Flitched beams

## Flitched beam

A beam made up of two or more different materials assumed to be rigidly connected together and behave like a single member is known as Flitched beam (or) Composite beam.

Flitch beams are less expensive than solid steel beam designs. They are used to support heavy vertical loads while maintaining a strict construction budget.

Flitch beams are also very useful when adding additional load carrying capacity to an existing beam.

BEAM REPAIR BASICS


Figure: Flitched beam
2.9 Shear stress distribution

## Shear stress distribution:

The variation of shear stress along the depth of the beam is called shear stress distribution.

Maximum shear stress to the average shear stress for the rectangular section:

Qmax is 1.5 times the Qavg.

Shear stress for the solid circular section:
Qmax is $4 / 3$ times the Qave
Shear stress distribution for I-section:
$q=f / 21$ * (D2/4-y)
D-depth
$y$ - Distance from neutral axis
Maximum of minimum shear stress in a rectangular cross section:

Qmax $=3 / 2$ * $F /(b d)$
Shear stress distribution for I-section:
The shear stress distribution I-section is parabolic, but at the junction of web and flange, the shear stress changes abruptly. It changes from $F / 81$ [D2 -d2] to $B / b \times F / 81$ [D2-d2]
where $D=$ over all depth of the section
$d=$ Depth of the web
$b=$ Thickness of web
$B=$ Over all width of the section.
Shear stress distribution for unsymmetrical section
The shear stress distribution for unsymmetrical sections is obtained aftercalculating the position of N A.

Shear stress is max for Triangular section

In the case of triangular section, the shear stress is not max at N A. The shear stress is max at a height of $h / 2$.

Shear stress distribution diagram draw for composite section
The shear stress distribution diagram for a composite section, should be drawn by calculating the shear stress at important points.

Problems
Let us discuss a rectangular beam 100 mm wide and 250 mm deep is subjected to a maximum shear force of 50 KN . Determine, i)Averageshear stress, ii)Maximum shear stress and,iii)Shear stress at a distance of 25 mm above the neutral axis.


## Given:

$\mathrm{b}=100 \mathrm{mmd}=250 \mathrm{~mm}$
Max SF $=50 \mathrm{KN}=50 \times 10^{3} \mathrm{~N}$

## Solution:

i)Average shear stress
$\tau_{\mathrm{av}}=\frac{\mathrm{F}}{\text { Area }}=\frac{50 \times 10^{3}}{(100 \times 250)}=2 \mathrm{~N} / \mathrm{mm}^{2}$

## ii)Maximum shear stress

For a rectangular section,
Max shear stress,
$\tau_{\max }=1.5 \tau_{\mathrm{av}}=1.5 \times 2=3 \mathrm{~N} / \mathrm{mm}^{2}$
iii)Shear stress at a distance of 25 mm above the neutral axis

Using the equation, $\tau=F \times \frac{\mathrm{A} \bar{y}}{\mathrm{Ib}}$


Let MN be the layer at 25 mm above NA .
$\therefore \tau=\tau_{M N}$
$\mathrm{F}=50 \times 10^{3} \mathrm{~N}$
$A=$ Area of cross section of beam above MN layer.
$=100 \times 100=10,000 \mathrm{~mm}^{2}$

$$
\begin{gathered}
\mathrm{I}_{\mathrm{NA}}=\frac{\mathrm{bd}^{3}}{12} \\
=\frac{100 \times 250^{3}}{12}=1.302 \times 10^{5} \mathrm{~mm}^{4}
\end{gathered}
$$

$$
\overline{\mathrm{y}}=\text { Distance of centroid of area above MN layer. }
$$

$=25+\left(\frac{100}{2}\right)=75 \mathrm{~mm}$
$\mathrm{b}=$ Width of beam on MN layer $=100 \mathrm{~mm}$

$$
\therefore \tau_{\mathrm{MN}}=\frac{50 \times 10^{3} \times 10,000 \times 75}{1.302 \times 10^{5} \times 100}
$$

$=2.88 \mathrm{~N} / \mathrm{mm}^{2}$
Let us consider a beam of $\mathbf{1 0} \mathbf{~ c m}$ square cross section is used with a diagonal in a vertical position. If the vertical shear force at this section is 150 KN , i)Find the maximum shear stress and its location on the cross section. ii) Find the shear stress at the neutral axis. iii)Sketch the shear stress distribution.

