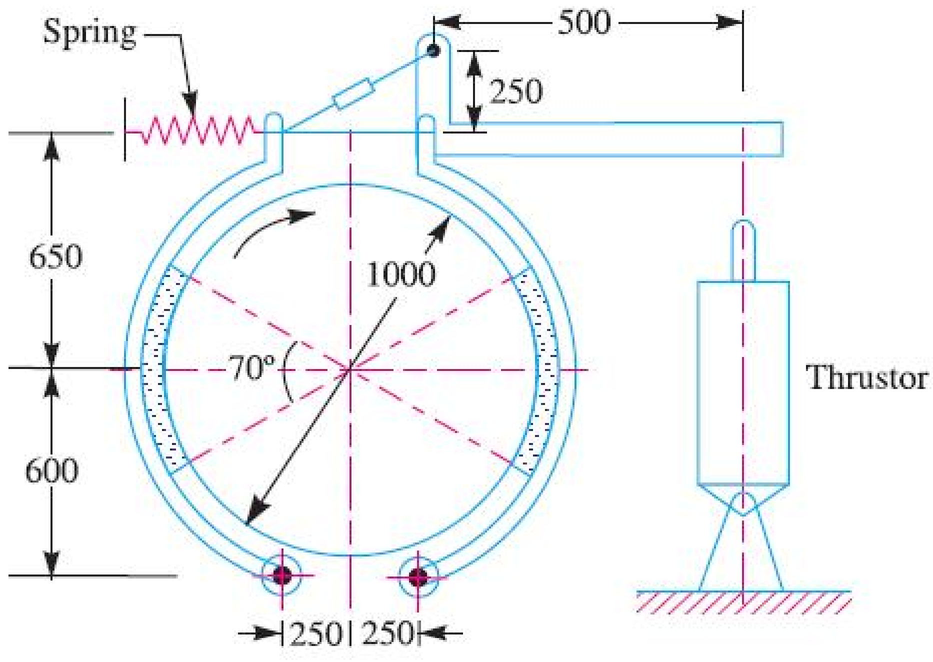
$$\therefore \quad F_t \times x = R_N \times 200$$

$$2400 \times x = 8000 \times 200 \quad \text{or} \quad x = 8000 \times 200 / 2400 = 667 \text{ mm} \quad \textbf{Ans.}$$

Double Block or Shoe Brake:

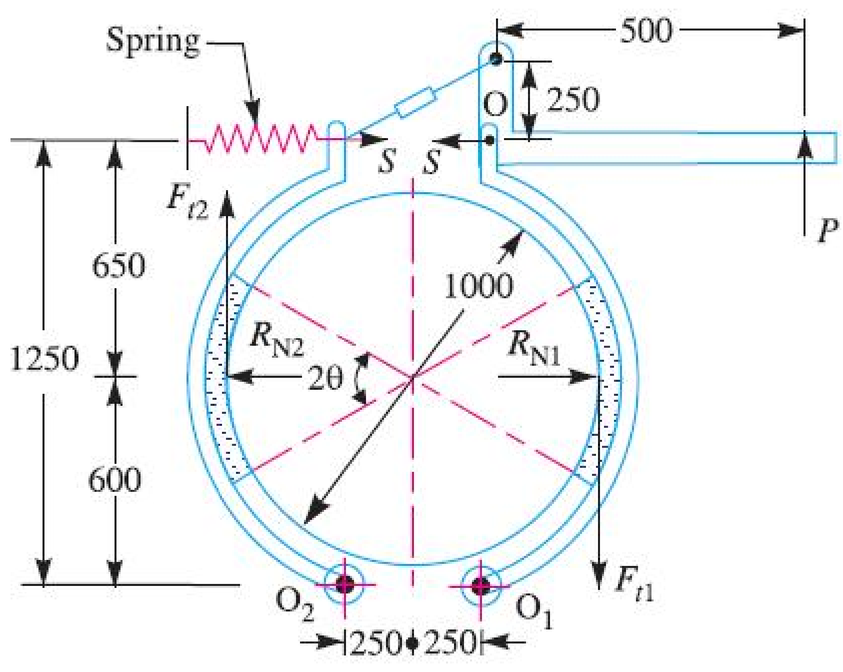


<u>Pbm 7:</u> A spring closed thrustor operated double shoe brake is to be designed for a maximum torque capacity of 3000 N-m. The brake drum diameter is not to exceed 1 metre and the shoes are to be lined with Ferrodo having a coefficient of friction 0.3. The other dimensions are as shown in Fig. 1. Find the spring force necessary to set the brake. 2. If the permissible stress of the spring material is 500 MPa, determine the dimensions of the coil assuming spring index to be 6. The maximum spring force is to be 1.3 times the spring force required during braking. There are eight active coils. Specify the length of the spring in the closed position of the brake. Modulus of rigidity is 80 kN / mm^2. 3. Find the width of the brake shoes if the bearing pressure on the lining material is not to exceed 0.5 N/mm^2. 4. Calculate the force required to be exerted by the thrustor to release the brake. (Nov/Dec 2007)



All dimensions in mm.

<u>Given Data:</u> TB = 3000 N-m = 3 × 106 N-mm ; d = 1 m or r = 0.5 m = 500 mm ; μ = 0.3 ; 2θ = 70° = 70 × π / 180 = 1.22 rad



All dimensions in mm.

1. *Spring force necessary to set the brake*

Let S = Spring force necessary to set the brake,

R_{N1} and F_{t1} = Normal reaction and the braking force on the right hand side shoe,

and R_{N2} and F_{t2} = Corresponding values for the left hand side shoe.

We know that deflection of the spring,

ion,

$$\delta = \frac{8 W_S . C^3 . n}{G.d} = \frac{8 \times 5736 \times 6^3 \times 8}{80 \times 10^3 \times 15} = 66 \text{ mm}$$

Taking moments about the fulcrum O_1 (Fig. 25.13), we have

$$S \times 1250 = R_{N1} \times 600 + F_{t1} (500 - 250)$$

$$= \frac{F_{t1}}{0.32} \times 600 + 250 F_{t1} = 2125 F_{t1} \qquad ...(\because R_{N1} = F_{t1} / \mu)$$

Again taking moments about the fulcrum O_2, we have

$$S \times 1250 + F_{t2} (500 - 250) = R_{N2} \times 600 = \frac{F_{t2}}{0.32} \times 600 = 1875 F_{t2} \qquad ...(\because R_{N2} = F_{t2} / \mu)$$

or $\quad 1875 F_{t2} - 250 F_{t2} = S \times 1250 \quad$ or $\quad 1625 F_{t2} = S \times 1250$

$\therefore \quad F_{t2} = S \times 1250 / 1625 = 0.77 S$ N

We know that torque capacity of the brake (T_B),

$$3 \times 10^6 = (F_{t1} + F_{t2})\, r = (0.59 S + 0.77 S)\, 500 = 680 S$$

$\therefore \quad S = 3 \times 10^6 / 680 = 4412$ N **Ans.**

2. *Dimensions of the spring coil*

Given : $\tau = 500$ MPa $= 500$ N/mm^2 ; $C = D/d = 6$; $n = 8$; $G = 80$ kN/mm^2 $= 80 \times 10^3$ N/mm^2

Let $\quad D$ = Mean diameter of the spring, and

$\quad d$ = Diameter of the spring wire.

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

Since the maximum spring force is 1.3 times the spring force required during braking, therefore maximum spring force,

$$W_S = 1.3 S = 1.3 \times 4412 = 5736 \text{ N}$$

We know that the shear stress induced in the spring (τ),

$$500 = \frac{K \times 8 W_S . C}{\pi d^2} = \frac{1.2525 \times 8 \times 5736 \times 6}{\pi d^2} = \frac{109\,754}{d^2}$$

$\therefore \quad d^2 = 109\,754 / 500 = 219.5 \quad$ or $\quad d = 14.8$ say 15 mm **Ans.**

and $\quad D = C.d = 6 \times 15 = 90$ mm **Ans.**

The length of the spring in the closed position of the brake will be its free length. Assuming that the ends of the coil are squared and ground, therefore total number of coils,

$$n' = n + 2 = 8 + 2 = 10$$

∴ Free length of the spring,

$$L_F = n'.d + \delta + 0.15\,\delta$$
$$= 10 \times 15 + 66 + 0.15 \times 66 = 226 \text{ mm } \textbf{Ans.}$$

3. *Width of the brake shoes*

Let b = Width of the brake shoes in mm, and

p_b = Bearing pressure on the lining material of the shoes.

