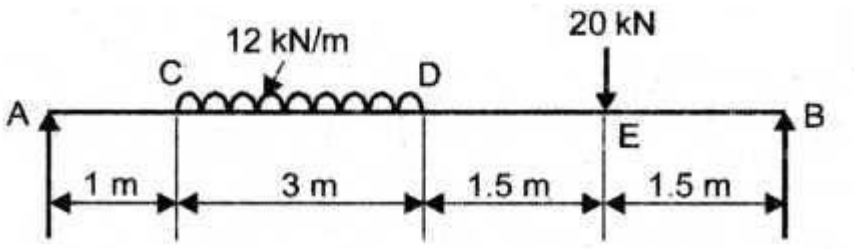
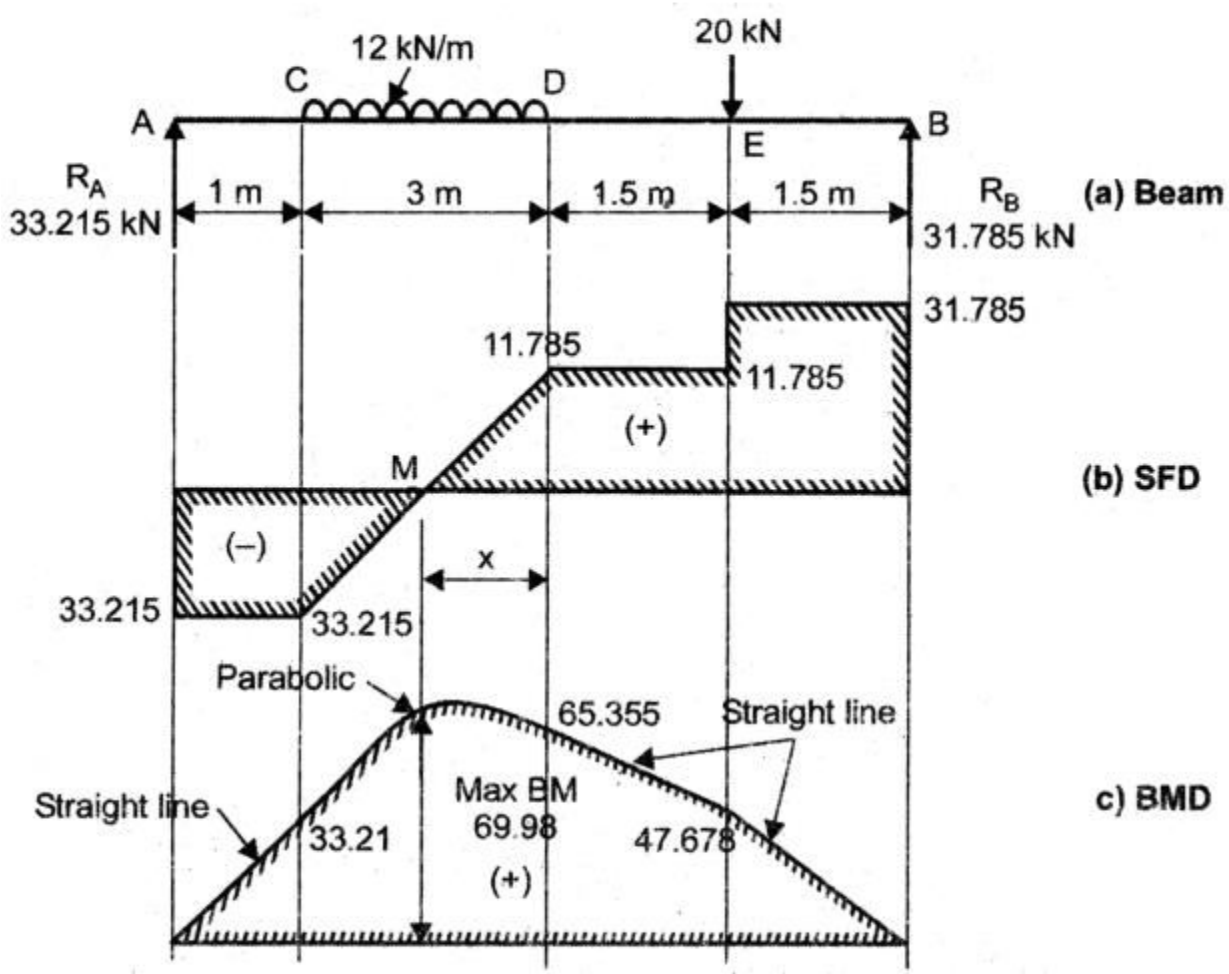
Simply supported beams and over hanging beams

Let us see an example of shear force and bending moment diagram



Solution:



Given beam is simply supported. Hence support reactions are to be determined before finding Shear force and Bending moment values at various points.

