$\frac{d^2\theta}{dt^2}$ = Angular acceleration rad/sec^2

T - Applied torque Nm.

J - Moment of inertia

B - Rotational frictional coefficient

k = Stiffness of spring.

Torque balance equation

T - applied torque

T_j - opposing torque

$T_j \propto \frac{d^2\theta}{dt^2}$

$T_j = J\frac{d^2\theta}{dt^2}$

By Newton's second law

$$\boxed{T = T_j = J\frac{d^2\theta}{dt^2}}$$

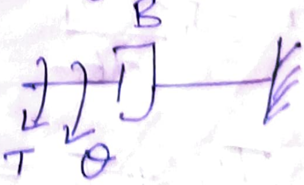
$m\frac{d^2x}{dt^2}$

$J\frac{d^2\theta}{dt^2}$

The dash pot will offer an opposing torque $\propto$ angular velocity

T - Applied torque

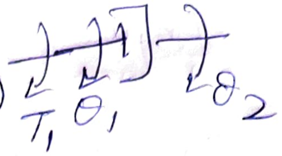
T_b - opposing torque



$T_b \propto \frac{d\theta}{dt}$, $T_b = B\frac{d\theta}{dt}$

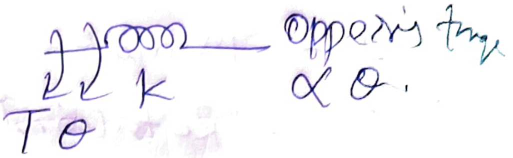
$$\boxed{T = T_b = B\frac{d\theta}{dt}}$$

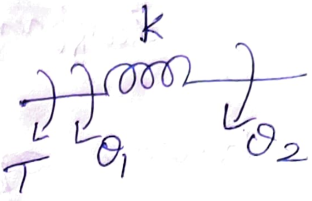
$$T_b = B\frac{d}{dt}(\theta_1-\theta_2)$$



$$T_k \propto \theta$$

$$\boxed{T_k = k\theta}$$



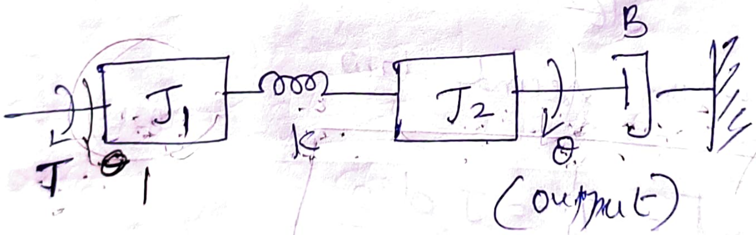


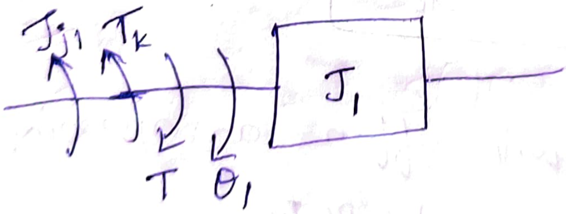
$$K_K \propto k(\theta_1-\theta_2)$$

$$\boxed{T_K = k(\theta_1-\theta_2)}$$

Problem.

Write the differential equations governing the mechanical rotational system.





M → J
x = θ
B = B
k = k
f = T
f(t) = T

$$T_{j1} = J_1\frac{d^2\theta_1}{dt^2}, \quad T_k = k(\theta_1-\theta)$$

$$T_{j1} + T_k = T$$

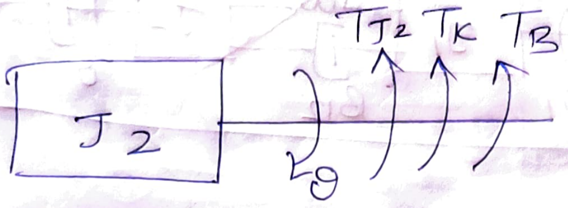
$$J_1\frac{d^2\theta_1}{dt^2} + k(\theta_1-\theta) = T$$

$$J_1\frac{d^2\theta_1}{dt^2} + k\theta_1 - k\theta = T \quad (1)$$

L.T.