$= 0.5$ N/mm^2 ...(Given)

We know that projected bearing area for one shoe,

$$A_b = b\,(2r.\sin\theta) = b\,(2 \times 500 \sin 35°) = 574\,b \text{ mm}^2$$

We know that normal force on the right hand side of the shoe,

$$R_{N1} = \frac{F_{t1}}{\mu'} = \frac{0.59\,S}{0.32} = \frac{0.59 \times 4412}{0.32} = 8135 \text{ N}$$

and normal force on the left hand side of the shoe,

$$R_{N2} = \frac{F_{t2}}{\mu'} = \frac{0.77\,S}{0.32} = \frac{0.77 \times 4412}{0.32} = 10\,616 \text{ N}$$

We see that the maximum normal force is on the left hand side of the shoe. Therefore we shall design the shoe for the maximum normal force *i.e.* R_{N2}.

We know that bearing pressure on the lining material (p_b),

$$0.5 = \frac{R_{N2}}{A_b} = \frac{10\,616}{574\,b} = \frac{18.5}{b}$$

∴ $b = 18.5 / 0.5 = 37$ mm **Ans.**

4. *Force required to be exerted by the thrustor to release the brake*

Let P = Force required to be exerted by the thrustor to release the brake.

Taking moments about the fulcrum of the lever O, we have

$$P \times 500 + R_{N1} \times 650 = F_{t1}\,(500 - 250) + F_{t2}\,(500 + 250) + R_{N2} \times 650$$

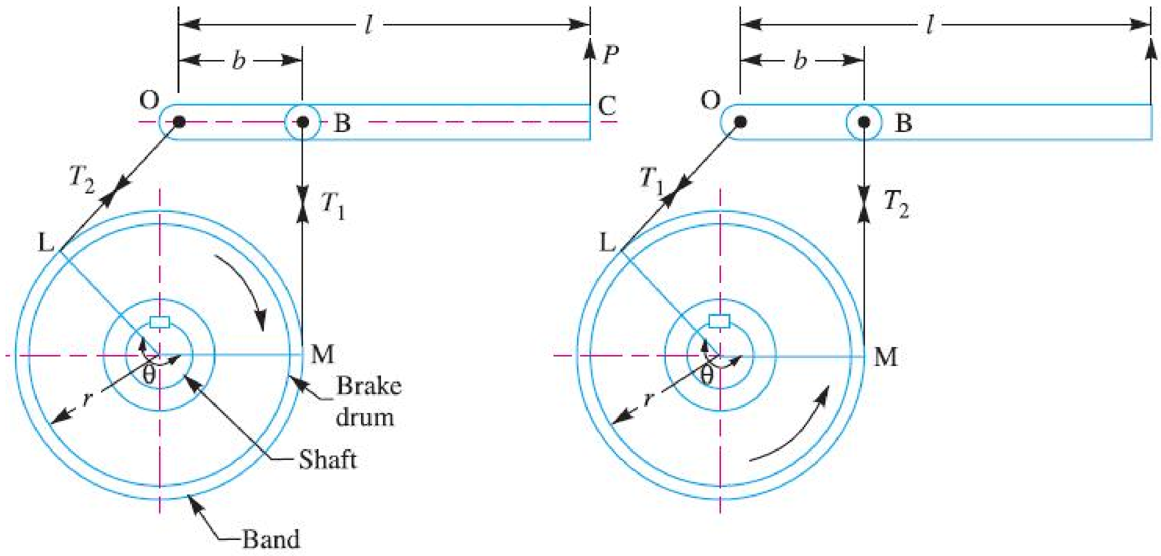
$$P \times 500 + 8135 \times 650 = 0.59 \times 4412 + 250 + 0.77 \times 4412 \times 750 + 10\,616 \times 650$$

...(Substituting $F_{t1} = 0.59\,S$ and $F_{t2} = 0.77\,S$)

$$P \times 500 + 5.288 \times 10^6 = 0.65 \times 10^6 + 2.55 \times 10^6 + 6.9 \times 10^6 = 10.1 \times 10^6$$

$$\therefore \quad P = \frac{10.1 \times 10^6 - 5.288 \times 10^6}{500} = 9624 \text{ N } \textbf{Ans.}$$

SIMPLE BAND BRAKE:



(*a*) Clockwise rotation of drum. (*b*) Anticlockwise rotation of drum.

Let T_1 = Tension in the tight side of the band,

T_2 = Tension in the slack side of the band,

θ = Angle of lap (or embrace) of the band on the drum,

μ = Coefficient of friction between the band and the drum,

r = Radius of the drum,

t = Thickness of the band, and

r_e = Effective radius of the drum = $r + t / 2$.

