Bending stress distribution

2.7.1 Bending stress distribution and shear stress distribution for a rectangular section

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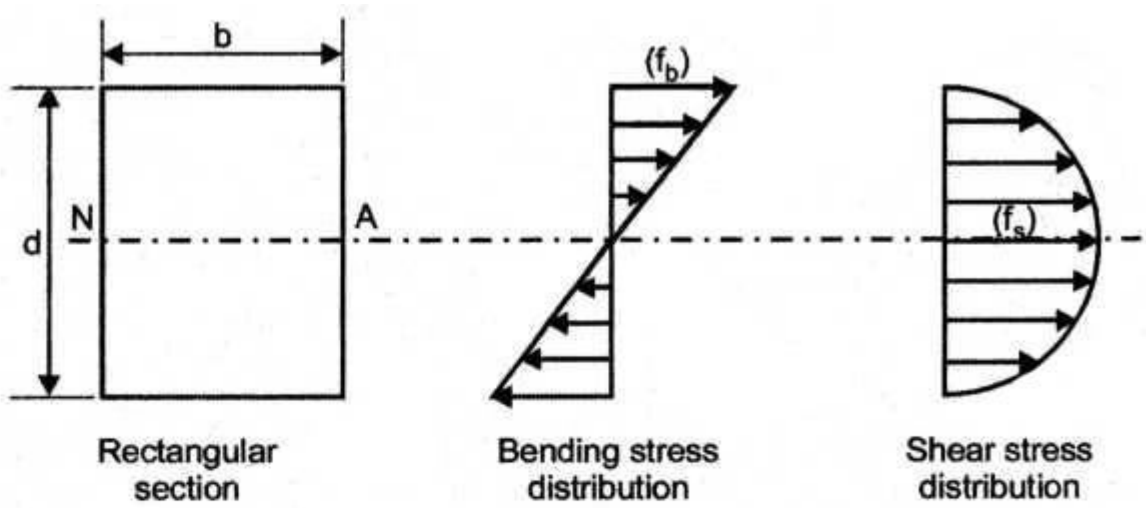


Problems

Let us solve a timber of rectangular section which support a load of 20 KN, uniformly distributed over a span of 3.6 m when beam is simply supported, If the depth of section is to be twice the breadth and the stress in the timber is not to exceed 7 N/mm^2. find the dimensions of the cross section.

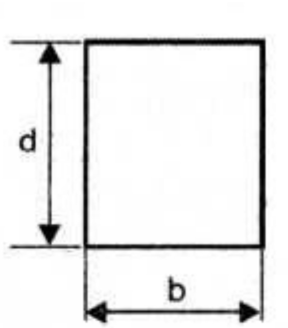
Given :

Total load= 20 N over a span of 3.6 m

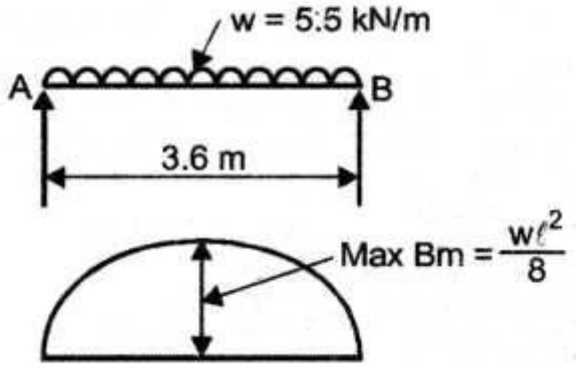
∴udl = $\frac{20}{3.6}$ = 5.55 k Nm

depth = 2 × breadth∴ d = 2b

Max.stress,$(f_b)_{max}$ = 7N/mm^{2}Find:b = ?d = ?



