Elastic constants

Elastic constants

The elastic constants are

1) Modulus of elasticity (or) Young's modulus, E

2) Modulus of Rigidity (or) Shear modulus, G (or) C (or) N.

3) Bulk Modulus, K

4) Poisson's ratio.

1.5.1. Relationship between modulus of elasticity and modulus of Rigidity

The Relation between elastic constants are,

$$1)\ E = 3k\left(1 - \frac{2}{m}\right)$$

$$2)\ E = 2C\left(1 + \frac{1}{m}\right)$$

Where E = Modulus of elasticity

C = Modulus of Rigidity

K = Bulk modulus

$\frac{1}{m} =$ Poisson's ratio.

1.5.2. Bulk modulus and Modulus of Rigidity

Bulk modulus:

When a body is subjected to three mutually perpendicular like and direct stresses of equal magnitude, the ratio of direct stress to the corresponding Volumetric strain is constant for a given material with in the elastic limit, the constant is known as Bulk modulus. It is denoted by the symbol 'K'.

∴ Bulk Modulus,

$$K = \frac{\text{Direct stress}}{\text{Volumetric strain}} = \frac{\sigma}{\left(\frac{\delta V}{V}\right)}$$

Modulus of Rigidity:

When a material is subjected to shear, within the elastic limit the shear stress is proportional to the shear strain and bears a constant, known as Shear Modulus (or) Modulus of Rigidity. It is denoted by the symbol G (or) N.

∴ Shear Modulus,

$$G = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{\tau}{\phi}$$

Problems

Let us discuss about bulk modulus problem with the given problem statement that the Young's modulus of the material is given by 100 KN/mm2 and its rigidity modulus is given by 40 KN/mm2. Determine its Bulk modulus.

Given E = 100 KN/mm^2, G = 40 KN/mm^2,K = ?.

Solution:

Relation between E and G,

$$E = 2G\left(1 + \frac{1}{m}\right)$$

$\therefore$ 100 = 2 × 40 $\left(1 + \frac{1}{m}\right)$

Solving, $\frac{1}{m} = 0.25$

Relation between E and K, $E = 3K\left(1 - 2/m\right)$

Now, substituting the value of $\frac{1}{m}$,

100 = 3K[1 – (2 × 0.25)](or)3K = 200

orK = 66.67 KN/mm^2 **(Ans)**

Here we solve Modulus of elasticity problem with the given problem statement that the modulus of rigidity of a material is 38 KN/mm^2. A 10 mm diameter rod of the material is subjected to an axial tensile force of 5 KN and the change in its diameter is observed to be 0.002 mm. Calculate the Poisson's ratio, Modulus of elasticity and Bulk modulus of material.

Solution:

Given,C = 38 KN/mm^2 = 38 × 10^3 N/mm^2

d = 10 mm,

P = 5 KN = 5 × 10^3 N,

$\delta d = 0.002$ mm.

Find: $\frac{1}{m}$, ***E and K***

Stress, $\sigma = \frac{P}{A} = \frac{5 \times 10^3}{\pi/4 \times 10^2} = 63.66 \text{ N/mm}^2$

Young's Modulus, $E = \sigma/e$

$$\text{(or)} \qquad \sigma = \frac{\sigma}{E} = \frac{63.66}{E}$$

Lateral strain,

$$e_d = \frac{\text{Change in diameter}}{\text{Original diameter}}$$

$$= \frac{0.002}{10} = 2 \times 10^{-4}.$$

Using the relation, Poisson's ratio $= \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$

(or) $\frac{1}{m} = \frac{e_d}{e} = \frac{2 \times 10^{-4}}{63.66/E}$

But $$E = 2G\left(1+\frac{1}{m}\right)$$

or $$E = 2\times 38\times 10^{3}\left(1+\frac{2\times 10^{-4}}{63.66/E}\right)$$

$$= 76\times 10^{3}\left(1+\frac{2\times 10^{-4}\,E}{63.66}\right)$$

$$= 76\times 10^{3}\left(\frac{63.66+2\times 10^{-4}\,E}{63.66}\right)$$

$= 1194\ (63.66 + 2 \times 10^{-4}\ E)$

$= 76010 + 0.2388\ E$

or $0.7612\ E = 76010$

$\therefore E = \dfrac{76010}{0.7612} = 99855\ N/mm^2$

$\therefore$ Poisson's ratio, $$\frac{1}{m} = \frac{2\times 10^{-4}}{\left(\frac{63.66}{99855}\right)}$$

