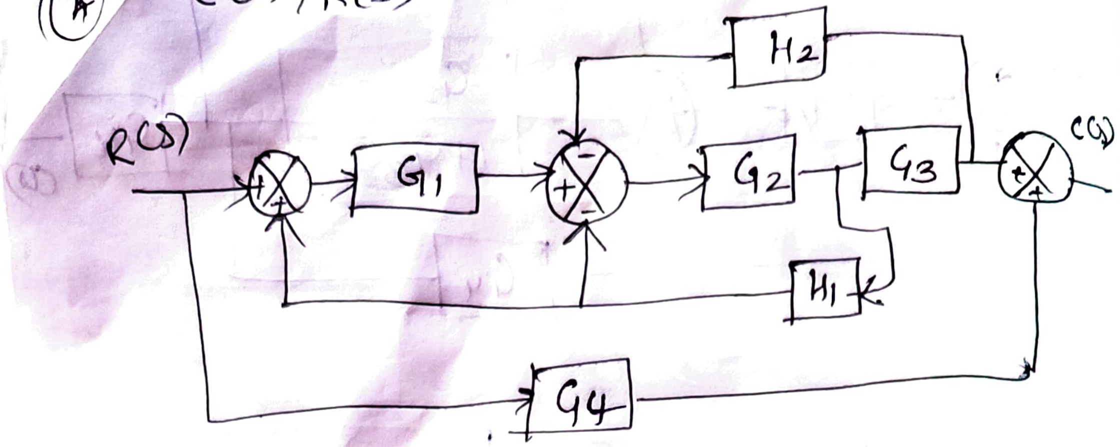
(4) C(s)/R(s)



1. splitting the summing points
2. eliminating the feedback path
3. shifting the branch point after the block
4. combining the blocks in cascade & eliminating the feedback path

Signal flow graph.

Terms

Node: Node is a point representing a variable or signal

Branch: Directed line segment joining two nodes.

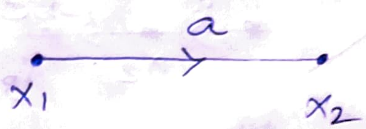
Input node: It has only outgoing branches

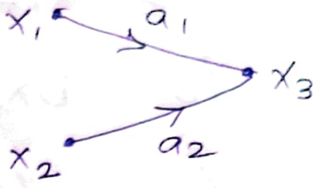
o/p node: It has only incoming branches

mixed node: Both incoming & outgoing branches.

Transmittance: The gain acquired by the signal when it travels from one node to another node.

Rules. (1) Incoming signal to a node through a branch is given by the product of a signal at previous node and gain of the branch.



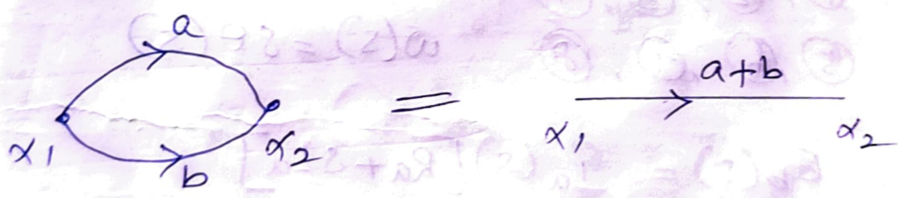


$$x_2 = a x_1$$

$$x_3 = a_1 x_1 + a_2 x_2$$

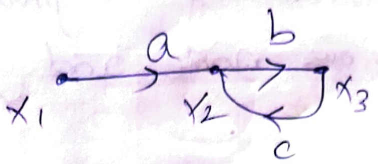
Rule (2) cascade branches can be combined to give a single branch whose transmittance is equal to the product of individual branch transmittance.

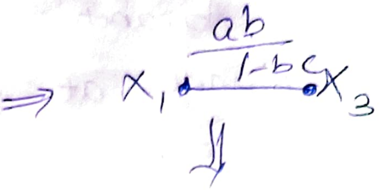
Rule (3) Parallel branches is sum of individual branch transmittance.

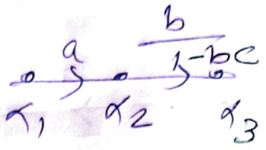


Rule (4) A mixed node can be eliminated by xplying the transmittance of outgoing branch

Rule 5. Loop elimination







$$x_2 = a x_1 + c x_3$$

$$x_3 = b x_2$$

$$x_3 = b(a x_1 + c x_3)$$

$$x_3 = ab x_1 + bc x_3$$

$$x_3 - bc x_3 = ab x_1$$

$$x_3(1-bc) = ab x_1$$

$$\frac{x_3}{x_1} = \frac{ab}{1-bc}.$$

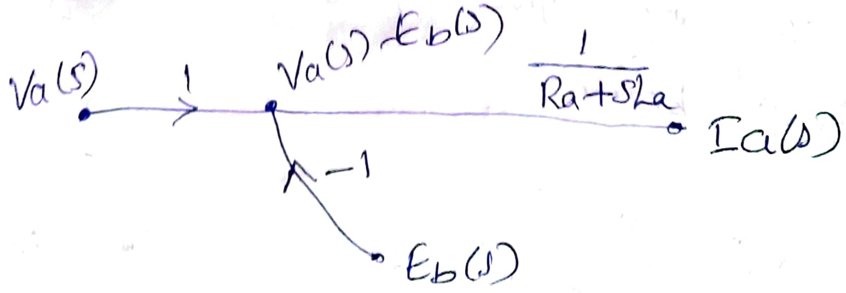
Problem 1

Construct a signal flow graph for armature controlled DC motor.