Let R_A = Reaction at A R_B = Reaction at B

To find support Reactions

Applying $\sum V$ = (↑ = ↓)

$R_a + R_B = 20 + (15 \times 3)$

= 65 KN.... (i)

Applying $\sum M_A = 0$ (↺ = ↻)

$$\left\{15 \times 3 \times \left(1 + \frac{3}{2}\right)\right\} + (20 \times 5.5) = R_b \times 7$$

Solving, R_B = 31.785 KN

Substituting R_b in equation (i)

$R_a = 65 - R_b = 65 - 31.785 = 33.215$ KN

Shear Force:

$(SF)_B = + R_b = + 31.785$ KN

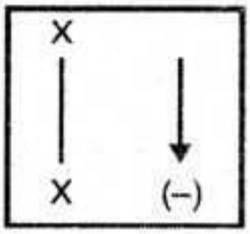
$(SF)_e = 31.785 - 20 = 11.785$ KN

$(SF)_D = 31.785 - 20 = 11.785$ KN

$(SF)_C = 31.785 - 20 - (15 \times 3) = -33.215$ KN

$(SF_a)_r = (SF)_C$ (Since there is no load in AC region)

$= -33.215$ KN



Bending Moment:

$(BM)_B = 0$

$(BM)_E = +(31.785 \times 1.5) = 47.678\ K\ N_m$

$(BM)_d = (31.785 \times 3) - (20 \times 1.5) = 65.355\ K\ N_m$

$(BM)_C = (31.785 \times 6) - (20 \times 4.5) - (15 \times 3 \times \frac{3}{2})$

$= 33.21$ k Nm

$(BM)_A = 0$ since the End A is simply supported.

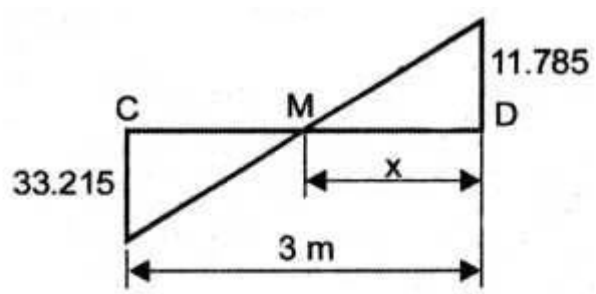
The shear force diagram and Bending moment diagram are shown in Figure (b) and (c) respectively.

Form Figure(c), it is evident that the maximum Bending moment occurs in the region CD where the shear force is zero (or where the shear force changes its sign).

To determine the maximum bending moment, the location of point of maximum Bending Moment is to be determined first.

To find the point of zero shear force (or Max. BM).

Consider the shear force diagram in the region CD as shown below.



Let the point of zero shear force (or maximum BM) occurs at M, at a distance of x from D. Now, from similar triangles,

$$\frac{11.785}{x} = \frac{33.215}{(3-x)}$$

or33.215x=11.785(3 – x)

=35.355 – 11.785 x

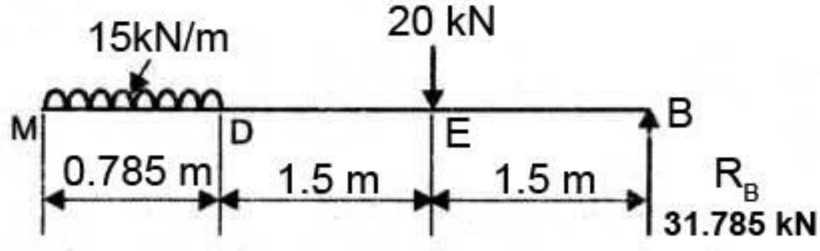
or (33.215x + 11.785x) =35.355

or45x=35.355

orx = $\frac{35.355}{45}$ = 0.785 m

To find the maximum Bending moment

To find the magnitude of maximum Bending moment consider the right side of the beam up to the point M as shown below and determine the Bending Moment at M.