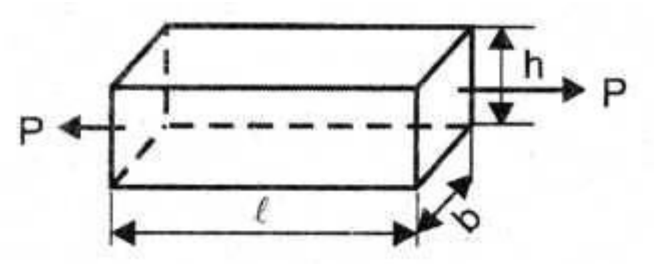
$$= \frac{2\times 10^{-4}}{6.375\times 10^{-4}}$$

$= 0.313$

Let us learn about Poisson's ratio with the given problem statement that the bar of cross section 8mm × 8mm is

subjected to an axial pull of 7000 N. The lateral dimension of the bar is found to be changed to 7.9985 × 7.9985 mm. If the modulus of rigidity of the material is 0.8 × 10^5 N/mm^2, determine the Poisson's ratio and modulus of elasticity.

Given :



P = 7000N;b = 8 mm;h = 8 mm

b_1 = 7.9985 mm, h_1 = 7.9985 mm

Solution:

$\delta_b = b - b_1$

= 8 – 7.9985

= 1.5×10^{-3} and

$\delta_h = h - h_1$

= 8 – 7.9985

= 1.5×10^{-3}

C = 0.8×10^5 N/mm^2; $\frac{1}{m}$ = ?E = ..?

stress, $\sigma = \frac{P}{A} = \frac{7000}{8 \times 8} =$ 109.37 N/mm^2

Young modulus, $E = \frac{\sigma}{e}$ or $e = \frac{\sigma}{E} = \frac{109.37}{E}$

Lateral strain = $\frac{\text{Change in breadth}}{\text{Original breadth}}$

$$= \frac{1.5 \times 10^{-3}}{8} = 1.875 \times 10^{-4}$$

Poisson's ratio, $\frac{1}{m} = \frac{\text{Laterral Strain}}{\text{Linear Strain}}$

$$\frac{1}{m} = \frac{1.875 \times 10^{-4}}{e}$$

But $E = 2G \left[1 + \frac{1}{m}\right]$

$$\therefore \; E = 2 \text{ x } 0.8 \text{ x } 10^5 \left[1 + \frac{1.875 \text{ x } 10^{-4}}{e}\right]$$

or $$E = 16 \times 10^4 \left[1 + \frac{1.875 \times 10^{-4}}{\left(\frac{109.37}{E} \right)} \right]$$

$$= 16 \times 10^4 \left[1 + \frac{1.875 \times 10^{-4} E}{109.37} \right]$$

$$= 16 \times 10^4 \left[\frac{109.37 + 1.875 \times 10^{-4} E}{109.37} \right]$$

$= 1463(109.37 + 1.875 \times 10^{-4}$ E)

E = $16 \times 10^4 + 1.875 \times 10^{-4}$ E

0.999 E = 16×10^4

∴E = 160160 N/mm^2(Ans)

∴Poisson's ratio, $$\frac{1}{m} = \frac{1.875 \times 10^{-4}}{\left(\frac{109.37}{160160} \right)}$$

= 0.274

1.6 Volumetric strains

Volumetric strain

When a body is subjected to a single force on a system of forces, the ratio of change in volume to the original volume of a body is called Volumetric strain. It is denoted by e_v.

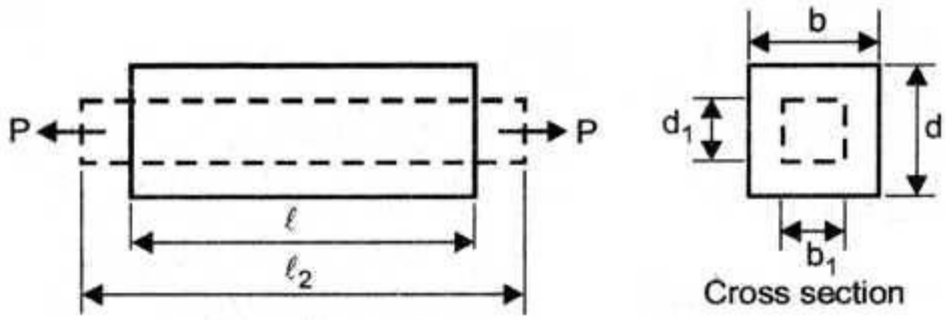
Volumetric strain $= \frac{\text{Change in volume}}{\text{Original volume}}$

$$e_v = \frac{\delta_v}{V}$$

1.6.1. Longitudinal strain and lateral strain

Longitudinal and lateral strain

When a member is subjected to an axial force,the strain along the line of axial force is known as longitudinal strain and the strain along the lateral dimensions of the body is known as lateral strain. In a rectangular bar subjected to tensile force as shown below.



i.e., longitudinal strain $= \frac{\text{Change in length}}{\text{Original length}}$

longitudinal strain, $e_1 = \frac{l_2 - l_1}{l_1}$ $l_1 < l_2$

lateral strain = change in breadth (or) depth / Original breadth (or) depth.

lateral strain, $e_b = \frac{b - b_1}{b}$ and b1 < b

lateral strain, $e_d = \frac{d - d_1}{d}$. d1 < d

1.6.2. Poisson's ratio

Poisson's ratio

Within the elastic limit, the ratio of lateral strain to longitudinal strain is known as Poisson's ratio.

i.e., Poisson's ratio $= \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$

Poisson's ratio is constant for any material. It is generally denoted by the symbol 1/m. It has no unit.

Problems

Let us solve poisson's ratio problem with the given problem statement that a steel rod 5m long and 25 mm in diameter is subjected to an axial tensile load of 50 KN. Determine the change in length, diameter and volume of the rod. Take E = 2 × 10^5 N/mm^2 and Poisson's ratio = 0.30

Given,l = 5000 mm,d = 25 mm

P = 50 × 10^3 N,E = 2 × 10^5 N/mm^2

$$\frac{1}{m} = 0.30,$$

δl = ?δd=?δv = ?.

Solution:

Change in length, $\delta\ell = \dfrac{P\ell}{AE}$

$$= \frac{50 \times 10^3 \times 5000}{\left(\pi/4 \times 25^2\right) \times 2 \times 10^5}$$

= 2.546 mm (Ans)