$$J_1 s^2 \theta_1(s) + k\theta_1(s) - k\theta(s) = T(s)$$

$$(J_1 s^2 + k)\theta_1(s) - k\theta(s) = T(s) \quad (2)$$



$$T_{j2} = J_2 \frac{d^2\theta}{dt^2}, \quad T_k = k(\theta - \theta_1), \quad T_B = B\frac{d\theta}{dt}$$

$$T_{j2} + T_K + T_B = 0$$

$$J_2 \frac{d^2\theta}{dt^2} + \overbrace{k\theta - k\theta_1}^{k(\theta - \theta_1)} + B\frac{d\theta}{dt} = 0 \quad (3)$$

L.T

$$J_2 s^2 \theta(s) + k\theta(s) - k\theta_1(s) + Bs\,\theta(s) = 0$$

$$[J_2 s^2 + k + Bs]\,\theta(s) = k\,\theta_1(s)$$

$$\theta_1(s) = \frac{[J_2 s^2 + Bs + k]\,\theta(s)}{k} \quad (4)$$

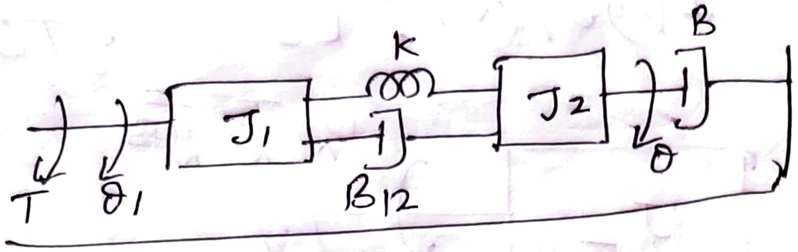
Sub (4) in (2)

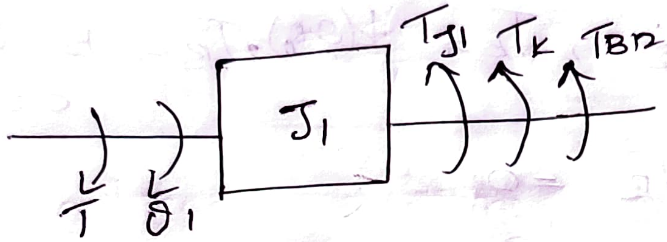
$$(J_1 s^2 + k)\left[\left[\frac{J_2 s^2 + Bs + k}{k}\right]\theta(s)\right] - k\theta(s) = T(s)$$

$$\frac{\theta(s)}{T(s)} = \frac{k}{(J_1 s^2 + k)(J_2 s^2 + Bs + k) - k^2} \quad (5)$$

Problems (2)

(3) Write the differential equations of rotational system. T.F $\theta(s)/T(s)$





$$T_{j1} = J_1 \frac{d^2\theta_1}{dt^2}, \quad T_{b12} = B_{12} \frac{d(\theta_1 - \theta)}{dt}$$

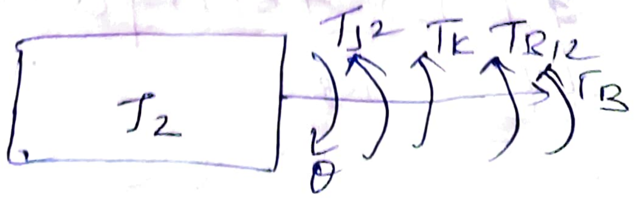
$$T_K = K(\theta_1 - \theta)$$

$$J_1 \frac{d^2\theta_1}{dt^2} + B_2 \frac{d(\theta_1 - \theta)}{dt} + K(\theta_1 - \theta) = T$$