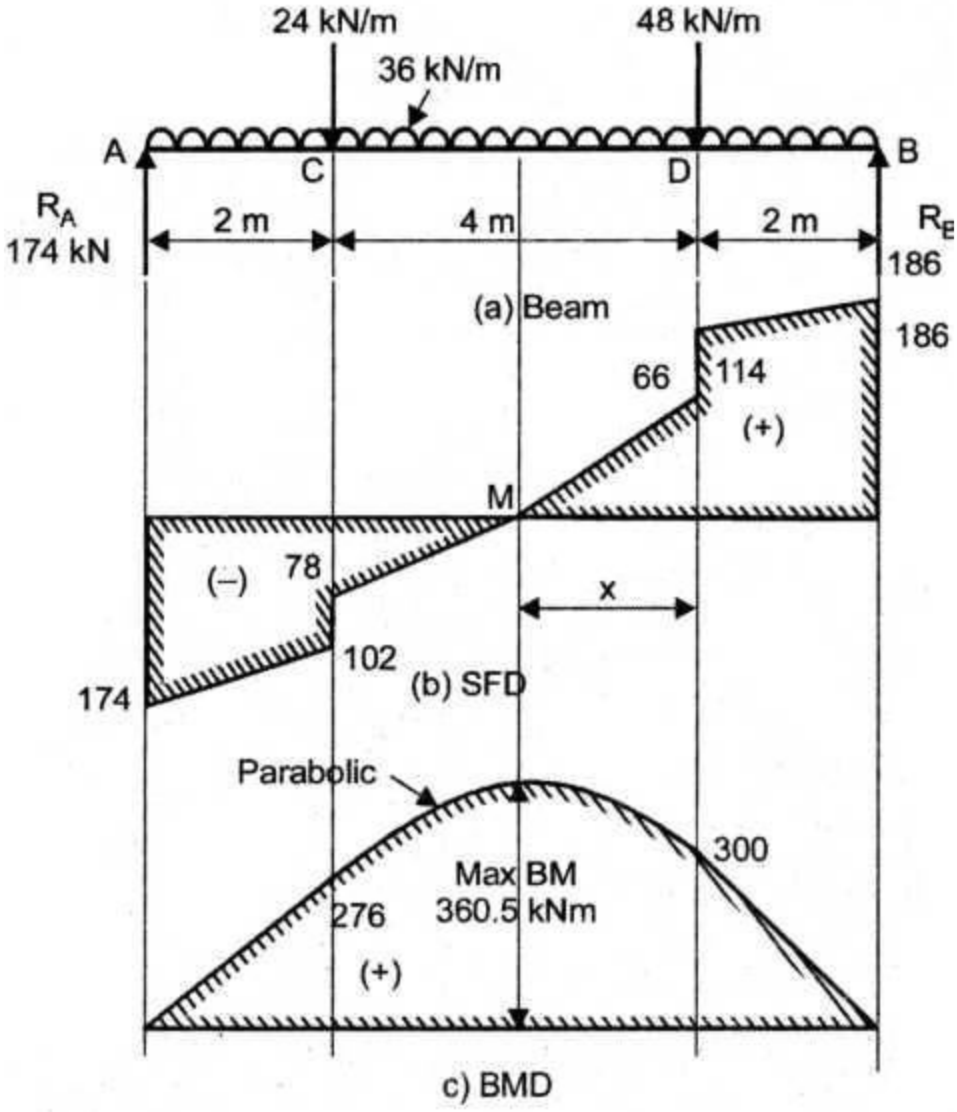


Maximum Bending Moment = (BM) at M

$\therefore$ $(BM)_m = (31.785 \times 3.785) - (20 \times 2.285) - (15 \times 0.785 \times \frac{0.785}{2})$

= 69.98 k Nm.