## Solution:

Let $A B C D$ be the square cross section having diagonal $B D$ in vertical position.


Neutral axis (NA) is passing through the diagonal AC. Clearly the diagonals $A C$ and $B D$ are equal, to $\sqrt{2} a=\sqrt{2} \times 10=14.142 \mathrm{~cm}$.
$\therefore \mathrm{I}_{\mathrm{NA}}=2 \times$ M.I of triangle about base

$$
=2 \times \frac{\mathrm{bh}^{3}}{12}
$$

b = diagonal AC
$=14.142 \times 10$
$=141.42 \mathrm{~mm}$

$$
\mathrm{h}=\frac{\mathrm{b}}{2}=\frac{141.42}{2}
$$

$=70.71 \mathrm{~mm}$

$$
=2 \times\left[\frac{141.42 \times 70.71}{12}\right]
$$

$=8.33 \times 10^{6} \mathrm{~mm}^{4}$
To Find $\tau_{n a}$


Using the relation,

$$
\begin{aligned}
& \tau=\frac{\mathrm{FA} \overline{\mathrm{y}}}{\mathrm{Ib}} \\
& \tau_{\mathrm{NA}}=\frac{150 \times 10^{3} \times\left(\frac{1}{2} \times 141.42 \times 70.71\right) \times\left(\frac{1}{3} \times 70.71\right)}{8.33 \times 10^{6} \times 141.42} \\
&=\frac{150 \times 10^{3} \times 5000 \times 23.57}{8.33 \times 10^{6} \times 141.42} \\
&=15 \mathrm{~N} / \mathrm{mm}^{2}
\end{aligned}
$$

To find the maximum shear stress:


For a Triangular section,
Max. shear stress,

$$
\begin{aligned}
& \tau_{\max }=\frac{4 \mathrm{~F}}{\mathrm{~b}^{4}} \times \frac{3 \mathrm{~b}}{8}\left(3 \mathrm{~b}-4 \times \frac{3 \mathrm{~b}}{8}\right) \\
& =\frac{4 \times 150 \times 10^{3}}{(141.42)^{4}} \times \frac{3 \times 141.42}{8}\left\{(3 \times 141.42)-\left(\frac{4 \times 3 \times 141.42}{8}\right)\right\} \\
& =\left(1.5 \times 10^{-3}\right) \times 53.03(424.26-212.13) \\
& =16.87 \mathrm{~N} / \mathrm{mm}^{2}
\end{aligned}
$$

Which occurs at $3 / 8 \mathrm{~b}=\left(\frac{3 \times 141.42}{8}\right)=53.03 \mathrm{~mm}$ from Apex.


Let us consider a T-section of a simply supported beam has the width of flange $\mathbf{= 1 0 0} \mathbf{~ m m}$, overall depth $=\mathbf{1 0 0} \mathbf{~ m m}$, thickness of flange and stem $=20 \mathrm{~mm}$. Determine the maximum stress in beam when a bending moment of 12 K Nm is acting on the section. Also calculate the shear stress at the neutral axis and at the junction of web and flange when shear force of 50 KN acting on the beam.


## Given:

$\mathrm{BM}=12 \mathrm{KNm}$
$=12 \times 10^{6} \mathrm{~N} \mathrm{~mm}$
(SF) = 50 KN
$=50 \times 10^{3} \mathrm{~N}$

## Solution:

i)Location of Centroid
$\mathrm{a}_{1}=100 \times 20=2000 \mathrm{~mm}^{2}$
$y_{1}=100-\left(\frac{20}{2}\right)=90 \mathrm{~mm}$
$\mathrm{a}_{2}=20 \times 80=1600 \mathrm{~mm}^{2}$

$$
\mathrm{y}_{2}=\frac{80}{2}=40 \mathrm{~mm}
$$

$$
\begin{gathered}
\therefore \bar{y}=\frac{a_{1} y_{1}+a_{2} y_{2}}{a_{1}+a_{2}} \\
=\frac{(2000 \times 90)+(1600 \times 40)}{2000+1600}=67.77 \mathrm{~mm}
\end{gathered}
$$

