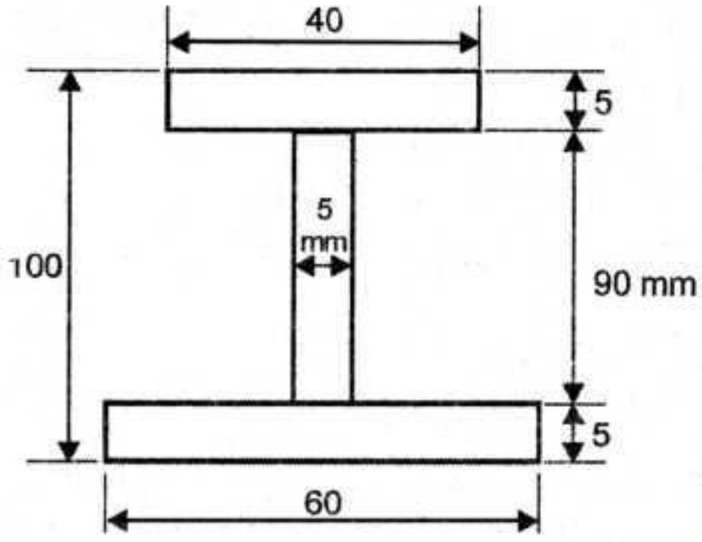
Here we see a simply supported beam of span 6m and of I section has the top flange 40 mm × 5 mm, Bottom flange of 60 mm × 5 mm total depth of 100 mm and web thickness 4 mm. It carries an udl of 2 k Nm over the full span. Calculate the maximum tensile stress and maximum compressive stress produced.

Solution:

The cross section of the beam is shown below.

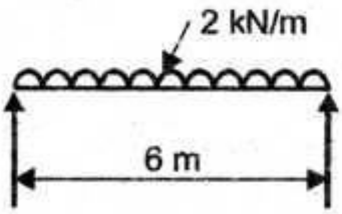


The beam is simply supported and uniformly loaded as shown below.

$$\therefore \text{Max BM} = \frac{\omega l^2}{8}$$

$$= \frac{2 \times 6^2}{8} = 9 \text{ k Nm}$$

= 9 × 106 N mm.



To find I_{NA} and Y_{max}

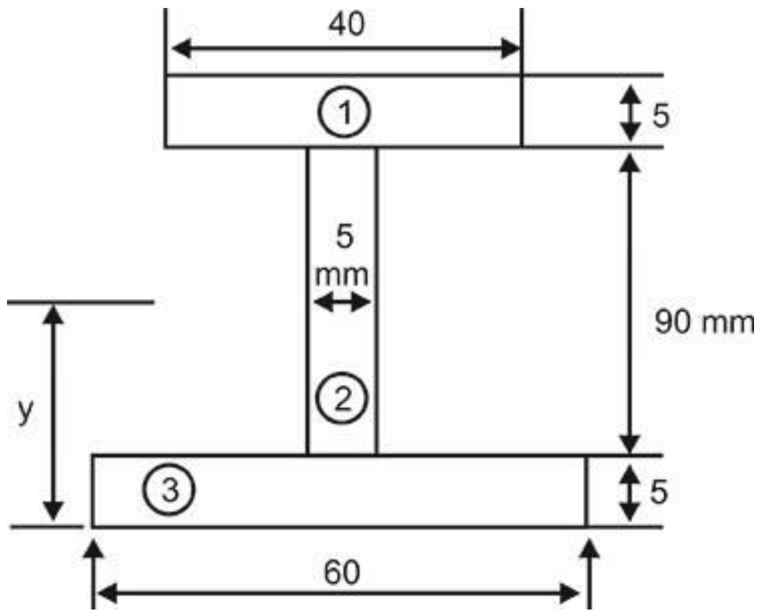
To find $\bar{y}$

$a_1 = 40 \times 5$

$= 200\ mm^2$

$y_1 = 95 + \frac{5}{2}$

$= 97.5$ mm



$a_2 = 5 \times 90$

$= 450\ mm^2$

$y_2 = 5 + \frac{90}{2} = 50\ mm^2$; $y_3 = \frac{5}{2} = 2.5$ mm

$a = 60 \times 5 = 300\ mm^2$

$$\therefore \qquad \bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(200 \times 97.5) + (450 \times 50) + (300 \times 2.5)}{200 + 450 + 300}$$

= 45 mm

Moment of Inertia I_{NA}

$I_{NA} = I_1 + I_2 + I_3$Here $I_G = I_{xx}$

$I1 = IG1 + A1\,\overline{h_1}^2 \quad \vec{h}_1 = y1 \sim \vec{y}$

$$= \frac{40 \times 5^3}{12} + \{200 \times (97.5 - 45)^2\}$$

$= 551666\ mm^4$

$I_2 = I_{G2} + A_2\,\overline{h_2}^2 \quad \vec{h}_2 = \vec{y} \sim y_2$

$$= \frac{5 \times 90^3}{12} + \{450 \times (50 - 45)^2\}$$

$= 315 \times 10^3\ mm^4$

$I_3 = I_{G3} + A_3\,\overline{h_3}^2$

$$= \frac{60 \times 5^3}{12} + \{300 \times (45 - 2.5)^2\} = 542{,}500 \text{ mm}^4$$

$\therefore I_{NA} = I_1 + I_2 + I_3$

$= (551666) + (315 \times 10^3) + (542500)$

$= 1409166 \text{ mm}^4$

With respect to the neutral axis, top fibers will be subjected to compression and bottom fibers will be subjected to tensile since the beam is simply supported.