Solution:



Max BM = $\frac{wl^2}{8}$

$$= \frac{5.5 \times (3.6)^2}{8}$$

= 8.91 k Nm

ButMax BM = Maximum Bending stress × Section modulus

i.e.,M = $(f_b)_{max} \times Z$

= $(f_b)_{max} \times \frac{I_{NA}}{y_{max}}$

$$= (f_b)_{max} \times \frac{\left(bd^3/12 \right)}{\frac{d}{2}}$$

$\because$ For rectangle $I_{NA} = \frac{bd^3}{12}$

$$y_{max} = \frac{d}{2}$$

Substituting d = 2b and $(f_b) = 7\ N/mm^2$

$$M = 7 \times \left\{ \frac{b(2b)^3}{12} \times \frac{2}{(2b)} \right\}$$

$$= 7 \times \left\{ \frac{b \times 8b^3}{12} \times \frac{2}{2b} \right\} = 7 \times \left(\frac{8b^3}{12} \right)$$

$M = 4.67\ b^3$

or$8.91 \times 10^6 = 4.67\ b^3$ ($\because$ M = 8.91 k Nm = 8.91×10^6 N mm)

Solving,b = 124.02 say 125 mm

$\therefore$ Depth of beam, d = 2b = 2 × 125 = 250 mm

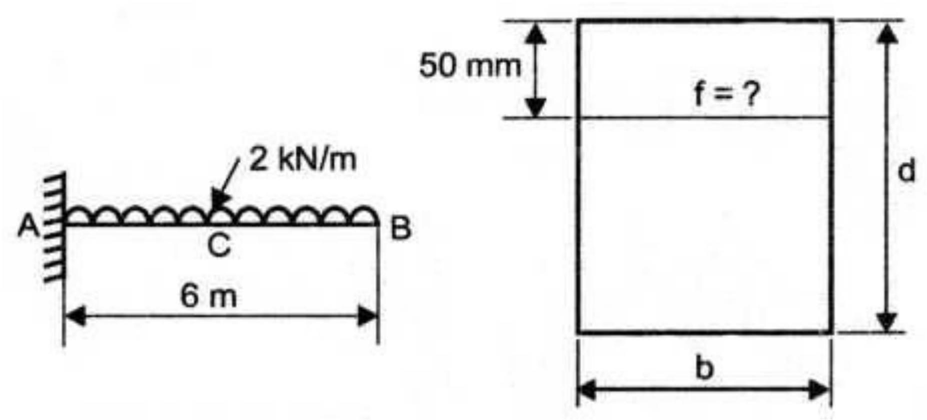
$\therefore$ Size of the beam = 125 mm × 250 mm

Let us solve A beam of rectangular cross section 50 mm wide and 150 mm deep which used as a cantilever 6m long and subjected to a uniformly distributed load of 2KN/m over the entire length. Determine the bending stress at 50 mm from the

top fiber, at the mid-span of the beam. Also calculate the maximum bending stress.

Given:b =50 mm;d = 150 mm

Solution:



Bending stress at mid span of beam is required. Hence, find the bending moment at mid span.

BM at Mid span = $-(2 \times 3 \times \frac{3}{2}) = -9$ k Nm

= 9 k Nm (Hogging BM)

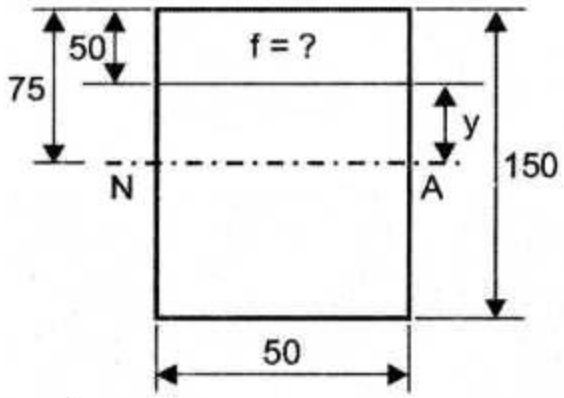
Bending stress at 50 mm from top fiber is required.