eq. 1, 2, 3, 4
5, 6, 7, 8 $\quad \omega(s) = s\theta(s)$

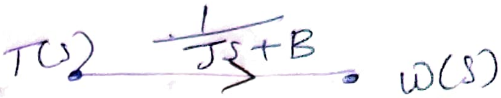
$$V_a(s) - E_b(s) = I_a(s)[R_a + sL_a]$$

$$I_a(s) = \frac{1}{R_a + sL_a}[V_a(s) - E_b(s)]$$



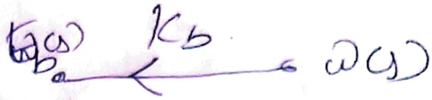
$T(s) = k_t I_a(s)$

$T(s) = \omega(s)[Js+B]$



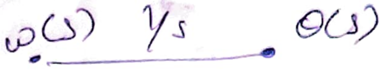
$$\omega(s) = \frac{T(s)}{Js+B}$$

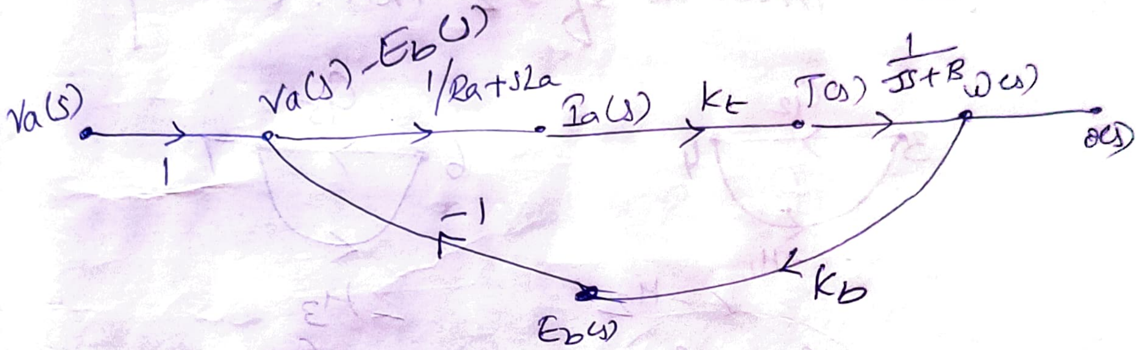
$E_b(s) = K_b \, \omega(s)$



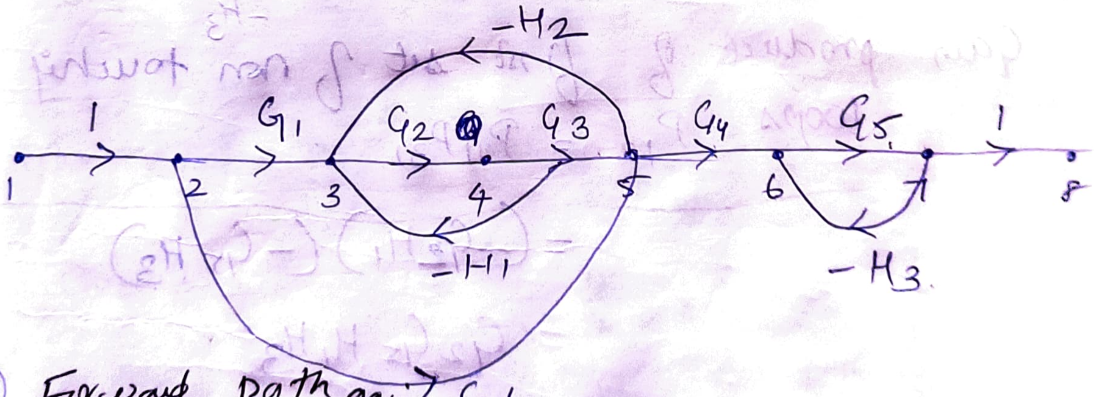
$\omega(s) = s\theta(s)$

$$\theta(s) = \frac{1}{s}\omega(s)$$





② Overall tranfer function - find.



① Forward path gains G6

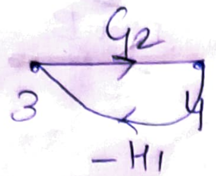
Two forward paths $K = 2$.

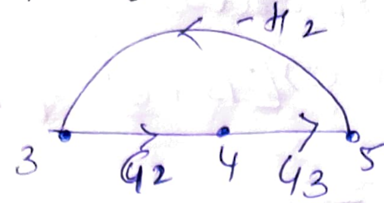
Let forward path gain be P_1 & P_2.

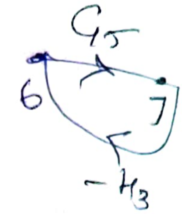
$P_1 = G_1 G_2 G_3 G_4 G_5$

$P_2 = G_4 G_5 G_6$.

② Individual loop gains.





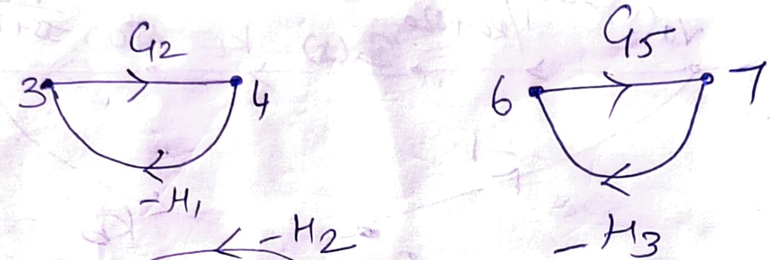


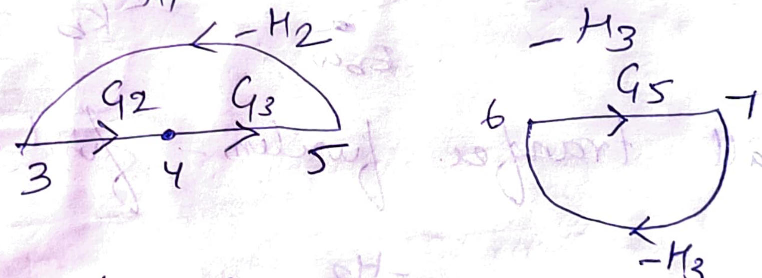
$P_{11} = -G_2H_1$

$P_{21} = -G_2G_3H_2$

$P_{31} = -G_5H_3$

③ Gain products of 2 non-touching loops





Gain product of first set of non touching loops $P_{12} = P_{11}P_{31}$

$= (-G_2H_1)(-G_5H_3)$

$= G_2G_5H_1H_3$

$P_{22} = P_{21}P_{31}$

$= G_2G_3G_5H_2H_3$.