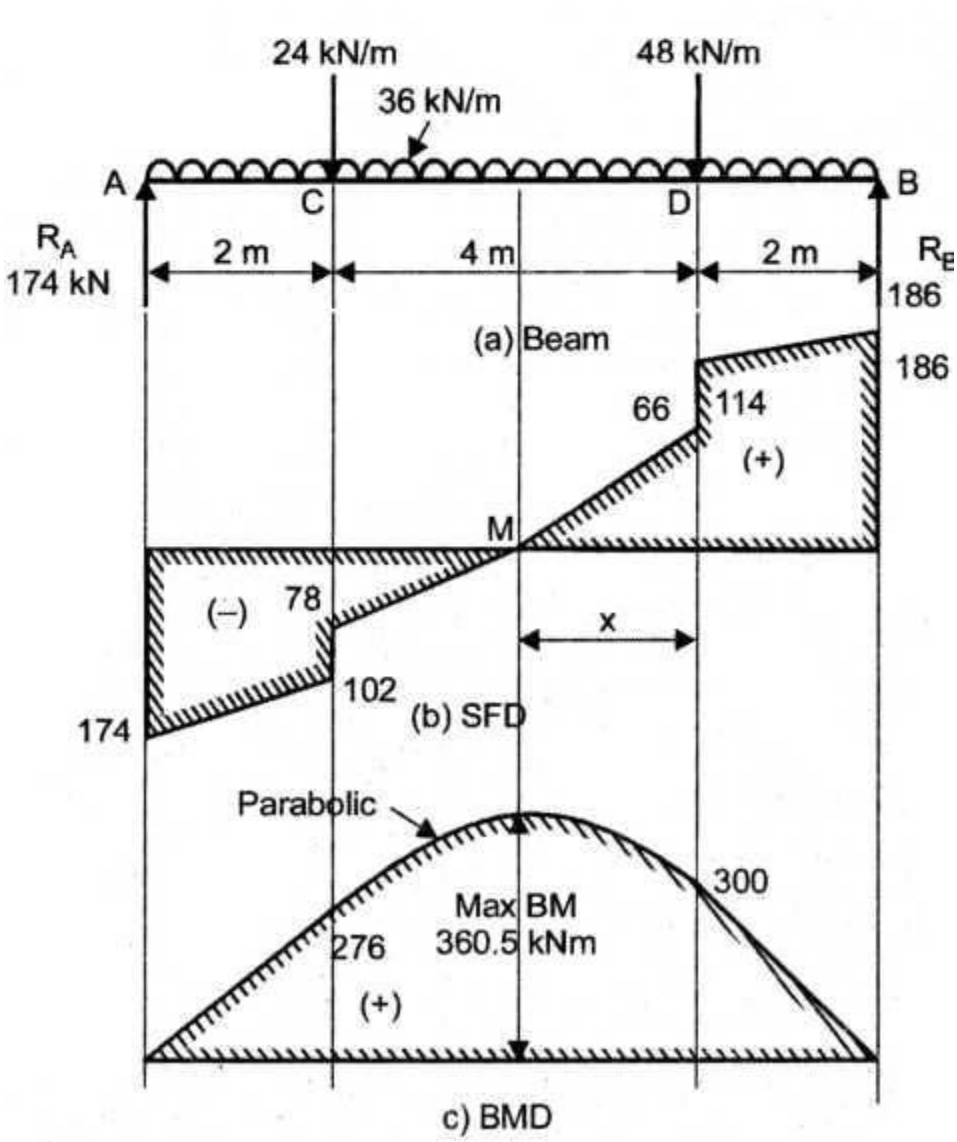
Here we solve a simply supported beam of span 8m long is subjected to two concentrated loads of 24 KN and 48 KN at 2m and 6m from left support respectively. In addition it carries a UDL of 36 KN/m over the entire span. Draw shear force and Bending moment diagrams. Mark the salient points.



Solution:

The simply supported beam with the given loads are shown in figure (a).

Shear force and Bending moment diagrams are shown in figure (b) and (c) respectively.



Support Reactions

Applying$\sum V = 0 (\downarrow = \uparrow)$

$R_a + R_B = 24 + 48 + (36 \times 8)$

= 360 kN....(1)

Applying $\sum M_A = 0$

$(24 \times 2) + (48 \times 6) + \left(36 \times 8 \times \frac{8}{2}\right) = R_B \times 8$

$48 + 288 + 1152 = 8R_B$

Solving, $R_B = 186$ KN

Substituting R_B in equation (1), $R_A = 360 - 186$

= 174 KN

Shear Force:

$(SF)_b = 186$ KN

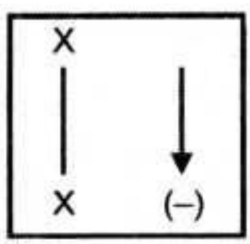
$(SF_d)_r = 186 - (36 \times 2) = 114$ KN

$(SF)_d = 186 - (36 \times 2) - 48 = 66$ KN

$(SF_c)_r = 186 - 48 - (36 \times 6) = -78$ KN

$(SF)_r = 186 - 48 - (36 \times 6) - 24 = -102$ KN

$(SF)_a = -174$ KN



Bending Moment:

$(BM)_b = 0$

$(BM)_d = (186 \times 2) - \left(36 \times 2 \times \frac{2}{2}\right) = 300$ kNm

$(BM)_C = (186 \times 6) - (48 \times 4) - \left(36 \times 6 \times \frac{6}{2}\right) = 276$ kNm

$(BM)_a = 0$.

Position of Maximum Bending Moment

Bending Moment is maximum where the shear force changes its sign i.e., zero shear force. Referring the shear force diagram, let the shear force is zero at M in the region CD, at a distance of x meter from D.

To find x: Consider the SFD in the region CD as shown below.