We know that limiting ratio of the tensions is given by the relation,

$$\frac{T_1}{T_2} = e^{\mu.\theta} \quad \text{or} \quad 2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta$$

and braking force on the drum

$$= T_1 - T_2$$

∴ Braking torque on the drum,

$$T_B = (T_1 - T_2)\, r \qquad \text{...(Neglecting thickness of band)}$$

$$= (T_1 - T_2)\, r_e \qquad \text{...(Considering thickness of band)}$$

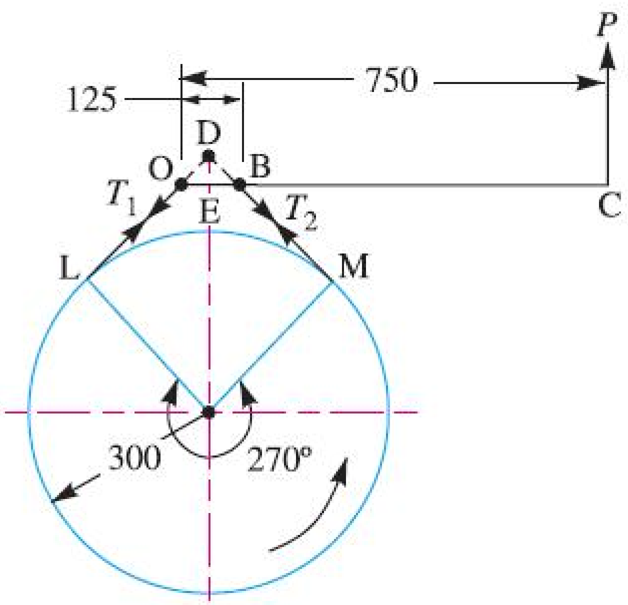
$$P.l = T_1.b \qquad \text{...(for clockwise rotation of the drum)}$$

$$P.l = T_2.b \qquad \text{...(for anticlockwise rotation of the drum)}$$

Pbm 8 : A simple band brake operates on a drum of 600 mm in diameter that is running at 200 r.p.m. The coefficient of friction is 0.25. The brake band has a contact of 270°, one end is fastened to a fixed pin and the other end to the brake arm 125 mm from the fixed pin. The straight brake arm is 750 mm long and placed perpendicular to the diameter that bisects the angle of contact. (a) What is the pull necessary on the end of the brake arm to stop the wheel if 35 kW is being absorbed? What is the direction for this minimum pull ? (b) What width of steel band of 2.5 mm thick is required for this brake if the maximum tensile stress is not to exceed 50 MPa?

Solution. Given : d = 600 mm or r = 300 mm ; N = 200 r.p.m. ; μ = 0.25 ; θ = 270° = 270 × π/180 = 4.713 rad ; Power = 35 kW = 35 × 103 W ; t = 2.5 mm ; σt = 50 MPa = 50 N/mm^2

Since one end of the band is attached to the fixed pin O, therefore the pull P on the end of the brake arm will act upward and when the wheel rotates anticlockwise, the end of the band attached to O will be tight with tension T_1 and the end of the band attached to B will be slack with tension T_2.



All dimensions in mm.

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.25 \times 4.713$$

$$= 1.178$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = 1.178 / 2.3 = 0.5123$$

$$\frac{T_1}{T_2} = 3.25 \qquad \ldots(i)$$

...(Taking antilog of 0.5123)

Let $\quad T_B$ = Braking torque.

We know that power absorbed,

$$35 \times 10^3 = \frac{2\pi N.T_B}{60} = \frac{2\pi \times 200 \times T_B}{60} = 21\ T_B$$

$$\therefore \quad T_B = 35 \times 10^3 / 21 = 1667 \text{ N-m} = 1667 \times 10^3 \text{ N-mm}$$

We also know that braking torque (T_B),

$$1667 \times 10^3 = (T_1 - T_2)\ r = (T_1 - T_2)\ 300$$

$$\therefore \quad T_1 - T_2 = 1667 \times 10^3 / 300 = 5557 \text{ N}$$

From equations (*i*) and (*ii*), we find that

$$T_1 = 8027 \text{ N} ; \text{ and } T_2 = 2470 \text{ N}$$

Now taking moments about O, we have

$$P \times 750 = T_2 \times {}^*OD = T_2 \times 62.5\sqrt{2} = 2470 \times 88.4 = 218\ 348$$

$$\therefore \quad P = 218\ 348 / 750 = 291 \text{ N} \quad \textbf{Ans.}$$

NOTE:

* OD = Perpendicular distance from O to the line of action of tension T_2.