Poisson's ratio $= \dfrac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$

i.e., $\dfrac{1}{m} = \dfrac{(\delta d/d)}{(\delta\ell/\ell)}$

or $0.30 = \dfrac{\delta d}{25} \times \dfrac{5000}{2.546}$

Solving, $\delta d = 3.819 \times 10^{-3}$ mm

To find δV

Initial volume, $V_1 = \ell \times \pi/4\, d^2$

$$= 5000 \times \pi/4 \times 25^2$$

$= 2.453 \times 10^6\ mm^3$

Final volume, V_2 = Final length $\times \pi/4 \times$ (Final dia)^2

ButFinal dia = Initial dia – Change in dia

$= 25 - 3.819 \times 10^{-3}$

= 24.99 mm

Final length = Initial length + Change in length

= 5000 + 2.546

= 5002.546 mm

$\therefore V_2 = 5002.546 \times \pi/4 \times (24.99)^2$

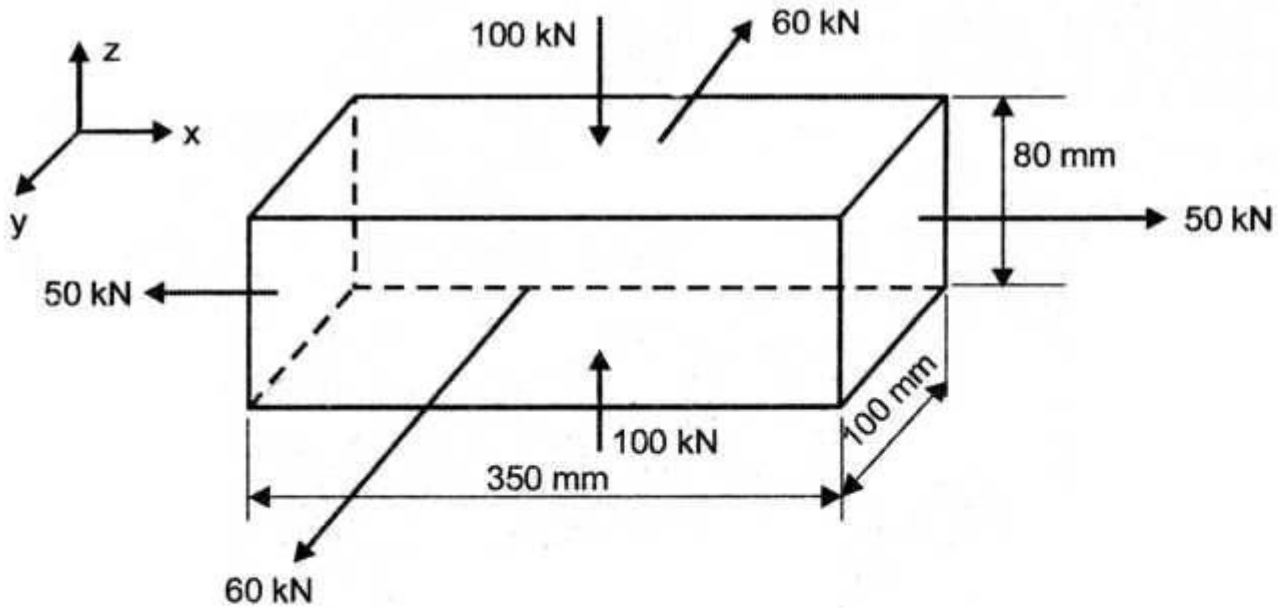
$= 2.452 \times 10^6$ mm^3

$\therefore$ Change in volume, $\delta V = V_1 - V_2$

$= (2.453 \times 10^6) - (2.452 \times 10^6)$

= 1000 mm^3

We discuss about A Rectangular block 350 mm long, 100 mm wide and 80mm thick is subjected to axial load as follows. 50 KN tensile in the direction of length, 100 KN compression in the direction of thickness and 60 KN tensile in the direction of breadth. Determine the change in volume, bulk modulus, modulus of rigidity. Take E = 2×10^5 N/mm^3 and Poisson's ratio = 0.25.



Given :

P_x = 50 KN; (tensile)

P_y = 60 KN; (tensile)

P_z = –100 KN; (compression)

E = 2×10^5 N/mm^2;

$$\frac{1}{m} = 0.25$$

Solution :

Dimensions:

x = 350 mm; y = 100 mm; z = 80 mm

i) Stresses in x, y and z direction

Stress in x direction, $\sigma_x = \frac{P_x}{y \times z}$

$$\therefore\ \sigma_x = \frac{50 \times 10^3}{100 \times 80} = 6.25\ \text{N/mm}^2$$

similarly, stress in y direction, $\sigma_y = \frac{P_y}{x \times z}$

$$\therefore\ \sigma_y = \frac{60 \times 10^3}{350 \times 80} = 2.143\ \text{N/mm}^2$$

Stress in z direction, $\sigma_z = \frac{P_z}{x \times y}$

$$\therefore\ \sigma_z = \frac{-100 \times 10^3}{350 \times 100} = -2.86\ \text{N/mm}^2$$

ii) Volumetric strain

Volumetric strain, $e_v = \frac{\delta_v}{V}$

But $\frac{\delta_v}{V}$ for the body subjected to three mutually perpendicular forces is given by an equation,

$$\frac{\delta_v}{V} = \frac{1}{E}(\sigma_x + \sigma_y + \sigma_z)\left(1 - \frac{2}{m}\right)$$

(Take (+) for tensile and (–) for compressive stress)

Hence for this problem,

$$\frac{\delta_v}{V} = \frac{1}{E}(6.25 + 2.143 - 2.86)[l - (2 \times 0.25)]$$

$$= \frac{1}{2 \times 10^5}(5.533) \times (0.5)$$

$= 1.383 \times 10^5$

iii) Change in volume

$$\frac{\delta_v}{V} = 1.383 \times 10^{-5}$$

Change in volume, $\delta_v = 1.383 \times 10^{-5} \times V$

$= 1.383 \times 10^{-5} \times (350 \times 100 \times 80)$

$= 38.724\ mm^3$

iv) Bulk Modulus

Using the equation, $E = 3K\left(1 - \frac{2}{m}\right)$

or

$$K = \frac{E}{3\left(1 - \frac{2}{m}\right)}$$

$$= \frac{2 \times 10^5}{3[1 - (2 \times 0.25)]} = 133333\ N/mm^2$$

v) Modulus of Rigidity

Using the equation, $E = 2C\left(1 + \frac{1}{m}\right)$

or $C = \frac{E}{2\left(1 + \frac{1}{m}\right)}$

$= \frac{2 \times 10^5}{2[1 + 0.25]}$ = 80,000 N/mm^2