From similar triangles

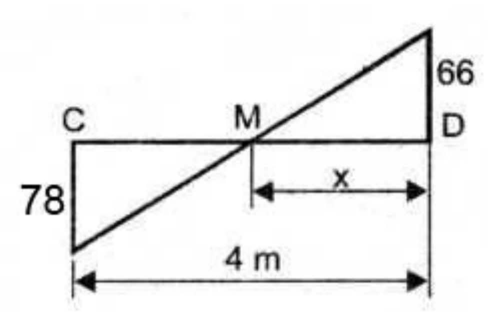
$$\frac{66}{x} = \frac{78}{(4-x)}$$

or $78x = 66(4 - x)$

or $78x = 264 - 66x$

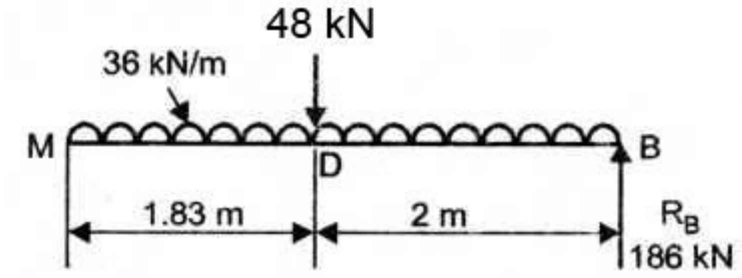
or $144x = 264$

or $x = \frac{264}{144} = 1.833$ m



To find Magnitude of Maximum Bending Moment

Bending moment is maximum at M. Consider the right side of beam up to the point M as shown below.

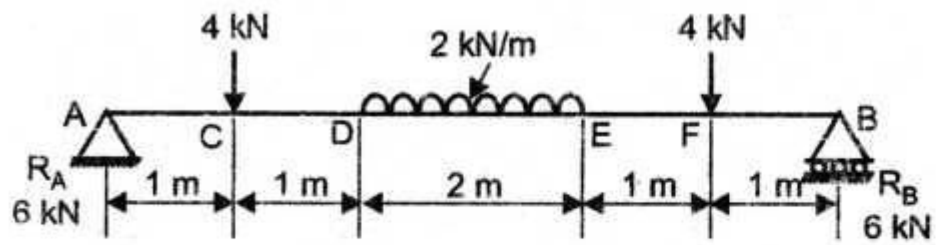


Maximum BM = BM at M

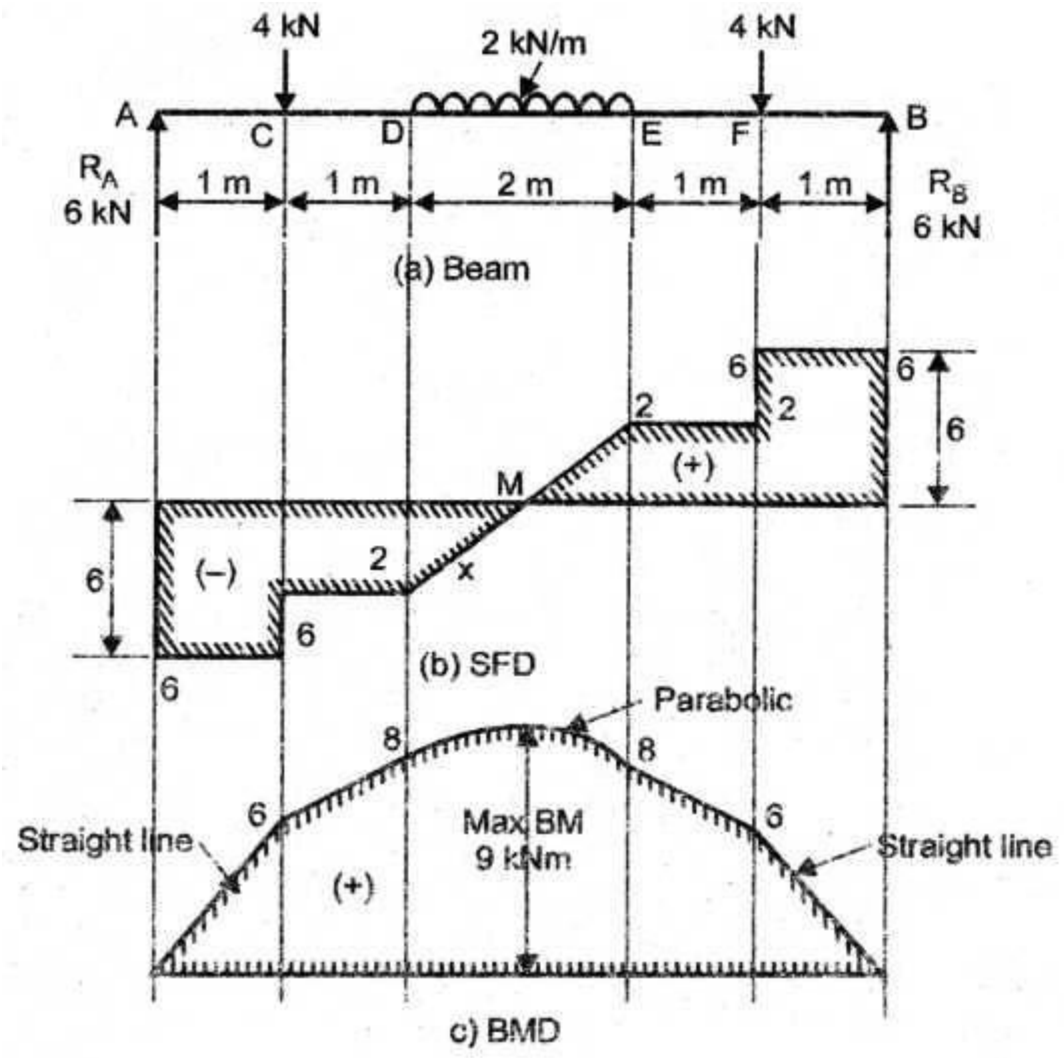
$$\therefore (BM)_M = (186 \times 3.83) - (48 \times 1.83) - (36 \times 3.83 \times \frac{3.83}{2})$$