④ calculation of Δ and Δ_K

$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + (P_{12} + P_{22})$

$= 1 - (-G_2H_1 - G_2G_3H_2 - G_5H_3) + (G_2G_5H_1H_3$
$+ G_2G_3H_2H_3G_5$

$\boxed{\Delta = \text{"}}$

$\Delta_1 = 1$ since (first forward path) there is no part of graph which has not touching the path "

$\Delta_2 = 1 - P_{11} = 1 + G_2H_1$

⑤ Transfer function

$$T = \frac{1}{\Delta} \sum_K P_K \Delta_K$$

$$= \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2)$$

$$= \frac{G_2G_4G_5[G_1G_3 + G_6/G_2 + G_6H_1]}{1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3}$$

---

mason's formula for gain.

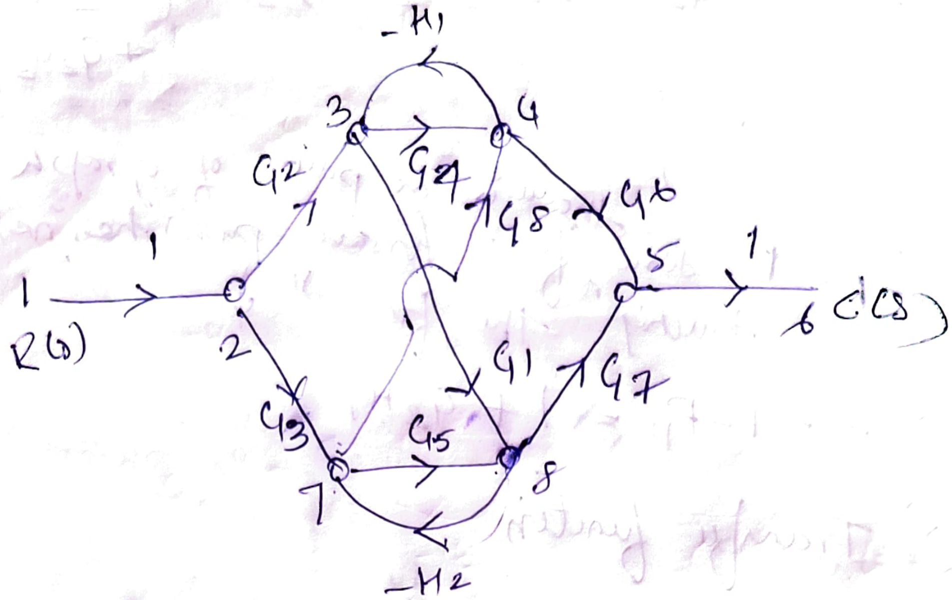
Overall gain $T = \frac{1}{\Delta} \sum_K P_K \Delta_K$

P_K - forward path gain of K^{th} forward path

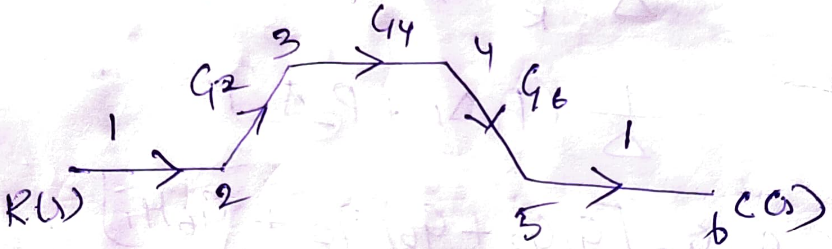
Δ - 1 - (Sum of individual loop gain)
+ (Sum of gain products of all possible of two non-touching loops)
- sum of 3 non-touching loops

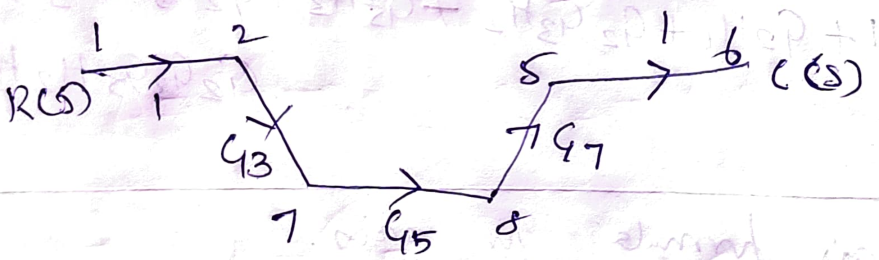
$\Delta_K = \Delta$ for that part of graph not touching K^{th} path

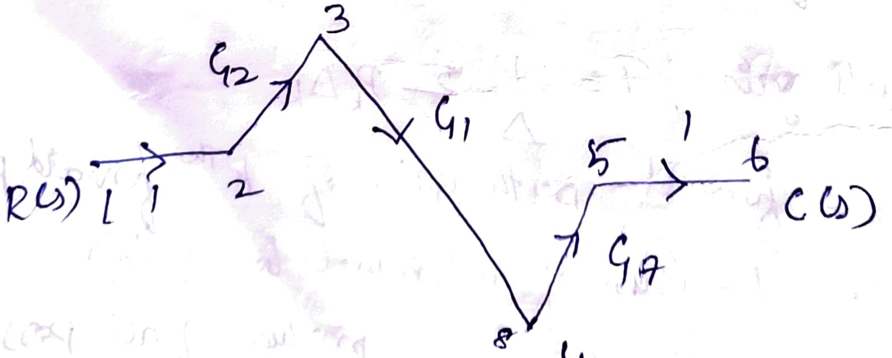
(3) Overall gain of the System.