LT

$$J_1 s^2 \theta_1(s) + B_2 s[\theta_1(s) - \theta(s)] + K[\theta_1(s) - \theta(s)] = T(s)$$

$$\theta_1(s)[J_1 s^2 + B_2 s + K] - \theta(s)[sB_2 + K] = T(s) \quad \leftarrow (1)$$



$$T_{j2} = J_2 \frac{d^2\theta}{dt^2} , \quad T_{b12} = B_{12} \frac{d(\theta - \theta_1)}{dt}$$

$$T_b = B \frac{d\theta}{dt} , \quad T_k = k(\theta - \theta_1)$$

Newton's law.

$$\theta_1(s) = \frac{J_2 s^2 + s(B_{12} + B) + k}{[sB_{12} + k]} \theta(s) \quad (2)$$

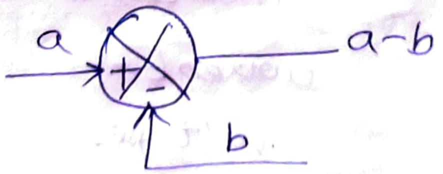
Sub (2) in (1)

$$\frac{\theta(s)}{T(s)} = \frac{B_{12} s + k}{(J_1 s^2 + sB_{12} + k)(J_2 s^2 + s(B_{12} + B) + k) - (sB_{12} + k)^2} \quad (3)$$

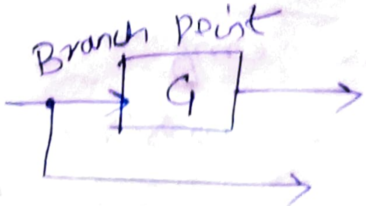
Block diagram.

Block

Summing point



Branch point



* Construct a block diagram for armature controlled dc motor.

$$V_a = i_a R_a + L_a \frac{di_a}{dt} + e_b \quad \text{—— (1)}$$

$$T = k_t i_a \quad \text{—— (2)}$$

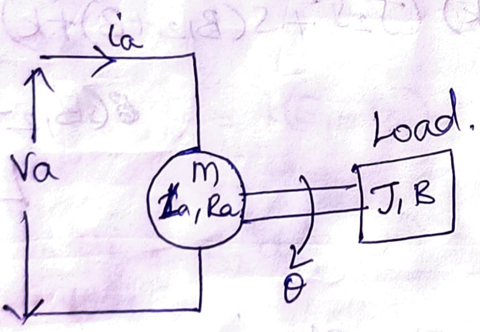
$$T = J \frac{d\omega}{dt} + B\omega \quad \text{—— (3)}$$

$$e_b = K_b \omega \quad \text{—— (4)}$$

$$\omega = \frac{d\theta}{dt} \quad \text{—— (5)}$$

Electrical systems

Obtain TF of Armature Controlled DC motor



R_a = armature resistance

L_a = " inductance

i_a = " current

V_a = " Voltage

e_b = back emf

K_t = Torque constant

T - Torque developed by motor

θ - Angular displacement

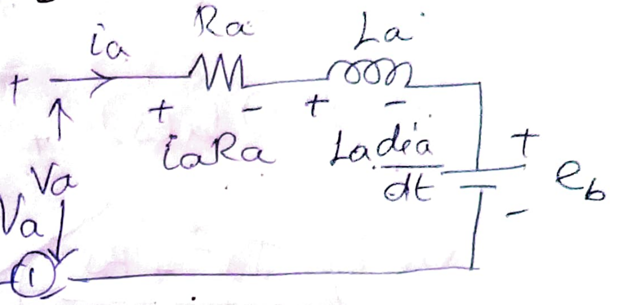
J - moment of inertia of motor load

B - frictional coefficient

K_b = back emf constant

Equivalent circuit

By KVL

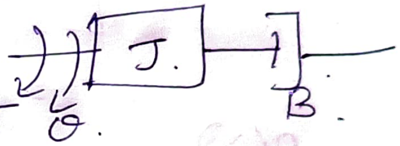


$$i_aR_a + L_a\frac{di_a}{dt} + e_b = V_a \quad \text{--- (1)}$$

$T \propto$ flux & current

$$T \propto i_a \Rightarrow T = K_t i_a \quad \text{--- (2)}$$

$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T \quad \text{--- (3)}$$



back emf is $\propto$ angular velocity -

$$e_b \propto \frac{d\theta}{dt}, \quad e_b = K_b\frac{d\theta}{dt} \quad \text{--- (4)}$$