$OE = EB = OB / 2 = 125 / 2 = 62.5$ mm, and $\angle DOE = 45°$

$\therefore \quad OD = OE \sec 45° = 62.5\sqrt{2}$ mm

(*b*) *Width of steel band*

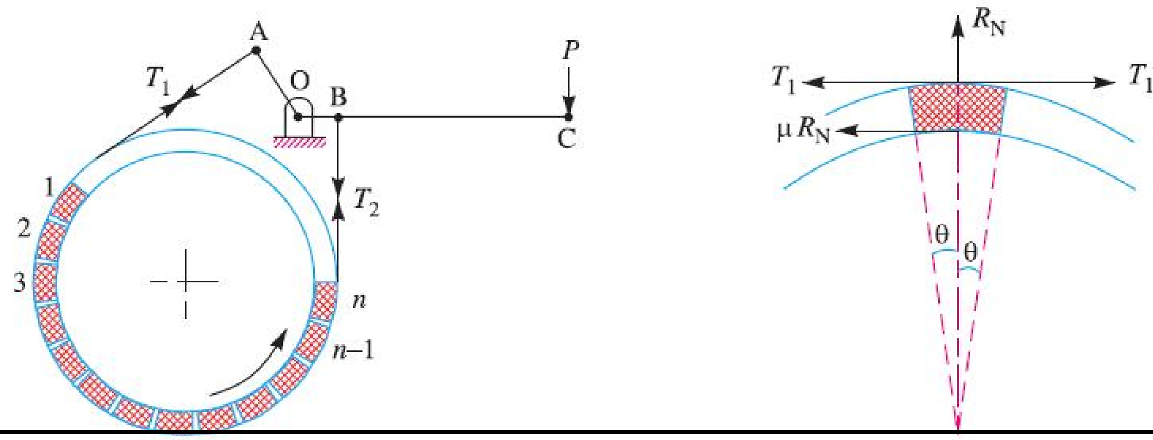
Let w = Width of steel band in mm.

We know that maximum tension in the band (T_1),

$$8027 = \sigma_t \times w \times t = 50 \times w \times 2.5 = 125\ w$$

$$\therefore \quad w = 8027/125 = 64.2 \text{ mm} \quad \textbf{Ans.}$$

BAND AND BLOCK BRAKE:



(*a*) (*b*)

$$\therefore \quad \frac{T_1}{T_2} = \frac{T_1}{T_1'} \times \frac{T_1'}{T_2'} \times \frac{T_2'}{T_3'} \times ... \times \frac{T_{n-1}}{T_2} = \left(\frac{1 + \mu \tan\theta}{1 - \mu \tan\theta}\right)^n \quad ...(iii)$$

Braking torque on the drum of effective radius r_e,

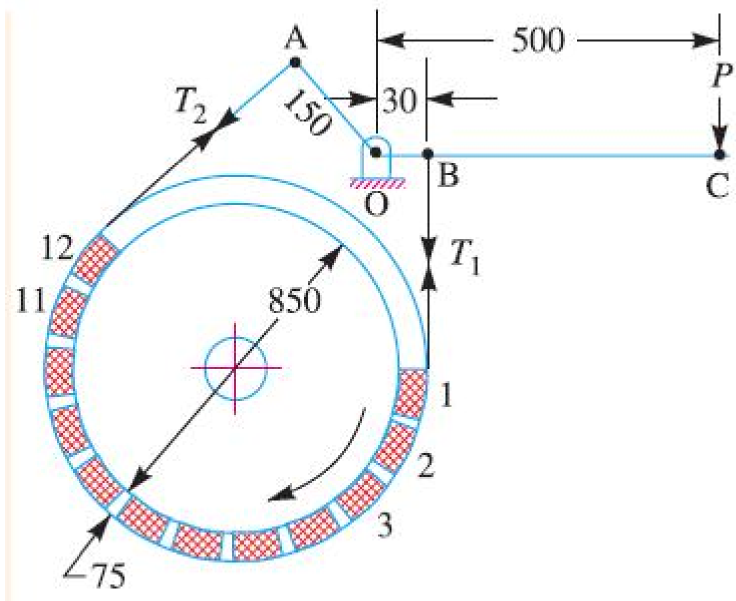
$$T_B = (T_1 - T_2)\, r_e$$

$$= (T_1 - T_2)\, r \quad \text{...(Neglecting thickness of band)}$$

Pbm 9: **In the band and block brake shown in Fig., the band is lined with 12 blocks each of which subtends an angle of 15° at the centre of the rotating drum. The thickness of the blocks is 75 mm and the diameter of the drum is 850 mm. If, when the brake is inaction, the greatest and least tensions in the brake strap are T_1 and T_2, show that**

$$\frac{T_1}{T_2} = \left(\frac{1 + \mu \tan 7\,^1/_2{}^\circ}{1 - \mu \tan 7\,^1/_2{}^\circ}\right)^{12}$$

where μ is the coefficient of friction for the blocks. With the lever arrangement as shown in Fig., find the least force required at C for the blocks to absorb 225 kW at 240 r.p.m. The coefficient of friction between the band and blocks is 0.4. (Nov / Dec 2008)

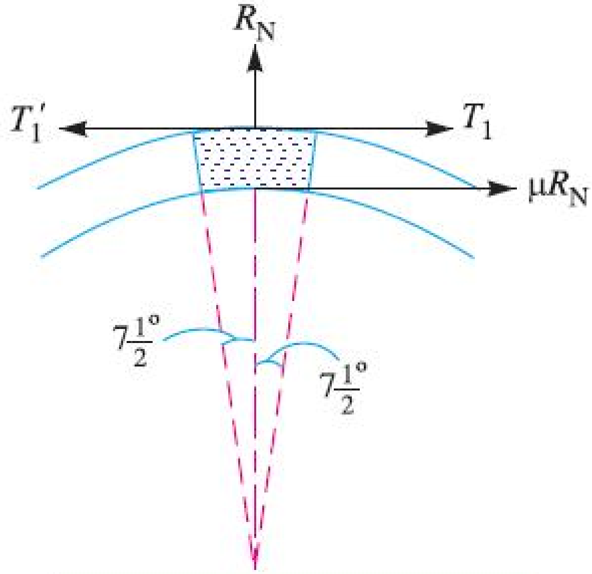


All dimensions in mm.

Given Data: n = 12 ; 2 θ = 15° or θ = 7 1/2° ; t = 75 mm = 0.075 m ; d = 850 mm = 0.85 m ;
Power = 225 kW = 225 × 103 W ; N = 240 r.p.m. ; μ = 0.4

Since *OA* > *OB*, therefore the force at *C* must act downward. Also, the drum rotates clockwise, therefore the end of the band attached to *A* will be slack with tension T_2 (least tension) and the end of the band attached to *B* will be tight with tension T_1 (greatest tension). Consider one of the blocks (say first block) as shown is Fig. This is in equilibrium under the action of the following four forces :

1. Tension in the tight side (T_1), **2.** Tension in the slack side (T_1') or the tension in the band between the first and second block, **3.** Normal reaction of the drum on the block (R_N), and **4.** The force of friction ($\mu.R_N$).



Resolving the forces radially, we have

$$(T_1 + T_1')\sin 7\tfrac{1}{2}° = R_N$$

Resolving the forces tangentially, we have

$$(T_1 - T_1')\cos 7\tfrac{1}{2}° = \mu.R_N$$

Dividing equation *(ii)* by *(i)*, we have

$$\frac{(T_1 - T_1')\cos 7\tfrac{1}{2}°}{(T_1 + T_1')\sin 7\tfrac{1}{2}°} = \mu \quad \text{or} \quad \frac{T_1 - T_1'}{T_1 + T_1'} = \mu \tan 7\tfrac{1}{2}°$$

$$\therefore \quad \frac{T_1}{T_1'} = \frac{1 + \mu \tan 7\tfrac{1}{2}°}{1 - \mu \tan 7\tfrac{1}{2}°}$$

Similarly, for the other blocks, the ratio of tensions $\frac{T_1'}{T_2'} = \frac{T_2'}{T_3'}$ etc., remains constant. Therefore for 12 blocks having greatest tension T_1 and least tension T_2 is

$$\frac{T_1}{T_2} = \left(\frac{1 + \mu \tan 7\tfrac{1}{2}°}{1 - \mu \tan 7\tfrac{1}{2}°}\right)^{12}$$

Least force required at C

Let P = Least force required at C.

