UNIT – I PROPERTIES OF MATTER

TOPIC – VI UNIFORM BENDING

2.18 UNIFORM BENDING – ELEVATION AT THE CENTRE OF THE BEAM LOADED AT BOTH ENDS

Theory

Let us consider a beam of negligible mass, supported symmetrically on the two knife edges A and B as shown. Let the length between A and B be 'l'. let equal weights W, be added to either end of the beam C and D.

Let the distance CA = BD = a.

Due to the load applied the beam bends from position F to E into an arc of a circle and produces as elevation 'x' from position F to E. let 'W' be the reaction produced at the points A and B which acts vertically upwards as shown in fig 2.19

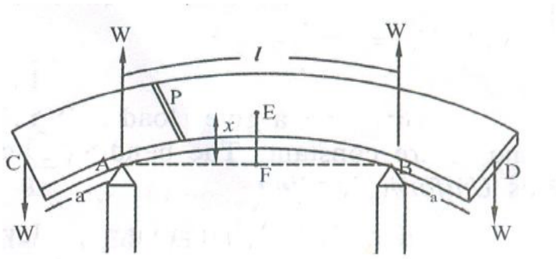

Fig: 2.19

Consider a point 'P' on the cross section of the beam. Then the forces acting on the part PC of the beam are

i. Force W at 'C'
ii. Reaction W at A as shown in fig 2.20.

Let the distance PC = a_1 and PA = a_2, then

The external bending moment about 'P' is

$$M_p = W \times a_1 - W \times a_2$$

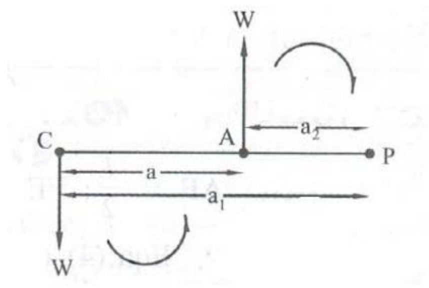

Fig 2.20

Here, the clockwise moment is taken as negative and anticlockwise moment is taken as positive.

External bending moment about P can be written as

$$M_p = W (a_1 - a_2)$$

$$M_p = Wa \qquad \text{--------------(1)}$$

We know the internal bending moment = $\frac{YI_g}{R}$ --------------(2)

Under equilibrium condition, External bending moment = Internal bending moment

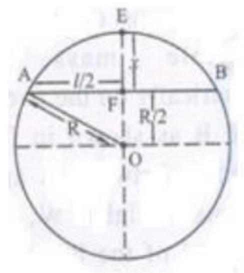

Fig 2.21

Since for a given load (W), Y, Ig, a and R are constant. The bending is called as Uniform Bending. Here it is found that the elevation 'x' forms an arc of the circle of radius 'R' as shown in fig 2.21. From the ΔAFO we can write

Since OF = FE, therefore we can write $OA^2 = AF^2 + FE^2$

Rearranging we can write

$$AF^2 = FE\left[\frac{OA^2}{FE} - FE\right] \qquad \text{----------------(4)}$$

Here AF = $\frac{1}{2}$; FE = x = $\frac{R}{2}$; OA=R

Eqn. (4) can be written as $$\left(\frac{l}{2}\right)^2 = x\left[\frac{R^2}{\frac{R}{2}} - x\right]$$

$$\frac{l^2}{4} = x[2R - x]$$

$$\frac{l^2}{4} = [2xR - x^2]$$

If the elevation 'x' is very small, then the term x^2 can be neglected.

We can write $$\frac{l^2}{4} = 2x\,R$$

(or) $$x = \frac{l^2}{8R}$$

Radius of curvature $$R = \frac{l^2}{8x} \qquad \text{-------------(5)}$$

Substituting the value of 'R' value in eqn. (3) we have

(or) $$W.a = \frac{YI_g}{\left(\frac{l^2}{8x}\right)}$$

(or) $$W.a = \frac{8YI_g x}{l^2}$$

Rearranging eqn. (6)

The Elevation of point 'E' above 'A' is $x = \frac{Wal^2}{8YI_g}$

2.19 YOUNG'S MODULUS BY UNIFORM BENDING

Statistical method

Description

It consists of a beam, symmetrically supported on the two knife edges A and B. Two weight hangers are suspended on either side of the beam at the position 'C' and 'D'. The distance between AC and BD are adjusted to be equal. A pin is fixed vertically at the centre of the beam as shown in fig 2.22. A travelling microscope is placed in front of the whole set up for finding the position of the pin.

Procedure:

Taking the weight hanger as the dead load (W), the microscope is adjusted and the tip of the pin is made to coincide with the vertical cross wire. The reading is noted from the vertical scale of the microscope. Now the load on each hanger is increased in equal steps of 'm', '2m', 3m etc. kilogram and the corresponding readings are noted from the vertical scale of the microscope. The same procedure is repeated during unloading.

The readings are noted from the vertical scale of the microscope. The readings are tabulated in the tabular column. The mean elevation 'x' of the centre for M kg is found.

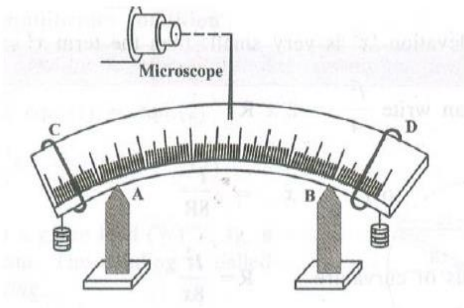

Fig. 2.22

The distance between the two knife edges is measured as 'l' and the distance from the point of suspension of the load to the knife edge is measured as 'a'.

Sl.No.	Load (M)	Microscope readings			Elevation (x)	(M/x)
		Increasing Load	Decreasing Load	Mean		
Unit	Kg	$\times 10^{-2}$m	$\times 10^{-2}$m	10^{-2}m	metre	Kg m^{-1}
1.	W			x_0		
2.	W+m			x_1		
3.	W+2m			x_2		
4.	W+3m			x_3		
5.	W+4m			x_4	$x_4 - x_0$	
6.	W+5m			x_5	$x_5 - x_1$	
7.	W+6m			x_6	$x_6 - x_2$	
8.	W+7m			x_7	$x_7 - x_3$	

Then we know the elevation produced is

$$x = \frac{Wal^2}{8YI_g} \qquad \text{-----------------(1)}$$

If 'b' is the breadth of the beam and 'd' is the thickness of the beam, then

For a rectangular bar geometrical moment of Inertia $I_g = \frac{bd^3}{12}$ ------------------(2)

Also we know, Weight W = Mg ------------------(3)

Substituting eqn.(2) and eqn.(3) in eqn.(1), we have

$$x = \frac{Mgal^2}{8Y\left(\frac{bd^3}{12}\right)} \qquad \text{-----------------(4)}$$

Rearranging eqn.(4) we can write, the Young's Modulus

$$Y = \frac{3Mgal^2}{2xbd^3}$$

(or) The Young's modulus $Y = \frac{3gal^2}{2xbd^3}\left(\frac{M}{x}\right)$ Nm^{-2}

Substituting the mean value of $\frac{M}{x}$ from the tabular column the Young's modulus Y of the material of the given beam can be calculated.

Graphical method (or) Dynamical method

A graph is drawn between load (M) along x axis and elevation (x) along y axis. It is found to be a straight line as shown in fig 2.23. The slope of the straight line gives the value of $\frac{x}{M}$. Eqn.(5) can be written as

$$Y = \frac{3gal^2}{2bd^3} = \frac{1}{Slope}$$

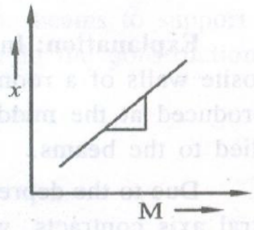

Fig. 2.23