L.T

$$I_a(s)R_a + L_a s I_a(s) + E_b(s) = V_a(s) \quad \text{--- (5)}$$

$$T(s) = K_t I_a(s) \quad \text{--- (6)}$$

$$Js^2\theta(s) + Bs\theta(s) = T(s) \quad \text{--- (7)}$$

$$E_b(s) = K_b s\theta(s) \quad \text{--- (8)}$$

equate (6) & (7)

$$K_t I_a(s) = Js^2\theta(s) + Bs\theta(s)$$

$$I_a(s) = \frac{(Js^2 + Bs)\theta(s)}{K_t} \quad \text{--- (9)}$$

Sub eq (8) & (9) in (5)

$$R_a I_a(s) + L_a s I_a(s) + K_b s \theta(s) = V_a(s)$$

$$(R_a + sL_a) I_a(s) + K_b s \theta(s) = V_a(s)$$

$$(R_a + sL_a)\left(\frac{Js^2 + Bs}{K_t}\right)\theta(s) + K_b s \theta(s) = V_a(s)$$

$$V_a(s) = \theta(s) \left[\frac{(R_a + sL_a)(Js^2 + Bs) + K_b K_t s}{K_t}\right]$$

$$\frac{\theta(s)}{V_a(s)} = \frac{K_t}{[(R_a + sL_a)(Js^2 + Bs) + K_b K_t s]} \quad (10)$$

Continuation

on LT (1)

$$V_a(s) = I_a(s) R_a + L_a s I_a(s) + E_b(s) \quad (6)$$

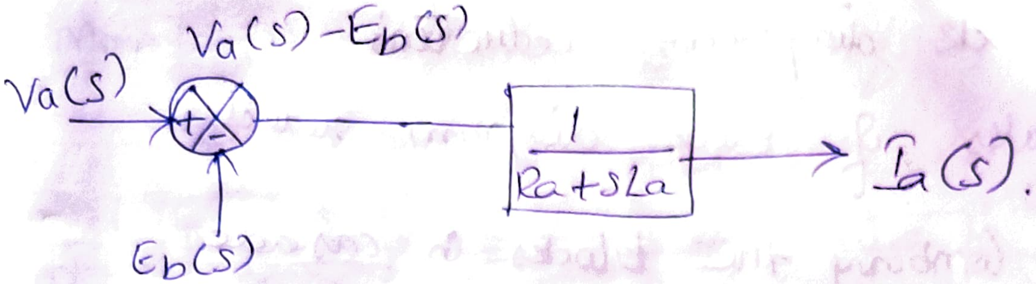
$V_a(s)$ & $E_b(s)$ are i/p

$I_a(s) \Rightarrow$ o/p

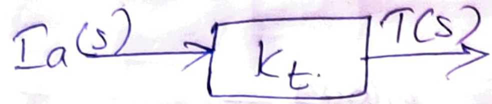
$$V_a(s) - E_b(s) = I_a(s)[R_a + sL_a]$$

$$I_a(s) = \frac{V_a(s) - E_b(s)}{R_a + sL_a}$$

//

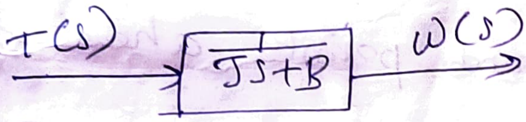


on LT of (2) $T(s) = K_t I_a(s)$



on L.T of (3) $T(s) = J s \omega(s) + B \omega(s)$

$$T(s) = [Js + B]\, \omega(s).$$



on L.T of (4) $E_b(s) = K_b\, \omega(s)$

on LT of (5) $\omega(s) = s\theta(s)$

$$\theta(s) = \frac{1}{s}\omega(s)$$