We know that diameter of band,

$$D = d + 2\,t = 0.85 + 2 \times 0.075 = 1 \text{ m}$$

and power absorbed $= \dfrac{(T_1 - T_2)\,\pi\,D.N}{60}$

$$\therefore \quad T_1 - T_2 = \frac{\text{Power} \times 60}{\pi DN} = \frac{225 \times 10^3 \times 60}{\pi \times 1 \times 240} = 17\,900 \text{ N} \qquad ...(iii)$$

We have proved that

$$\frac{T_1}{T_2} = \left(\frac{1 + \mu \tan 7\tfrac{1}{2}^\circ}{1 - \mu \tan 7\tfrac{1}{2}^\circ}\right)^{12} = \left(\frac{1 + 0.4 \times 0.1317}{1 - 0.4 \times 0.1317}\right)^{12} = 3.55 \qquad ...(iv)$$

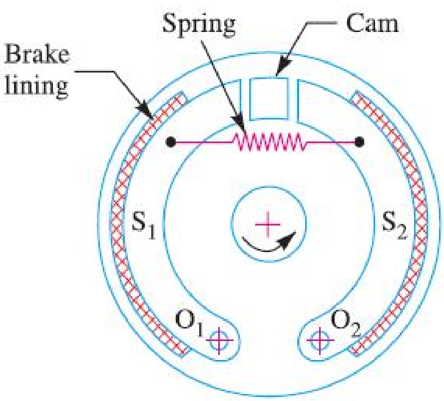
From equations *(iii)* and *(iv)*, we find that

$$T_1 = 24\,920 \text{ N} \;;\; \text{and } T_2 = 7020 \text{ N}$$

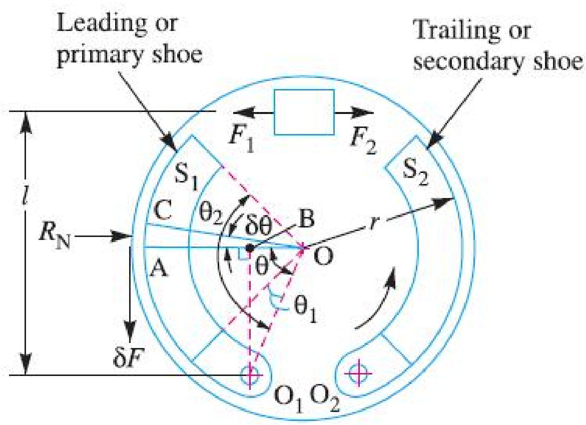
Now taking moments about O, we have

$$P \times 500 = T_2 \times 150 - T_1 \times 30 = 7020 \times 150 - 24\,920 \times 30 = 305\,400$$

$$\therefore \quad P = 305\,400 / 500 = 610.8 \text{ N} \textbf{ Ans.}$$



(*a*) Internal expanding brake.



(*b*) Forces on an internal expanding brake.

Now for leading shoe, taking moments about the fulcrum O_1,

$$F_1 \times l = M_N - M_F$$

and for trailing shoe, taking moments about the fulcrum O_2,

$$F_2 \times l = M_N + M_F$$

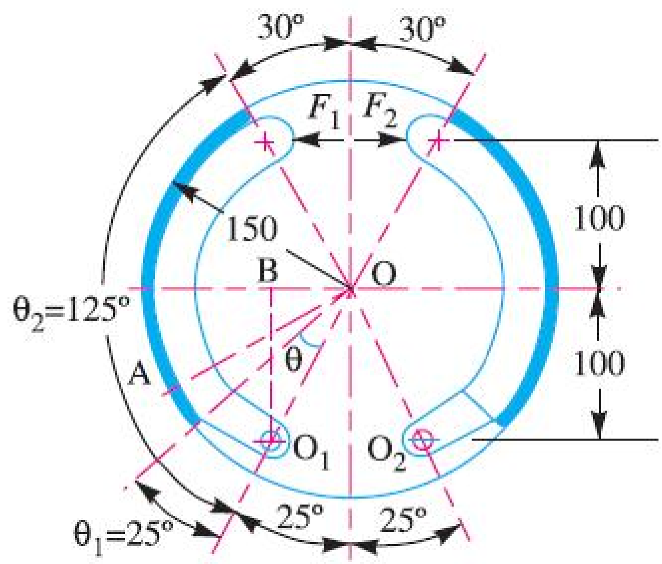
M_N = moment of Normal forces about the fulcrum O_1