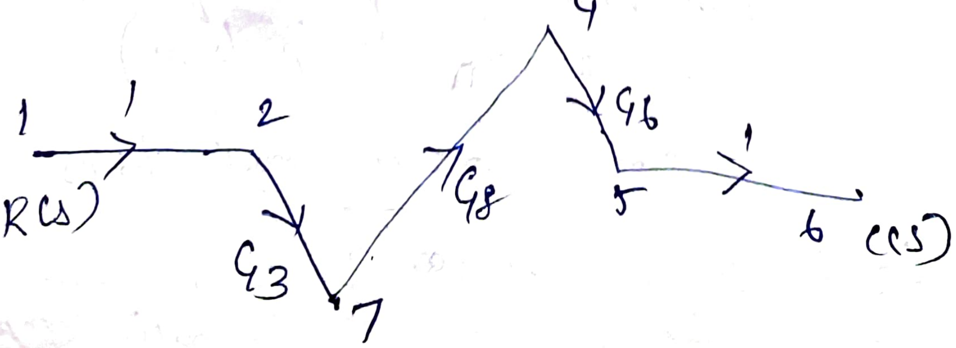


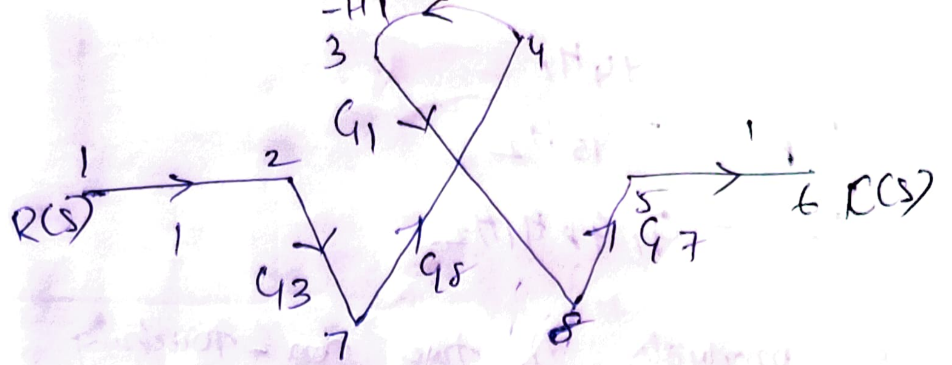
Forward path gains.

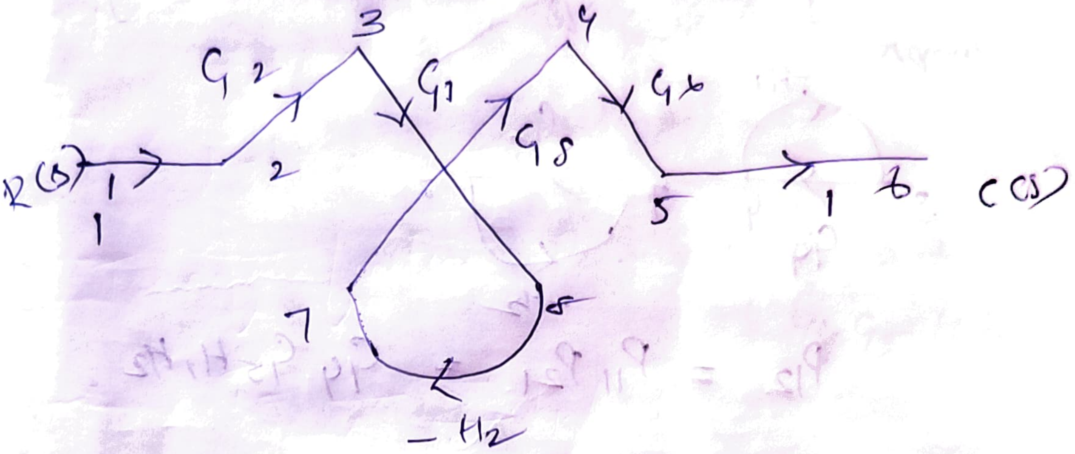












There are six forward paths $k=6$

Let the fwd path gains be P_1, P_2, P_3, P_4, P_5 & P_6

Gain of $P_1 = G_2 G_4 G_6$

$P_2 = G_3 G_5 G_7$

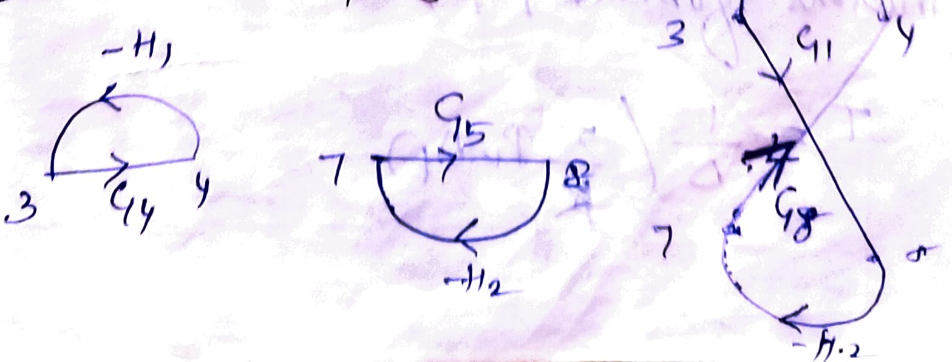
$P_3 = G_2 G_1 G_7$

$P_4 = G_3 G_8 G_6$

$P_5 = -G_3 G_1 G_8 G_7 H_1$

$P_6 = -G_2 G_1 G_8 G_6 H_2$

Individual loop gain

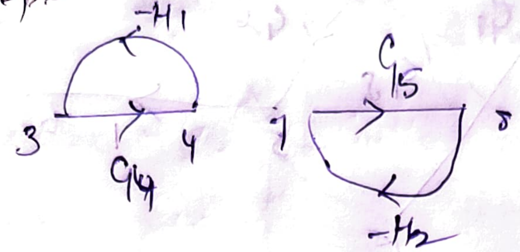


$P_{11} = -G_4 H_1$

$P_{21} = -G_5 H_2$

$P_{31} = G_1 G_8 H_1 H_2.$

(3) Gain products of two non-touching loops



$$P_{12} = P_{11} P_{21} = G_4 G_5 H_1 H_2.$$

Calculation of Δ & Δ_K

$$\Delta = 1 - (-G_4 H_1 - G_5 H_2 + G_1 G_8 H_1 H_2) + (G_4 G_5 H_1 H_2)$$

$$\Delta = 1 + G_4 H_1 + G_5 H_2 - G_1 G_8 H_1 H_2 + G_4 G_5 H_1 H_2$$

$$\Delta_1 = 1 - (-G_5 H_2) = 1 + G_5 H_2$$

$$\Delta_2 = 1 - (-G_4 H_1) = 1 + G_4 H_1$$

$$\Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 1.$$

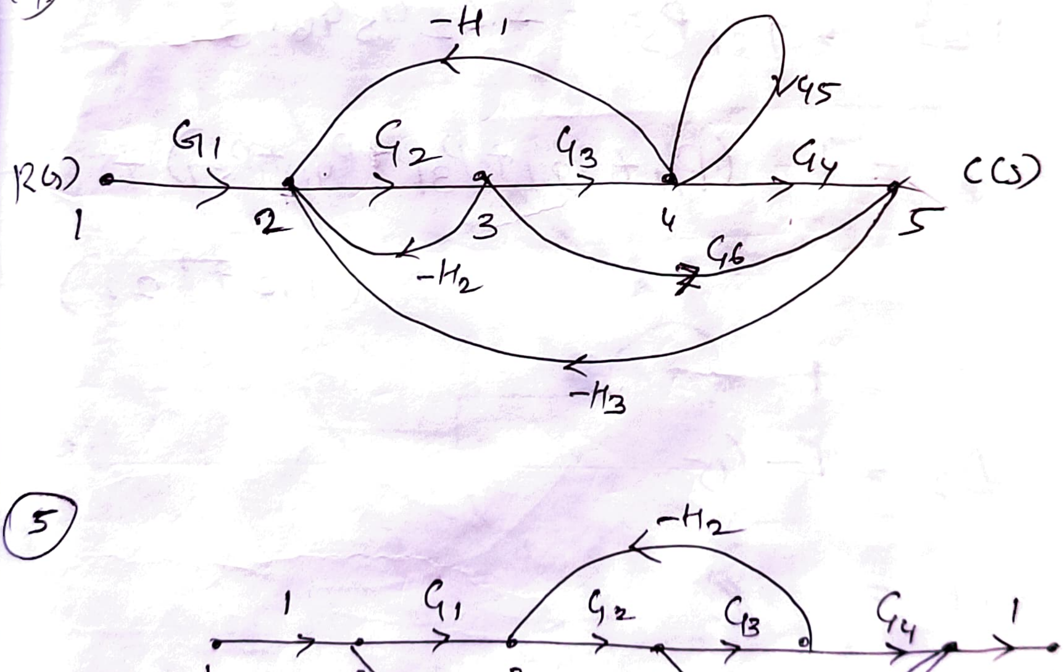
Mason's formula, T

$$T = \frac{1}{\Delta}\left(\sum_K P_K \Delta_K\right)$$

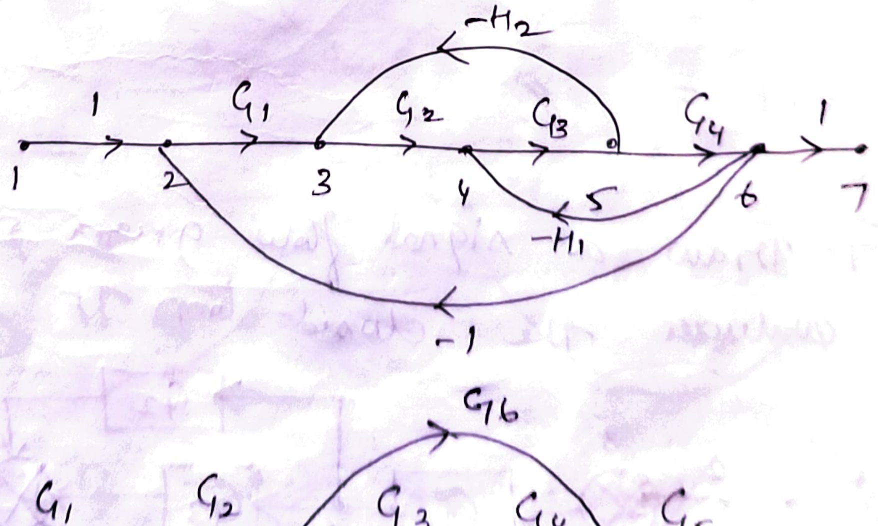
$$= \frac{1}{\Delta}\left(P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5 + P_6\Delta_6\right)$$

$$T.F = \frac{G_2G_4G_6(1+G_5H_2) + G_3G_5G_7(1+G_4H_1) + G_2G_1G_7 + G_3G_8G_6 - G_3G_1G_8G_7H_1 - G_2G_1G_8G_6H_2}{1 + G_4H_1 + G_5H_2 - G_1G_8H_1H_2 + G_4G_5H_1H_2}$$

