Asymptotic Notation & Properties:

→ An order of growth & Running time of alg gives a simple characterization of alg ef

→ Complexity is usually represented in O, Θ in f(n) notation.

→ To choose the best alg we need to check efficiency of each alg.

→ Asymptotic alg is a shorthand way t represent the time complexity.

→ Using Asymptotic notations, we can give time complexity as,

- ↳ Latest possible (or) Best case
- ↳ Slowest " (or) Worst case
- ↳ Average " (or) Average case.

Various notation used:

1. Big-oh (O)
2. Omega (Ω)
3. Theta (Θ)

Big-oh notation:

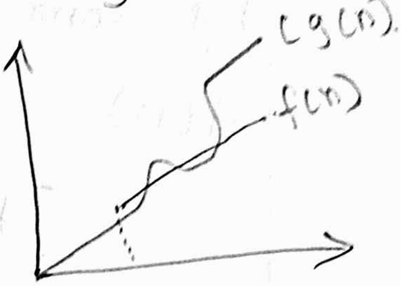
→ Its denoted by 'O'.

→ Its a method of representing the upper bound of alg running time.

- Using big-oh notations, we can give longest amount of time taken by the alg to complete.

Definition:

Let $f(n)$ & $g(n)$ be two non-negative functions. Let 'no' & constant 'c' are 2 integers. Such that n_0 denotes some value of I/P & $n > n_0$. Similarly 'c' is a constant such that $c > 0$,



we write,

$$\boxed{F(n) \leq c * g(n)}$$

then $f(n)$ is big-oh of $g(n)$. It's also denoted as $F(n) \in O(g(n))$

Ex: Consider the function $F(n) = 2n+2$ & $g(n) = n^2$. Then we have to find some constant c, so that $F(n) \leq c * g(n)$.

$n=1$:-

$F(n) = 2n+2$.

$= 2+2 = 4$.

$g(n) = n^2$

$= (1)^2 = 1$.

$F(n) > g(n)$.

$n=2$:-

$F(n) = 2(2)+2$

$= 4+2 = 6$

$g(n) = n^2$

$= (2)^2 = 4$

$F(n) > g(n)$.

$n=3$:-

$F(n) = 2(3)+2$

$= 6+2 = 8$

$g(n) = n^2$

$= (3)^2 = 9$

$F(n) < g(n)$. is true.

③ Omega Notation:

→ Its denoted by Ω

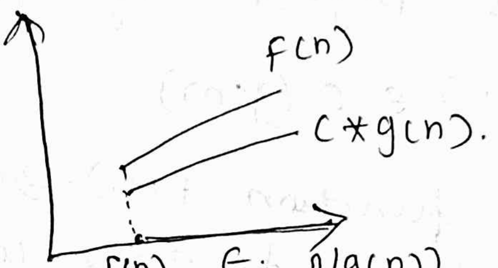
→ Its notation is used to represent the lower bound of alg's running time.

Definition:

A function $f(n)$ is said to be in $\Omega(g(n))$, if $f(n)$ is bounded below by some positive constant multiple of $g(n)$, such that,

$$\boxed{F(n) \geq c * g(n).}$$

→ Its denoted by $F(n) \in \Omega(g(n))$.



$f(n) \in \Omega(g(n))$

Ex: consider $F(n) = 2n^2+5$ & $g(n) = 7n$.

n=0:

$F(n) = 2(0)^2+5$
$= 5$

$g(n) = 7(0)$
$= 0.$

$F(n) > g(n).$

n=1:-

$F(n) = 2(1)^2+5$
$= 2+5 = 7$

$g(n) = 7(1)$
$= 7$

$F(n) = g(n).$

n=2:-

$F(n) = 2(3)^2+5$
$= 2(9)+5 = 18+5 = 23.$

$g(n) = 7(3)$
$= \cancel{14}.21$

$F(n) > g(n).$

∴ Hence for $n>3$, we get

$F(n) > c * g(n).$

③. Theta Notation:

→ Its denoted by Θ.

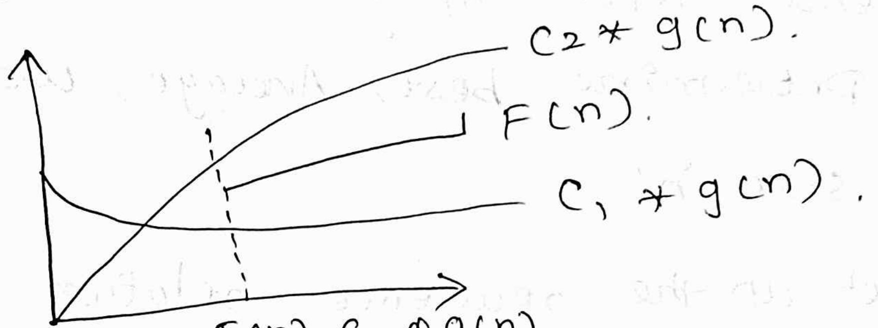
→ This notation is used to represent the upper & lower bound of alg's running time.

Def:

Let $f(n)$ & $g(n)$ be two non-negative functions. There are two positive constants namely C_1 & C_2, such that.

$$C_1 * g(n) \leq F(n) \leq C_2 * g(n).$$

→ Its denoted as $F(n) \in \Theta(g(n))$.



$F(n) \in \Theta g(n)$

Ex: $F(n) = 2n+8$ & $g(n) = 7n$ where $(n \geq 2)$

(i.e) $5n < 2n+8 \leq 7n$ for $n \geq 2$.

Here, $C_1 = 5$

$C_2 = 7$.

& $\boxed{n_0 = 2.}$