$M_N \quad = \frac{1}{2} p_1 . b . r . OO_1 \left[(\theta_2 - \theta_1) + \frac{1}{2}(\sin 2\theta_1 - \sin 2\theta_2)\right]$

M_F = Moment of frictional force about the fulcrum O_1

$M_F \quad = \mu . p_1 . b . r \left[r(\cos\theta_1 - \cos\theta_2) + \frac{OO_1}{4}(\cos 2\theta_2 - \cos 2\theta_1) \right]$

<u>Pbm 10:</u> Fig. shows the arrangement of two brake shoes which act on the internal surface of a cylindrical brake drum. The braking force F_1 and F_2 are applied as shown and each shoe pivots on its fulcrum O_1 and O_2. The width of the brake lining is 35 mm. The intensity of pressure at any point A is 0.4 sin θ N/mm^2, where θ is measured as shown from either pivot. The coefficient of friction is 0.4. Determine the braking torque and the magnitude of the forces F_1 and F_2. (Apr/May 2008)



All dimensions in mm.

Given Data: b = 35 mm; μ = 0.4; r = 150 mm; l = 200 mm; θ_1 = 25°; θ_2 = 125°

Since the intensity of normal pressure at any point is 0.4 sin θ N/mm^2, therefore maximum intensity of normal pressure,

$p_1 = 0.4 \text{ N/mm}^2$

We know that the braking torque for one shoe,

$= \mu . p_1 . b . r^2 (\cos\theta_1 - \cos\theta_2)$

$= 0.4 \times 0.4 \times 35 (150)^2 (\cos 25° - \cos 125°)$

$= 126\ 000\ (0.9063 + 0.5736) = 186\ 470$ N-mm

∴ Total braking torque for two shoes,

$T_B = 2 \times 186\ 470 = 372\ 940$ N-mm

Magnitude of the forces F_1 and F_2

From the geometry of the figure, we find that

$$OO_1 = \frac{O_1B}{\cos 25^\circ} = \frac{100}{0.9063} = 110.3 \text{ mm}$$

$$\theta_1 = 25^\circ = 25 \times \pi / 180 = 0.436 \text{ rad}$$

and $$\theta_2 = 125^\circ = 125 \times \pi / 180 = 2.18 \text{ rad}$$

We know that the total moment of normal forces about the fulcrum O_1,

$$M_N = \tfrac{1}{2}\, p_1 \,.\, b \,.\, r \,.\, OO_1 \,[(\theta_2 - \theta_1) + \tfrac{1}{2}(\sin 2\theta_1 - \sin 2\theta_2)]$$

$$= \tfrac{1}{2} \times 0.4 \times 35 \times 150 \times 110.3\,[(2.18 - 0.436) + \tfrac{1}{2}(\sin 50^\circ - \sin 250^\circ)\,]$$

$$= 115\,815 \left[1.744 + \frac{1}{2}(0.766 + 0.9397)\right] = 300\,754 \text{ N-mm}$$

and total moment of friction force about the fulcrum O_1,

$$M_F = \mu \,.\, p_1 \,.\, b \,.\, r \left[r\,(\cos\theta_1 - \cos\theta_2) + \frac{OO_1}{4}(\cos 2\theta_2 - \cos 2\theta_1)\right]$$

$$= 0.4 \times 0.4 \times 35 \times 150 \left[150\,(\cos 25^\circ - \cos 125^\circ) + \frac{110.3}{4}(\cos 250^\circ - \cos 50^\circ)\right]$$

$$= 840\,[150\,(0.9063 + 0.5736) + 27.6\,(-\,0.342 - 0.6428)]$$

$$= 840\,(222 - 27) = 163\,800 \text{ N-mm}$$

For the leading shoe, taking moments about the fulcrum O_1,

$$F_1 \times l = M_N - M_F$$

or $$F_1 \times 200 = 300\,754 - 163\,800 = 136\,954$$

$$\therefore\quad F_1 = 136\,954 / 200 = 685 \text{ N}$$ **Ans.**

For the trailing shoe, taking moments about the fulcrum O_2,

$$F_2 \times l = M_N + M_F$$

or $$F_2 \times 200 = 300\,754 + 163\,800 = 464\,554$$

$$\therefore\quad F_2 = 464\,554 / 200 = 2323 \text{ N}$$ **Ans.**