= 360.50 k Nm

We shall learn about bending moment diagram



Solution:



Support Reactions

Applying $\sum V = 0$

$R_a + r_b = 4 + 4 + (2 \times 2) = 12$ KN ....(1)

Applying $\sum M_A = 0$

$(4 \times 1) + (4 \times 5) + \left\{2 \times 2 \times \left(2 + \frac{2}{2}\right)\right\} = R_B \times 6$

$4 + 20 + 12 = 6R_b$

(or) $R_B = 6$ KN

Substituting R_B in equation (1), $R_A = 12 - 6 = 6$ KN.

Shear Force:

$(SF)_b = 6$ KN

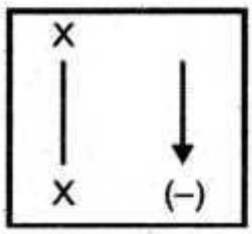
$(SF)_f = 6 - 4 = 2$ KN

$(SF)_E = 6 - 4 = 2$ KN

$(SF)_D = 6 - 4 - (2 \times 2) = -2$ KN

$(SF)_C = 6 - 4 - (2 \times 2) - 4 = -6$ KN

$(SF)_a = -6$ KN



Bending Moment:

$(BM)_B = 0$

$(BM)_F = (6 \times 1) = 6$ K N_m

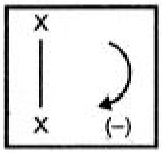
$(BM)_E = (6 \times 2) - (4 \times 1) = 8$ k Nm

$(BM)_D = (6 \times 4) - (4 \times 3) - \left(2 \times 2 \times \frac{2}{2}\right) = 8$ k Nm

$(BM)_C = (6 \times 5) - (4 \times 4) - \left\{2 \times 2 \times \left[1 + \frac{2}{2}\right]\right\}$

$= 6$ k Nm

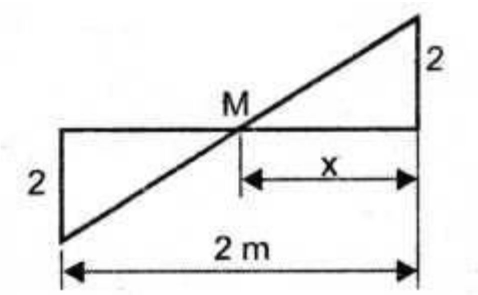
$(BM)_A = 0$



Shear force and Bending moment diagrams are shown in figure (b) and (c) respectively. Referring the shear force diagram, it is evident that the shear force changes its sign (i.e., zero shear force) in the region DE.

Location of zero Shear Force

Consider the SFD in the region DE.



Let the zero Shear force occurs at M From similar triangles,

$$\frac{2}{x} = \frac{2}{(2 - x)}$$

Solving, $x = 1$ m.

i.e., Zero SF occurs at midpoint of DE. i.e., at the midpoint of beam.

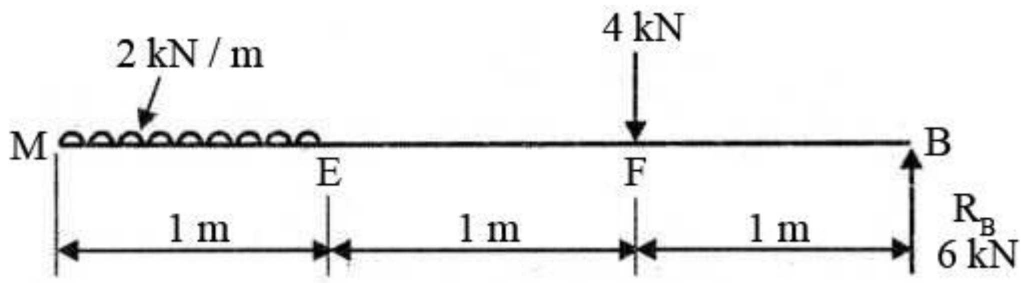
Note: In all symmetrically loaded beams.

i)Reactions are equal

ii)Zero shear force i.e., Maximum Bending moment occurs at midpoint of beam length.

To find Maximum Bending Moment

Consider the right side of beam up to the point M (i.e., up to mid point of beam).

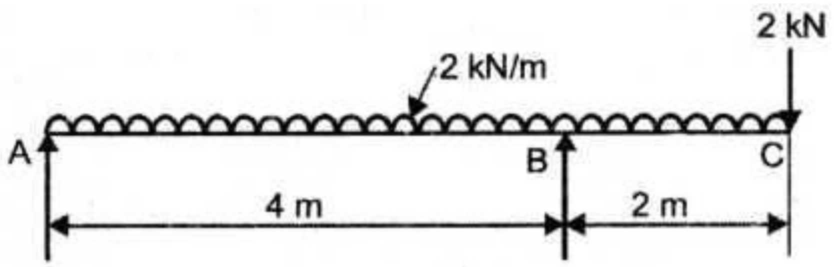


Maximum Bending moment = BM at M

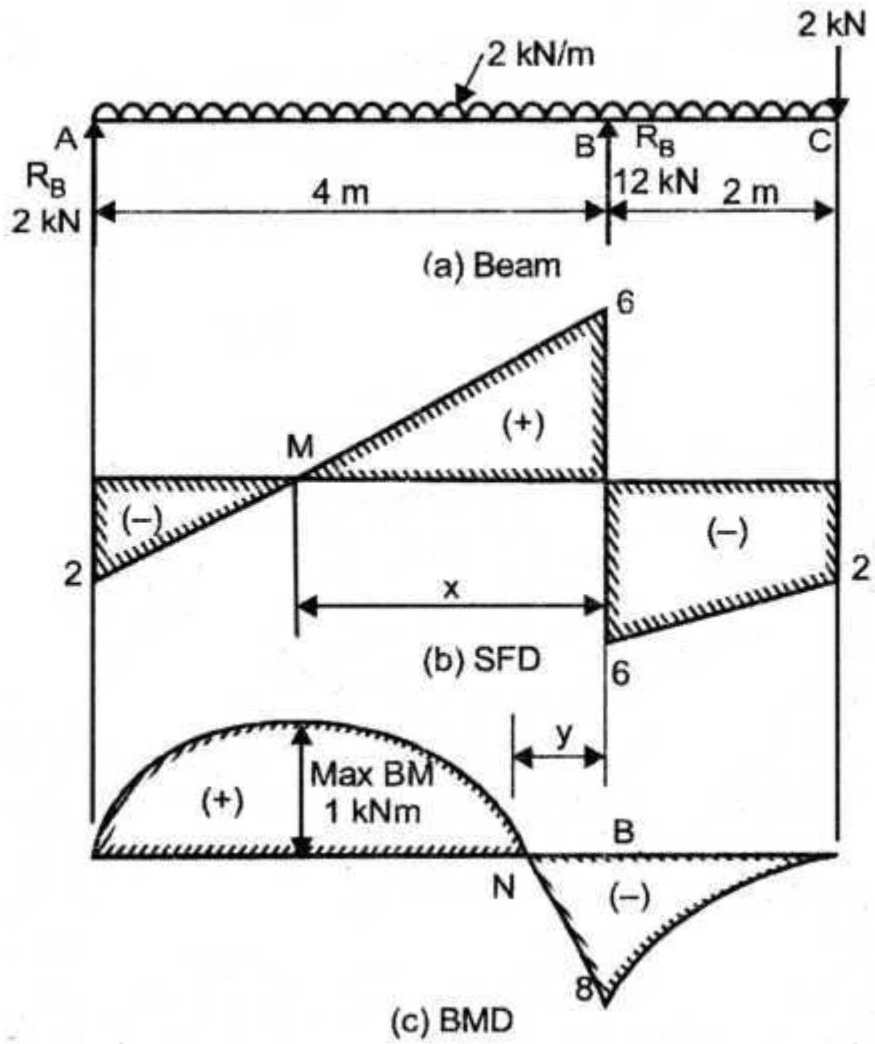
$\therefore (BM)_m = (6 \times 3) - (4 \times 2) - (2 \times 1 \times \frac{1}{2})$

= 9 k Nm

Let us draw SF and BM



Solution:



Support Reactions

Applying $\sum V = 0$

$R_a + R_b = 2 + (2 \times 6)$

$= 14$ KN....(1)

Applying $\sum M_A = 0$

$(2 \times 6) + (2 \times 6 \times \frac{6}{2}) = R_B \times 4$

or $12 + 36 = 4\ R_b$

$\therefore R_B = 12$ KN

Substituting the value of R_B in equation (1)

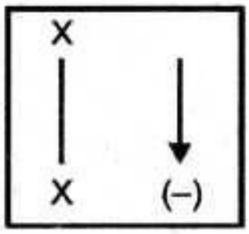
$R_A = 14 - R_b = 2$ KN

Shear Force:

$(SF)_C = -2$KN

$(SF_b)_r = -2 - (2 \times 2) = -6$ KN

$(SF)_B = -2 - (2 \times 2) + 12 = 6$ KN
$(SF_a)_r = -2 - (2 \times 6) + 12 = -2$ KN



Bending Moment:

$(BM)_C = 0$

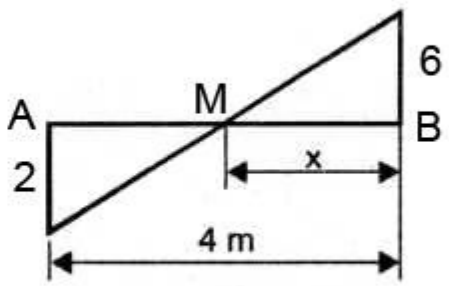
$(BM)_B = -(2 \times 2) - (2 \times 2 \times \frac{2}{2}) = -8$ k Nm

$(BM)_a = 0$

Referring the figure (b)the point of Max.BM i.e., zero shear force occurs in the region AB, at a distance of x from B.

Location of point of Max.BM

Taking SFD in the region AB from similar triangles



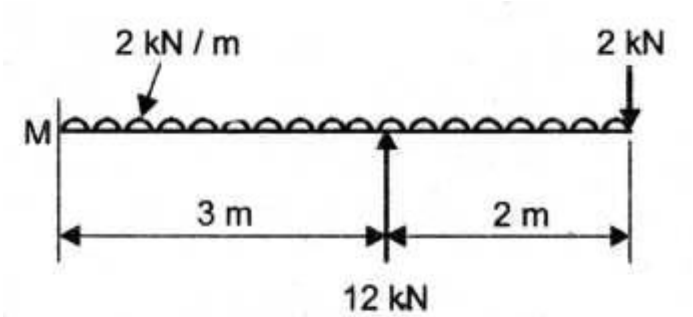
$$\frac{6}{x} = \frac{2}{(4-x)}$$

or2x = 24 – 6xor8x = 24

x = 3 m

Maximum Bending Moment:

Consider the right side of beam up to the point M.



Max BM = $(BM)_m$= – (2 × 5) – (2 × 5 × $\frac{5}{2}$) + (12 × 3)

= 1 k Nm

∴ Max.Positive BM = 1 k Nm occurs at 1 from the left support A

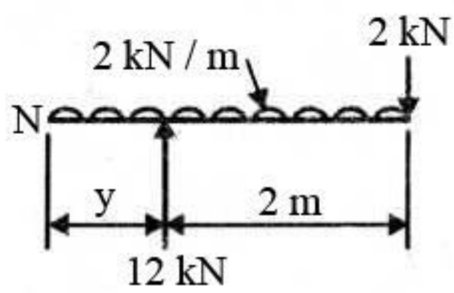
Max. Negative BM = 8 k Nm occurs at the support B.

To find the point of contraflexure

The point at which Bending moment is zero is known as point of contraflexure. Referring the Bending moment diagram, let the point of zero BM occurs at ‘N ’ at a distance ‘y’ m from B and towards A.

To find ***y***

Consider right side of beam up to the point N



$(BM)_n = 0$

But $(BM)_N = -2(2 + y) - \left\{2 \text{ x } (2+y) \text{ x } \frac{(2+y)}{2}\right\} + (12 \times y)$

$= -4 - 2y - \{(2 + y)^2\} + 12y$

$= -4 - 2y - (4 + y^2 + 4y) + 12y$

$= -4 - 2y - 4 - y^2 - 4y + 12y$

$(BM)_n = -y^2 + 6y - 8 = 0$

$\therefore$ or $y^2 - 6y + 8 = 0$

Or $y = \frac{6 + \sqrt{6^2 - (4 \text{ x } 1 \text{ x } 8)}}{2} = \frac{6+2}{2}$

= 4 m and 2 m

The point of zero BM occurs at 4 m from B and towards A (i.e., at the left support A itself) and 2 m from Band towards A (i.e., at the point N)