$\therefore (y_t) = 45$ mm and $(y_c) = 55$ mm

Maximum Bending tensile stress $(f_b)_t) = ?$

Using the relation

$$\frac{M}{I} = \frac{f}{y} \text{ (or)} f = \frac{M\,y}{I}$$

$$\text{or} (f_b)_t = \frac{M\,(y_t)}{I_{NA}} = \frac{9 \times 10^6 \times 45}{1409166} = 287.4 \text{ N/mm}^2$$

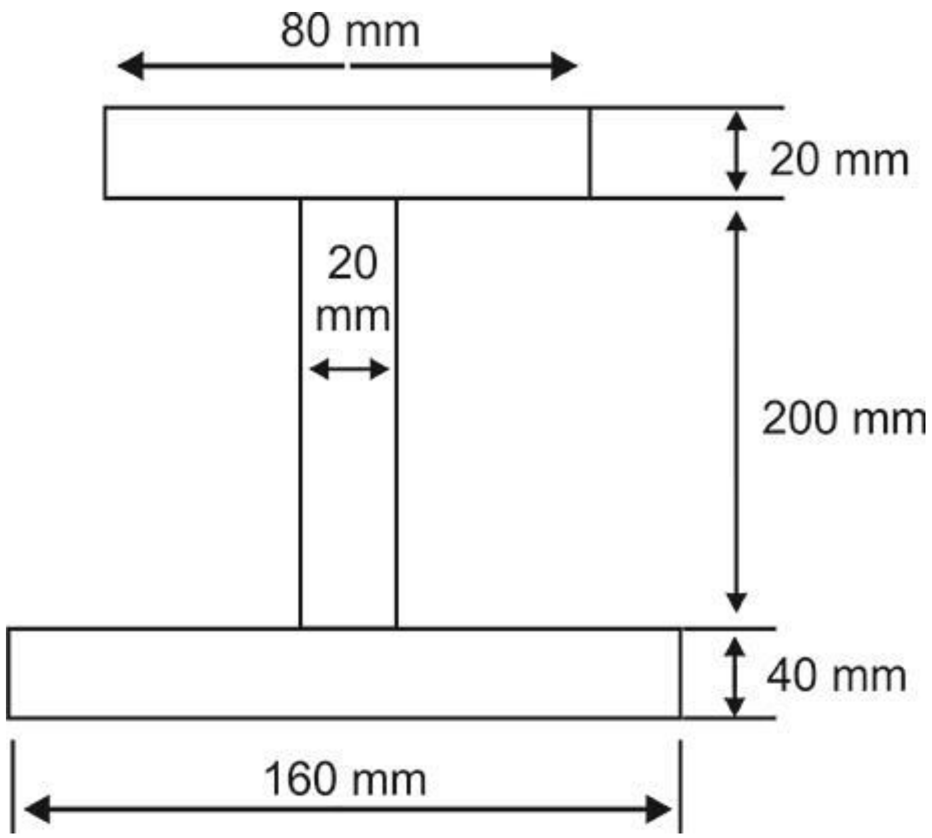
Maximum Bending compressive stress $(f_b)_c = ?$

From the equation

$$f = \frac{M\,y}{I}$$

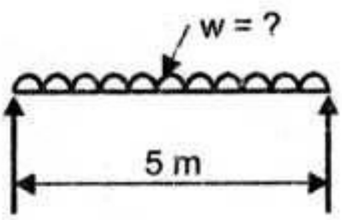
$$(f_b)_c = \frac{M\,(y_C)}{I_{NA}} = \frac{9 \times 10^6 \times 55}{1409166} = 351.27 \text{N/mm}^2$$

Let us discuss a cast iron beam I section as shown in figure below is simply supported for a span of 5m. If the tensile stress is not to exceed 20 N/mm^2, find the maximum compressive stress.