$$\therefore \quad y = \frac{150}{2} - 50$$

= 25 mm

$$I_{NA} = \frac{bd^3}{12} = \frac{50 \times 150^3}{12}$$

$= 14.06 \times 10^6\ mm^4$



Using the equation,

$$\frac{M}{I} = \frac{f}{y}$$ (∵ M = 9 k Nm = 9 × 10^6 N mm)

or $f = \frac{M\,y}{I} = \frac{9 \times 10^6 \times 25}{14.06 \times 10^6}$ = 16 N/mm^2

To find the Max. Bending stress

Using the equation,

$$\frac{M}{I} = \frac{f}{y}$$

But M = Max BM = (BM) at A

$= -(2 \times 6 \times \frac{6}{2}) = -36$ k Nm

= 36 × 10^6 N mm (Hogging)

$I = I_{NA} = 14.06 \times 10^6$; $y = y_{max} = \frac{150}{2}$ = 75 mm

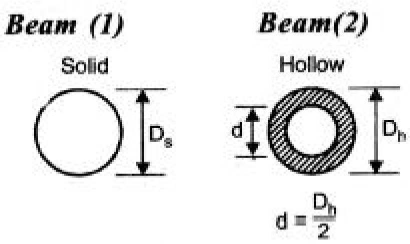
$f_{max} = \dfrac{M_{max} \times y_{max}}{I_{NA}}$

$= \dfrac{36 \times 10^6 \times 75}{14.06 \times 10^6}$

= 192 N mm^2

We see that two beams are simply supported over the same span and have the same flexural strength. Compare the weight of these two beams, if one of them is solid and the other is hollow circular with internal diameter half of the external diameter.

Solution:



Both the beams have same span and same flexural strength.

i.e.,$(BM)_1 = (BM)_2$

But Flexural strength = $(f_b)_{max} \times Z$

∴ Flexural strength of solid beam = Flexural strength of Hollow beam

i.e., $(f_b)_{max} \times Z_{solid} = (f_b)_{max} \times Z_{hollow}$

Since both the beams are of same material, $\mathbf{(f_b)_{max}}$ will be the same in both the beams.

i.e., $$Z_{Solid} = Z_{Hollow}$$

i.e., $$\frac{\pi}{32}(D_s)^3 = \frac{\pi}{32D_h}(D_h^4 - (0.5D_h)^4)$$

i.e., $$(D_s)^3 = \frac{D_h^4}{D_h}(1 - 0.0625)$$

$$(D_s)^3 = 0.9375\ (D_h)^3$$

or $$\frac{(D_h)^3}{(D_s)^3} = \frac{1}{0.9375} = 1.067$$

or $$\left(\frac{D_h}{D_s}\right)^3 = 1.067$$

or $\frac{D_h}{D_s} = \sqrt[3]{1.067}$ = 1.022

$D_s = 0.9787\ D_h$

To find the ratio of weight

$$\frac{\text{Weight of solid beam}}{\text{Weight of Hollow beam}} = \frac{\text{Density of solid beam} \times \text{Area of cross section of solid}}{\text{Density of hollow beam} \times \text{Area of cross section of hollow}}$$

$$= \frac{\text{Area of c/s of solid}}{\text{Area of c/s of hollow}}$$

(∵ Density is same)

$$= \frac{\frac{\pi}{4}(D_S)^2}{\frac{\pi}{4}(D_h{}^2 - d^2)}$$

$$= \frac{(D_S)^2}{(D_h)^2 - (0.5D_h)^2} \qquad \left(\because d = \frac{D_h}{2}\right)$$

$$= \frac{(D_S)^2}{(D_h)^2\ (1 - 0.25)}$$

$$= \frac{(D_S)^2}{0.75\ (D_h)^2}$$

SubstitutingD_s= 0.9787 D_h

$$\frac{(D_S)^2}{0.75\ (D_h)^2} = \frac{(0.9787\ D_h)^2}{0.75\ (D_h)^2} = \frac{(0.9787)^2 \times (D_h)^2}{0.75\ (D_h)^2}$$

= 1.277 **(Ans)**