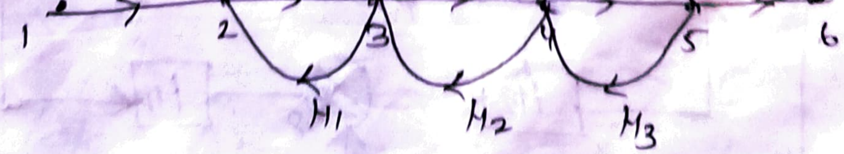
(4)



(5)



(6)



(4) $K = 2$

$P_1 = G_1 G_2 G_3 G_4$

$P_2 = G_1 G_2 G_6$

$P_{11} = -G_2 G_3 H_1$

$P_{21} = -H_2 G_2$

$P_{31} = -G_2 G_6 H_3$

$P_{41} = -G_2 G_3 G_4 H_3$

$P_{51} = G_5$

$P_{12} = P_{21} P_{51} = -G_2 G_5 H_2$

$P_{22} = P_{31} P_{51} = -G_2 G_5 G_6 H_3$.

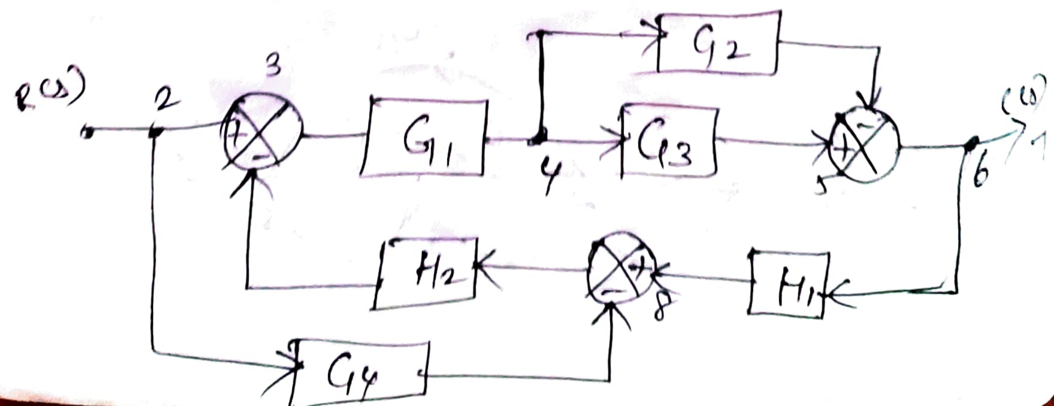
$\Delta = 1 - (P_{11} + P_{21} + P_{31} + P_{41} + P_{51}) + (P_{12} + P_{22})$

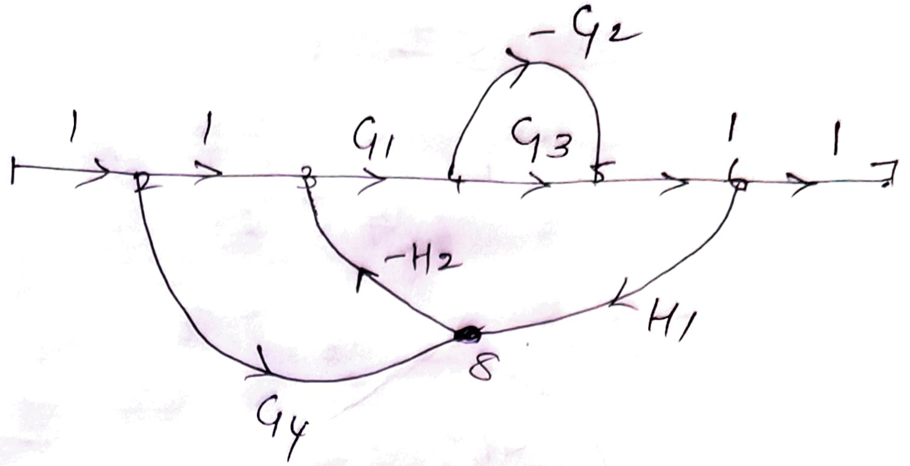
$\Delta_1 = 1$

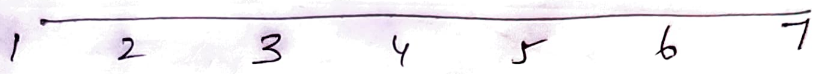
$\Delta_2 = 1 - G_5$

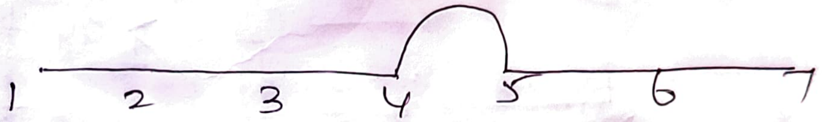
$TF = //.$

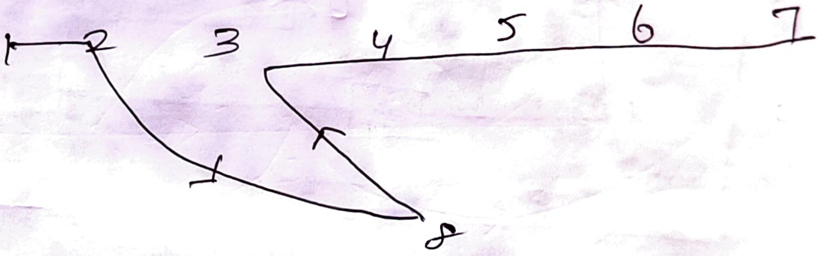
(7) Draw a signal flow graph & evaluate the closed loop TF

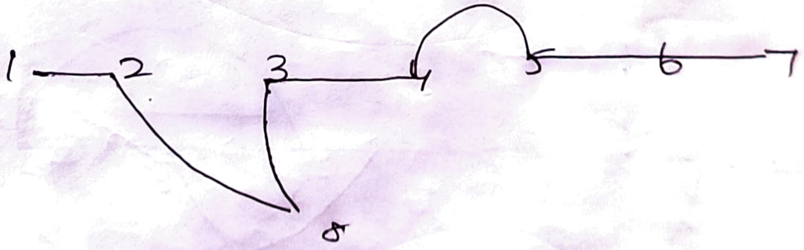


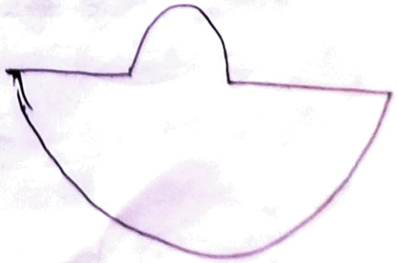










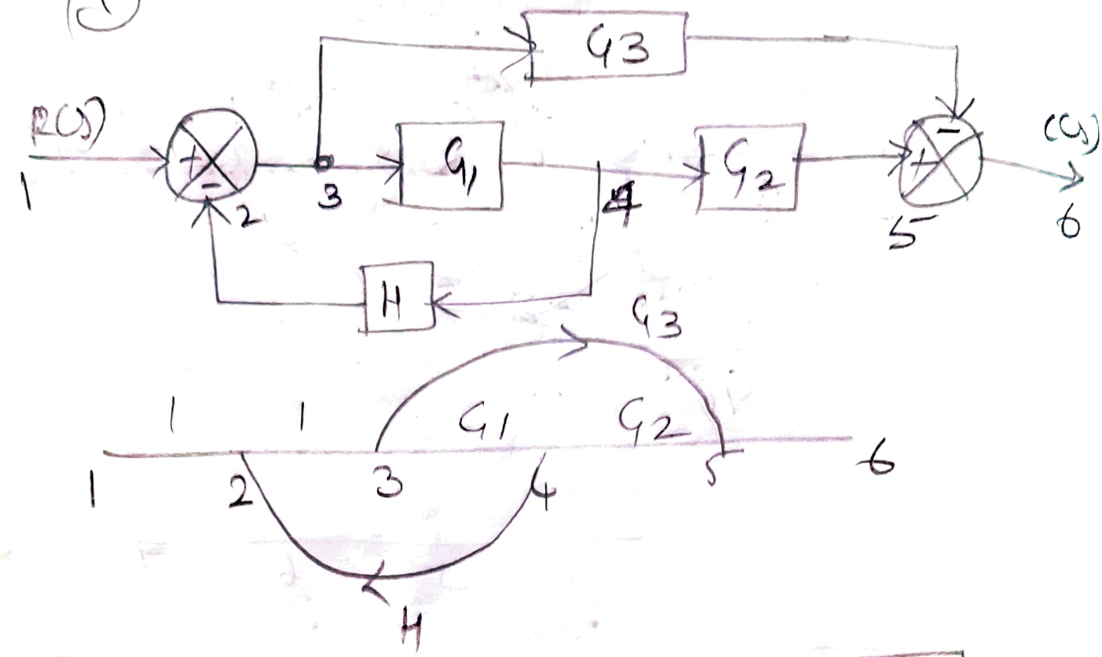


0. – non touching loops

$\Delta =$

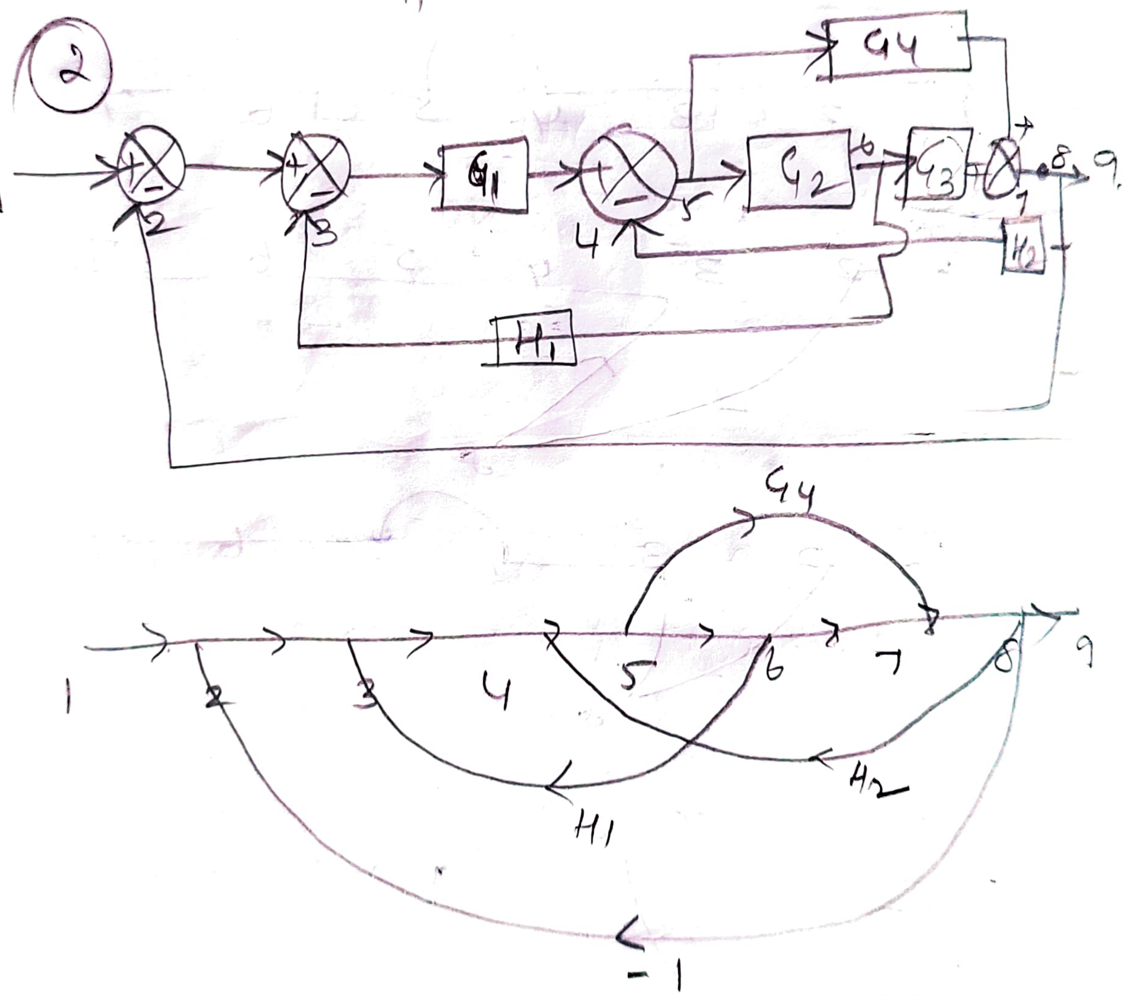
$\Delta_k = \Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 1.$

1



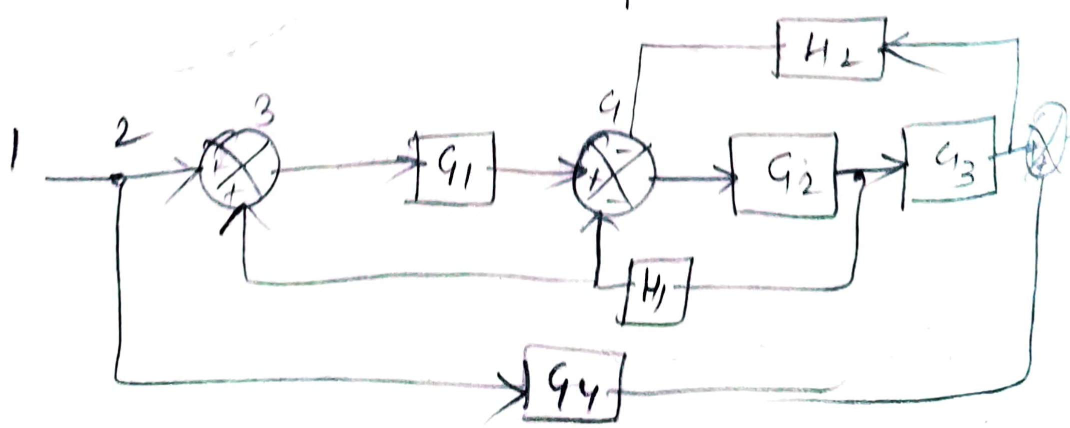
G3
R(s)
C(s)
G1
G2
H
1
2
3
4
5
6

2

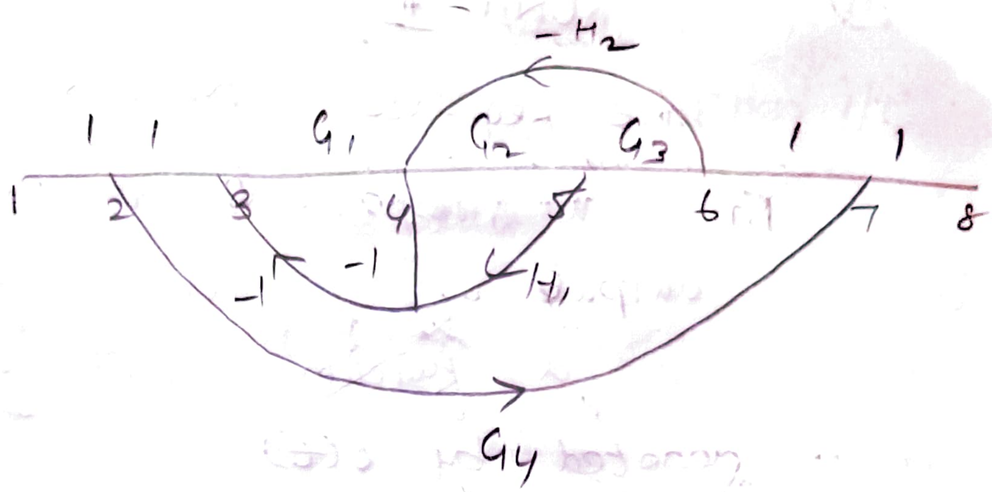


G4
G1
G2
G3
H2
H1
1
2
3
4
5
6
7
8
9
-1

3



H2
G1
G2
G3
H1
G4
1
2
3
4



Construct the block diagram of field controlled DC motor.

$$V_f = R_f i_f + L_f \frac{di_f}{dt} \quad \text{— (1)}$$

$$T = K_{tf} * i_f \quad \text{— (2)}$$

$$T = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} \quad \text{— (3)}$$

Take LT

$$V_f(s) = R_f I_f(s) + L_f s \dot{I}_f(s) \quad \text{— (4)}$$

$$I_f(s) = \frac{V_f(s)}{R_f + L_f s} \quad \text{— (5)}$$