Given:

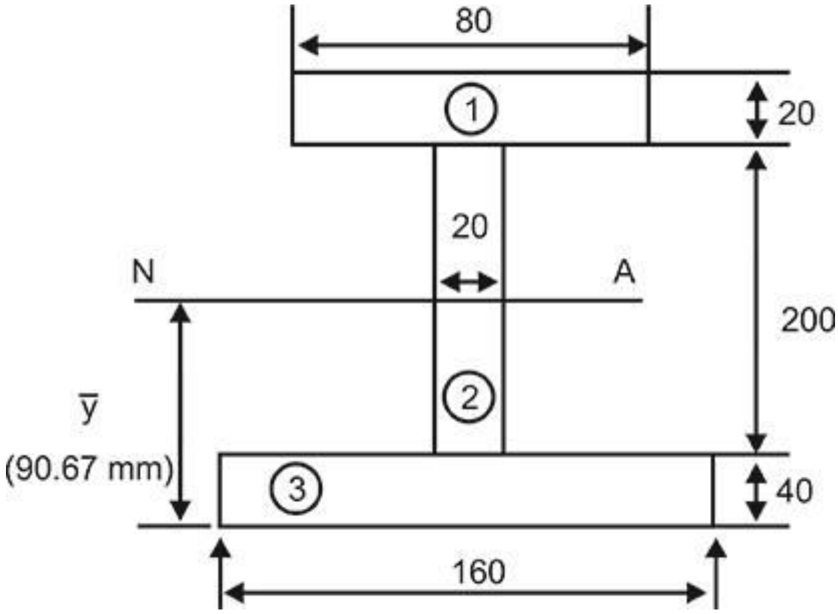
$(f_b)_t$= 20 N/mm^2$(f_b)_c$ = ?



Solution:

To Find I_{NA}

Location of Centroid



$a_1 = 80 \times 20 = 1600\ mm^2$; $y_1 = 40 + 200 + \frac{20}{2} = 250$ mm

$a_2 = 20 \times 200 = 4000\ mm^2$; $y_2 = 40 + \frac{200}{2} = 140$ mm

$a_3 = 160 \times 40 = 6400\ mm^2$; $y_3 = \frac{40}{2} = 20$ mm

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(1600 \times 250) + (4000 \times 140) + (6400 \times 20)}{1600 + 4000 + 6400}$$

= 90.67 mm

Since the beam is simply supported, with respect to neutral axis, above compression and below tension.

$\therefore (y_t)_{max} = 90.67$ mm

$(y_c)_{max} = 260 - 90.67 = 169.33$ mm

Now using the equation,

$$M = (f_b)_{max} \times \frac{I_{NA}}{y_{max}} \quad ....(1)$$

It is given maximum tensile stress = 20 N/mm^2

Hence $(f_b)_{max} = 20$ N/mm^2 and $(y_t)_{max} = 90.67$ mm

To find I_{NA}

$$I_{NA} = I_1 + I_2 + I_3 \qquad \boxed{\overline{h_1} = y_1 - \overline{y}}$$

$$I_1 = I_{G1} + A_1 \overline{h_1}^2$$

$$= \frac{80 \times 20^3}{12} + \{(80 \times 20) \times (250 - 90.67)^2\}$$

$= 40.671 \times 10^6$ mm^4

$$I_2 = I_{G2} + A_2 \overline{h_2}^2 \qquad \boxed{\overline{h_2} = y_2 \sim \overline{y}}$$

$$= \frac{20 \times 200^3}{12} + \{(20 \times 200) \times (140 - 90.67)^2\}$$

$= 23.067 \times 10^6\ mm^4$

$I_3 = I_{G3} + A_3 \overline{h_3}^2$ $\boxed{\overline{h_3} = y_3 \sim \overline{y}}$

$$= \frac{160 \times 40^3}{12} + \{(160 \times 40) \times (90.67 - 20)^2\}$$

$= 32.817 \times 10^6\ mm^4$

$\therefore I_{NA} = (40.671 \times 10^6) + (23.067 \times 10^6) + (32.817 \times 10^6)$

$= 96.555 \times 10^6\ mm^4$

Substitute $(f_b)max$, I_{NA} and $(y_t)max$ in equation (1)

$\therefore$**M = 20 ×** $\frac{96.555 \times 10^6}{90.67}$

$= 21.298 \times 10^6$ N mm

$$= \frac{21.298 \times 10^6}{10^6} kNm$$

= 21.298 k Nm

But for a simply supported beam with UDL on entire span, max.BM atcenter = $\frac{wI^2}{8}$

$$\therefore \quad \frac{wI^2}{8} = 21.298$$

$$\text{or } \frac{w \times 5^2}{8} = 21.298 \ (\because 1 = 5m)$$

Solvingw = 6.81 k Nm

∴ The safe UDL that the beam can carry = 6.81 k N/m

To Find Maximum Bending Compressive stress [$(f_b)_c$ max]

Using the relation,

$$\mathbf{M = (f_b)_{max} \times \frac{I_{NA}}{y_{max}}}$$

$$\text{of } (f_b)_c \text{ max} = \frac{M \times (yc)_{max}}{I_{NA}}$$

$$= \frac{21.298 \times 10^6 \times 169.33}{96.555 \times 10^6}$$

= 37.35 N/mm^2