## ii)Moment of Inertia, $I_{N A}$

$I_{N A}=I_{1}+I_{2}$
$\mathrm{I}_{1}=\mathrm{I}_{\mathrm{G} 1}+\mathrm{A}_{1} \overline{\mathrm{~h}}^{2}$
$=\frac{100 \times 20^{3}}{12}$

$$
+\left[(100 \times 20) \times(90-67.77)^{2}\right]
$$

$=1055012 \mathrm{~mm}^{4}$
$I_{2}=I_{G 2}+{\overline{A_{2}}}^{2}$

$$
=\frac{20 \times 80^{3}}{12}+\left[(20 \times 80) \times(67.77-40)^{2}\right]
$$

$=2087210 \mathrm{~mm}^{4}$
$\therefore I_{N A}=1055012+2087210=3142222 \mathrm{~mm}^{4}$
iii) Maximum Bending Stress (f)

Using the equation, $\frac{M}{I}=\frac{f}{y}$
Maximum bending tensile stress,
$\left(f_{b}\right)_{t}=\frac{\frac{M}{I_{N A}}}{} \times\left(y_{t}\right)_{\max }=\frac{\frac{12 \times 10^{6}}{3142222}}{} \times 67.77=258.81 \mathrm{~N} / \mathrm{mm}^{2}$
Maximum bending compressive stress,
$\left(f_{b}\right)_{c}=\frac{\frac{M}{I_{N A}}}{} \times\left(y_{c}\right)_{\max }=\frac{\frac{12 \times 10^{6}}{3142222}}{3} \times 32.23=123.08 \mathrm{~N} / \mathrm{mm}^{2}$
iv)Shear Stress ( $\tau$ )

Using the equation, $\tau=\frac{F A \bar{y}}{I_{b}}$
Shear stress at Neutral Axis

$F=50 \times 10^{3} \mathrm{~N}$
A = area of cross section upto neutral layer
$=(100 \times 20)+(12.23 \times 20)=2244 \mathrm{~mm}^{2}$

$$
\bar{y}=\frac{(100 \times 20 \times 22.23)+(12.23 \times 20 \times 6.115)}{(100 \times 20)+(12.23 \times 20)}
$$

$$
=\frac{45955}{2244}=20.48 \mathrm{~mm}
$$

$b=20 \mathrm{~mm} ; \mathrm{I}=\mathrm{I}_{\mathrm{NA}}=3142222 \mathrm{~mm}^{4}$

$$
\therefore \tau_{\mathrm{na}}=\frac{50 \times 10^{3} \times 2244 \times 20.48}{3142222 \times 20}=36.56 \mathrm{~N} / \mathrm{mm}^{2}
$$

## Shear stress at Junction of Web and Flange



Let the Junction be MM.
$A=100 \times 20=2000 \mathrm{~mm}^{2}$
$y=12.23+\left(\frac{20}{2}\right)=22.23 \mathrm{~mm}$
$\therefore\left(\tau_{\text {MM }}\right)_{\text {top }}=\frac{\frac{\text { FAy }}{I_{b}}}{} \quad$ (but substitute $\mathrm{b}=100 \mathrm{~mm}$ )

$$
=\frac{50 \times 10^{3} \times 2000 \times 22.23}{3142222 \times 100}=7.074 \mathrm{~N} / \mathrm{mm}^{2}
$$

$\therefore$ Similarly $\left(\tau_{\text {MM }}\right)_{\text {below }}=\frac{\frac{\text { FAy }}{}}{I_{b}}$ (but substitute $b=20 \mathrm{~mm}$ )

$$
=\frac{50 \times 10^{3} \times 2000 \times 22.23}{3142222 \times 20}
$$

$=35.37 \mathrm{~N} / \mathrm{mm}^{2}$
The distribution of shear stress across the section is shown below:


