Mechanical Translational systems.

Model → 3 basic elements → mass, Spring, dash pot.

Symbols.

x = displacement, m.

speed of something in given direction ← $v = \frac{dx}{dt}$ = velocity. m/sec

rate of change of velocity of an object ← $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$ = Acceleration m/sec^2.

f = applied force, N

f_m = opposing force by mass of body, N

(spring) f_K = " " " elasticity " ", N

(dash pot) f_b = " " " friction " ", N

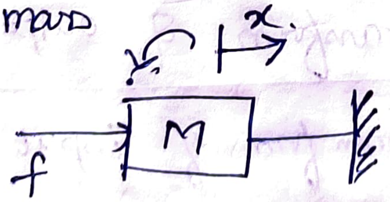
M = Mass, kg

K = Stiffness of spring, N/m

B = Viscous friction co-efficient N-sec/m

Force balanced equations of idealized elements

Ideal mass   friction & elasticity negligible



force applied, opposing force ∝ acceleration of body

Let

f = applied force

f_m = opposing force due to mass

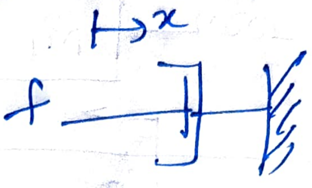
$f_m \propto$ acceleration $\frac{dx}{dt}$ = velocity

Newtons laws, $f = ma$ $\frac{dv}{dt}$ = acceleration

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

$$\boxed{f = f_m = M \cdot \frac{d^2x}{dt^2}} \quad \text{—} \; (1)$$

Ideal frictional element.

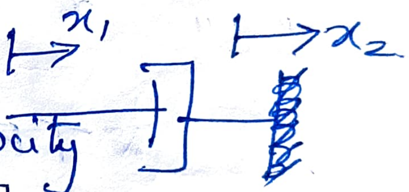


$f \propto f_b$.

$f_b \propto$ velocity

$$f_b \propto \frac{dx}{dt}$$

$$\boxed{f = f_b = B \frac{dx}{dt}} \quad \text{—} \; (2)$$



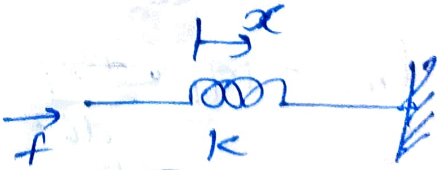
$f_b \propto$ differential velocity

$$\boxed{f = f_b = B \frac{d}{dt}(x_1 - x_2)} \quad \text{—} \; (3)$$

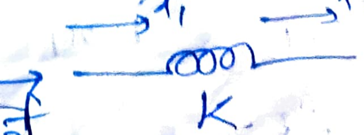
Ideal spring element

$f_k \propto$ displacement

$f_k \propto x \Rightarrow f_k = kx$



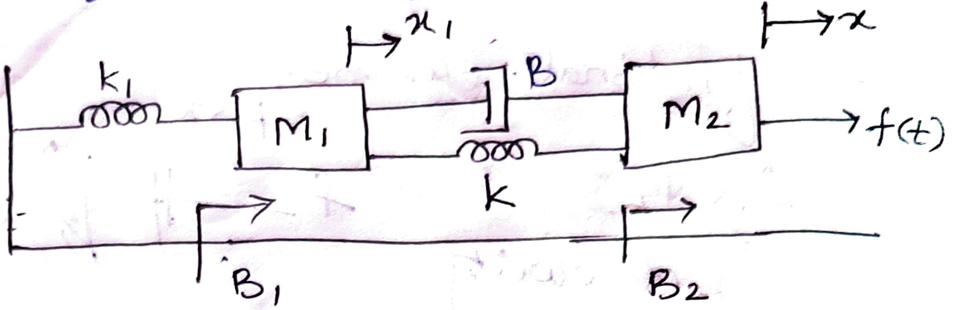
$$\boxed{f = f_k = kx} \quad \text{—} \; (4)$$



$$\boxed{f = f_K = K(x_1 - x_2)} \quad \text{—} \; (5)$$

Problem - 1

Write differential equation governing the mechanical system shown in fig & determine the transfer function.



$f(t)$ is input

x is displacement output

L.T of $f(t) = L[f(t)] = F(s)$

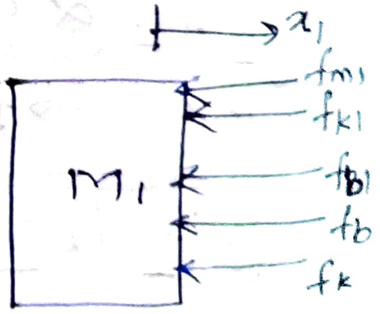
" " $x = L[x] = X(s)$

$$\therefore T.F = \frac{X(s)}{F(s)}$$

2 nodes, mass M_1 and M_2.

Let displacement of M_1 be x_1

free body diagram



$$f_{m1} = M_1 \cdot \frac{d^2x_1}{dt^2}$$

$$f_{b1} = B_1 \cdot \frac{dx_1}{dt}$$

$$f_{k1} = K_1 x_1$$

$$f_B = B \cdot \frac{d}{dt}(x_1 - x)$$

$$f_k = K(x_1 - x)$$

By Newtons law.

Sum of applied forces = Sum of opposing forces.

$$f_{m_1}+f_{b_1}+f_{k_1}+f_b+f_k=0.$$

de $$M_1\cdot\frac{d^2x_1}{dt^2}+B_1\cdot\frac{dx_1}{dt}+K_1x_1+B\cdot\frac{d}{dt}(x_1-x)+k(x_1-x)=0 \quad \text{--- (1)}$$

L.T $\quad Lx(t)=X(s) \quad L\left[\frac{dx(t)}{dt}\right]=sX(s) \quad L\frac{d^2x}{dt^2}=s^2X(s)$

$$M_1s^2X_1(s)+B_1sX_1(s)+Bs[X_1(s)-X(s)]$$
$$+K_1X_1(s)+k[X_1(s)-X(s)]=0$$

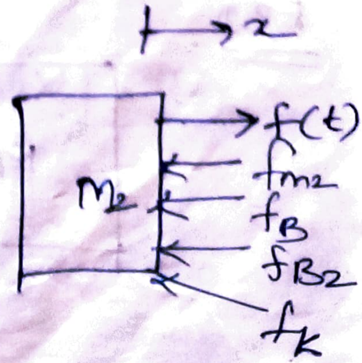
$$X_1(s)[M_1(s^2)+B_1s+K_1+K+Bs]$$
$$-[Bs\,X(s)+k\,X(s)]=0$$

$$X_1(s)[M_1s^2+B_1s+K_1+K+Bs]=X(s)[Bs+k]$$

$$X_1(s)=X(s)\frac{Bs+k}{M_1s^2+B_1s+K_1+K+Bs}$$

$$\boxed{X_1(s)=X(s)\frac{Bs+k}{M_1s^2+s(B_1+B)+(K_1+K)}} \quad \text{--- (2)}$$

Free body diagram of M_2



$$f_{m2}=M_2\cdot\frac{d^2x}{dt^2}$$

$$f_{b2}=B_2\frac{dx}{dt}$$

$$f_B=B\cdot\frac{d}{dt}(x-x_1)$$

$$f_k=k(x-x_1)$$

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Newtons' Law.

$$f_{m_2} + f_{b_2} + f_b + f_k = f(t)$$

i.e $M_2 \frac{d^2x}{dt^2} + B_2 \frac{dx}{dt} + B \frac{d}{dt}(x - x_1) + k(x - x_1) = f(t)$ — (3)

L.T

$$M_2 s^2 X(s) + B_2 s X(s) + Bs[X(s) - X_1(s)] + k[X(s) - X_1(s)] = F(s)$$

$$X(s)[M_2 s^2 + (B_2 + B)s + k] - X_1(s)[Bs + k] = F(s)$$ — (4)

Sub eq (2) in (4)

$$\frac{X(s)}{F(s)} = \frac{M_1 s^2 + (B_1 + B)s + (k_1 + k)}{[M_1 s^2 + (B_1 + B)s + (k_1 + k)] + [(B_2 + B)s + k + M_2 s^2] + (Bs + k)^2]}$$

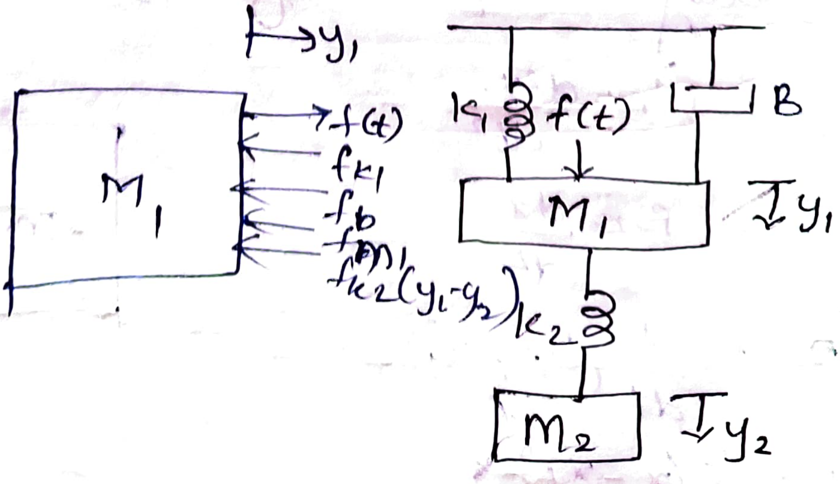
$\boxed{\text{T.F}}$ — (5)

Problem 2

Determine the transfer function $\frac{Y_2(s)}{F(s)}$

2 nodes

FBD



$f_{m1} = M_1 \cdot \frac{d^2y_1}{dt^2}$, $f_{k1} = k_1 y_1$

$f_b = B\frac{dy_1}{dt}$, $f_{k2} = k_2(y_1 - y_2)$

$f_{m1} + f_{k1} + f_b + f_{k2} = f(t)$

$$M_1\frac{d^2y_1}{dt^2} + B\frac{dy_1}{dt} + k_1y_1 + k_2(y_1 - y_2) = f(t) \quad \text{——} (1)$$

L.T

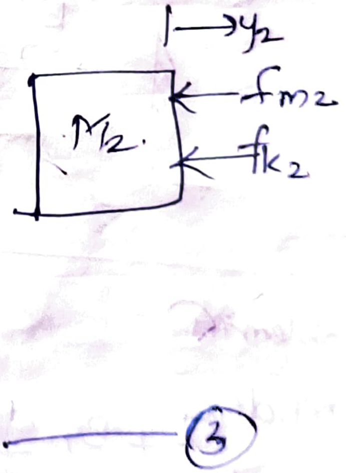
$$M_1S^2Y_1(S) + BSY_1(S) + K_1Y_1(S)$$
$$+ k_2[Y_1(S) - Y_2(S)] = F(S)$$

$$Y_1(S)[M_1S^2 + BS + (k_1 + k_2)] - Y_2(S)k_2 = F(S) \quad (2)$$

FBD M_2

$f_{m2} = M_2 \cdot \frac{d^2y_2}{dt^2}$

$f_{k2} = k_2(y_2 - y_1)$



$f_{m2} + f_{k2} = 0$

$$M_2 \cdot \frac{d^2y_2}{dt^2} + k_2(y_2 - y_1) = 0 \quad \text{——} (3)$$

L.T

$$M_2S^2Y_2(S) + K_2[Y_2(S) - Y_1(S)] = 0$$

$$Y_2(S)[M_2S^2 + k_2] - Y_1(S)k_2 = 0.$$

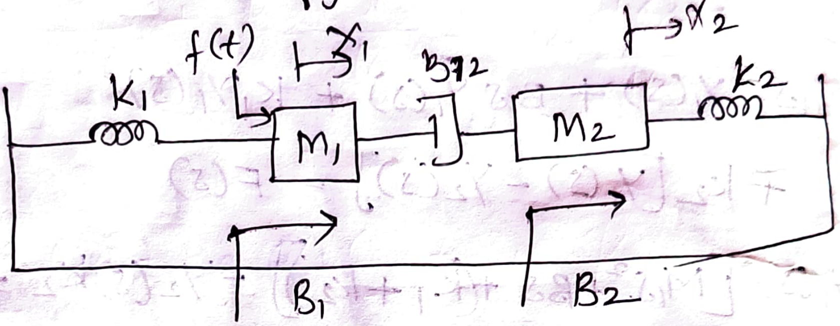
$$Y_1(S) = Y_2(S)\frac{M_2S^2 + k_2}{k_2} \quad \text{——} (4)$$

Sub (4) in (2)

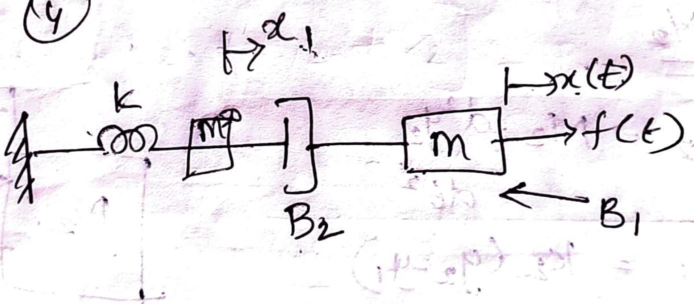
$$\frac{Y_2(S)}{F(S)} = \frac{K_2}{[M_1S^2+BS+(K_1+K_2)][M_2S^2+K_2]-K_2^2} \quad \text{--- (5)}$$

Problem (3)

Determine the T.F $\frac{X_1(S)}{F(S)}$ & $\frac{X_2(S)}{F(S)}$ for the system shown in fig.



Problem (4)



Problem (1)

$$M_1\frac{d^2x_1}{dt^2} + B_1\frac{dx_1}{dt} + B\frac{d}{dt}(x_1-x) + K_1x_1 + K(x_1-x) = 0 \quad \text{--- (1)}$$

$$X_1(S) = X(S)\frac{BS+K}{M_1S^2+(B_1+B)S+(K_1+K)} \quad \text{--- (2)}$$

$$M_2\frac{d^2x}{dt^2} + B_2\frac{dx}{dt} + B\frac{d}{dt}(x-x_1) + K(x-x_1) = f(t) \quad \text{--- (3)}$$

$$X(s)[M_2 s^2 + (B_2+B)s + k] - X_1(s)[Bs+k] = F(s) \quad \text{(4)}$$

Sub eq (2) in (4)

$$X(s)[M_2 s^2 + (B_2+B)s + k] - X(s)\frac{(Bs+k)^2}{M_1 s^2 + (B_1+B)s + (k_1+k)} = F(s)$$

$$X(s)\left[\frac{[M_1 s^2 + (B_1+B)s + (k_1+k)][M_2 s^2 + (B_2+B)s + k] - (Bs+k)^2}{M_1 s^2 + (B_1+B)s + (k_1+k)}\right] = F(s)$$

$$\frac{X(s)}{F(s)} = \frac{[M_1 s^2 + (B_1+B)s + (k_1+k)]}{[M_1 s^2 + (B_1+B)s + (k_1+k)][M_2 s^2 + (B_2+B)s + k] - (Bs+k)^2}$$

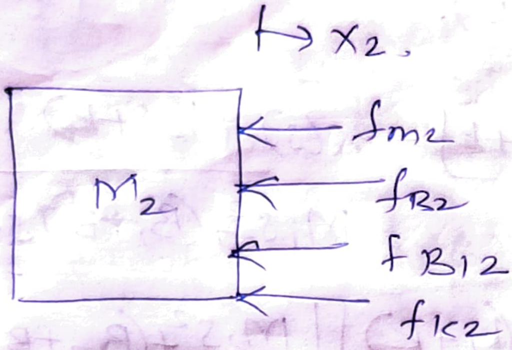
Problem (3)

f_{k1}, f_{B1}, $f(t)$, f_{B12}, f_{m1}, x_1.

$$M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx}{dt} + B_{12}\frac{d(x_1 - x_2)}{dt} + k_1 x_1 = f(t) \quad \text{(1)}$$

$$M_1 s^2 X_1(s) + B_1 s X(s) + B_{12} s[X_1(s) - X_2(s)] + k_1 X_1(s) = F(s)$$

$$X_1(s)\left[M_1 s^2 + (B_1 + B_{12})s + k_1\right] - B_{12} s X_2(s) = F(s) \quad (2)$$



$$M_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{dx_2}{dt} + B_{12} \frac{d(x_2 - x_1)}{dt} + k_2 x_2 = 0 \quad (3)$$

$$M_2 s^2 X_2(s) + B_2 s X_2(s) + B_{12} s\left[X_2(s) - X_1(s)\right] + k_2 X_2(s) = 0$$

$$X_2(s)\left[M_2 s^2 + B_2 s + B_{12} s + k_2\right] - B_{12} s X_1(s) = 0$$

$$X_2(s) = \frac{B_{12} s X_1(s)}{M_2 s^2 + B_2 s + B_{12} s + k_2} \quad (4)$$

sub (4) in (2)

$$X_1(s)\left[M_1 s^2 + (B_1 + B_{12})s + k_1\right] - \frac{(B_{12} s)^2 X_1(s)}{M_2 s^2 + B_2 s + B_{12} s + k_2} = F(s)$$

$$\frac{X_1(s)\left[\left(M_1 s^2 + (B_1 + B_{12})s + k_1\right)\left(M_2 s^2 + B_2 s + B_{12} s + k_2\right)\right] - (B_{12} s)^2 X_1(s)}{M_2 s^2 + B_2 s + B_{12} s + k_2} = F(s)$$

$$\frac{X_1(s)}{F(s)} = \frac{M_2 s^2 + (B_2 + B_{12}) s + K_2}{(M_1 s^2 + (B_1 + B_{12}) s + K_1)(M_2 s^2 + (B_2 + B_{12}) s + K_2) - (B_{12} s)^2} \quad \text{--- (5)}$$

from eq (4) $X_1(s) = ?$ --- (6)

sub (6) in (2)

$$\frac{X_2(s)}{F(s)} = \frac{B_{12} s}{[M_2 s^2 + (B_2 + B_{12}) s + K_2][M_1 s^2 + (B_1 + B_{12}) s + K_1] - (B_{12} s)^2} \quad \text{--- (7)}$$

Problem (4)

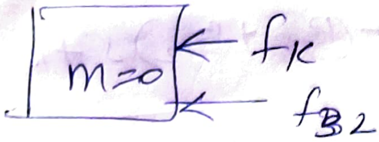
$f_m, f_{B_2}, f_{B_1}, f(t) \mapsto x(t)$

$f_m + f_{B_1} + f_{B_2} = f(t)$

$$M \frac{d^2 x}{dt^2} + B_1 \frac{dx}{dt} + B_2 \frac{d(x - x_1)}{dt} = f(t) \quad \text{--- (1)}$$

$$M s^2 X(s) + B_1 s X(s) + B_2 s X(s) - B_2 s X_1(s) = F(s)$$

$$[M s^2 + B_1 s + B_2 s] X(s) - B_2 s X_1(s) = F(s) \quad \text{--- (2)}$$



$$f_{b2} = B_2 \cdot \frac{d}{dt}(x_1 - x) \qquad fk = K x_1$$

$$B_2 s^2 X_1(s) - B_2 s X(s) + K X_1(s) = 0.$$

$$X_1(s) = \frac{B_2 s}{(B_2 s + K)} X(s) \quad \text{—— (3)}$$

Sub (3) in (2)

$$\frac{X(s)}{F(s)} = \frac{B_2 s + K}{[Ms^2 + (B_1 + B_2)s](B_2 s + K) - (B_2 s)^2}$$

Mechanical Rotational systems

3 elements → moment of inertia J of mass

dash pot with rotational frictional coefficient B.

Spring with stiffness K

List of symbols in rotational system

θ, angular displacement, rad

$\frac{d\theta}{dt}$, " Velocity, rad/sec