Mohr's circle of stress

Obliquity:

The angle made by the resultant stress with the normal of the oblique plane is known as obliquity. It is denoted by the symbol φ.

$$\therefore \tan\phi = \frac{\sigma_t}{\sigma_n}$$

where σ_t = tangential stress

σ_n = normal stress

Mohr's circle of stress:

Mohr's circle of stresses is a graphical method of finding normal, tangential and resultant stresses on an oblique plane.

Radius of Mohr's circle is equal to the maximum shear stress.

To find out the normal, resultant stresses and principle stress and their planes.

Procedure for Mohr's circle diagram

Plot the vertical face coordinates V(σ_{xx}, τ_{xy}).

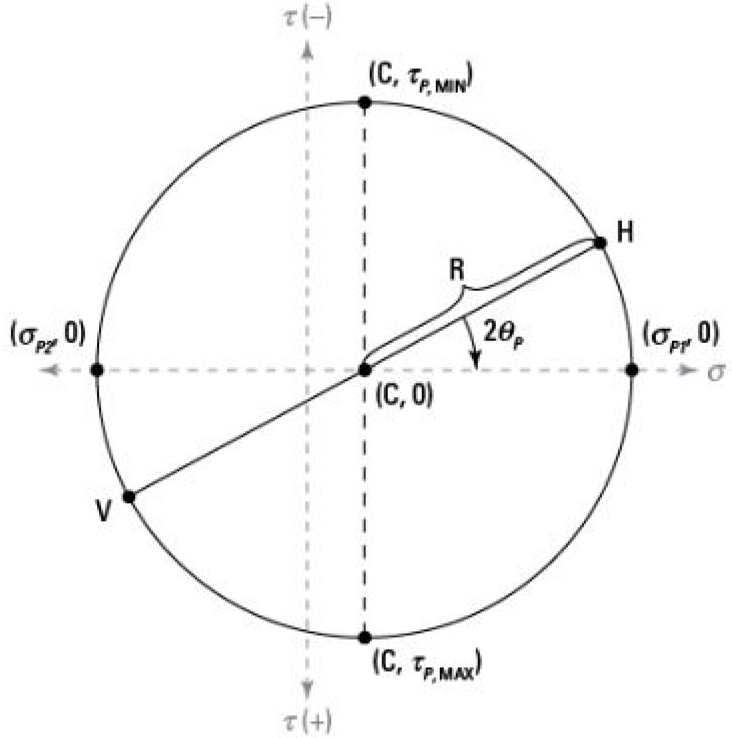
Plot the horizontal coordinates H(σ_{yy}, $-\tau_{xy}$)

You use the opposite sign of the shear stress from Step 1 because the shear stresses on the horizontal faces are creating a couple that balances (or acts in the opposite direction of) the shear stresses on the vertical faces.

Draw a diameter line connecting Points V (from Step 1) and H (from Step 2).

Sketch the circle around the diameter from Step 3.

The circle should pass through Points V and H as shown here.



Compute the normal stress position for the circle's center point (*C*).

$$C = \frac{\sigma_{xx} + \sigma_{yy}}{2}$$

Calculate the radius (*R*) for the circle.

$$R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \left(\tau_{xy}\right)^2}$$

Determine the principal stresses σ_{P1} and σ_{P2}.

$$\sigma_{P1,P2} = C \pm R$$

Compute the principal angles Θ_{P1} and Θ_{P2}.

You could also use equations directly (instead of Mohr's circle) to determine transformed stresses at any angle:

$$\sigma_{x1} = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\sigma_{xx} - \sigma_{yy}}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x1y1} = -\frac{\sigma_{xx} - \sigma_{yy}}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

To construct a Mohr's circle for strain or to use the transformation equations, substitute ε_{xx} for σ_{xx}, ε_{yy} for σ_{yy}, and $(0.5)\gamma_{xy}$ for τ_{xy} in the preceding equations.

Problems

Let us discuss about an elemental cube is subjected to tensile stresses of 30 N/mm^2 and 10 N/mm^2 acting on two mutually perpendicular planes and a shear stress of 10 N/mm^2 on these planes. Draw the Mohr's circle of stresses and determine the magnitude and direction of principal stresses and also the greatest shear stress.

Solution:

Graphical Solution (Mohr's circle method)

Assume 1 cm == 2 N/mm^2

$\therefore \sigma_1 = 30$ N/mm^2

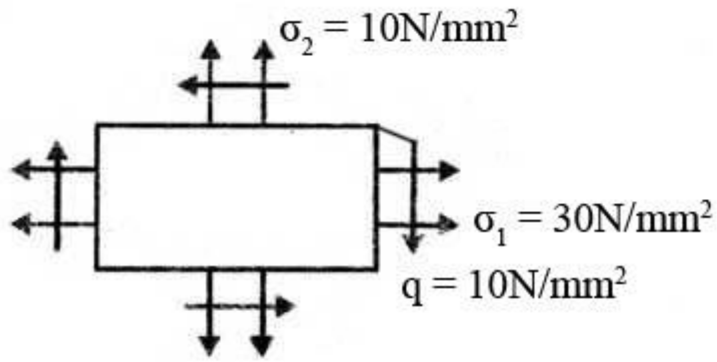
$$= \frac{30}{2} = 15 \text{ cm}$$

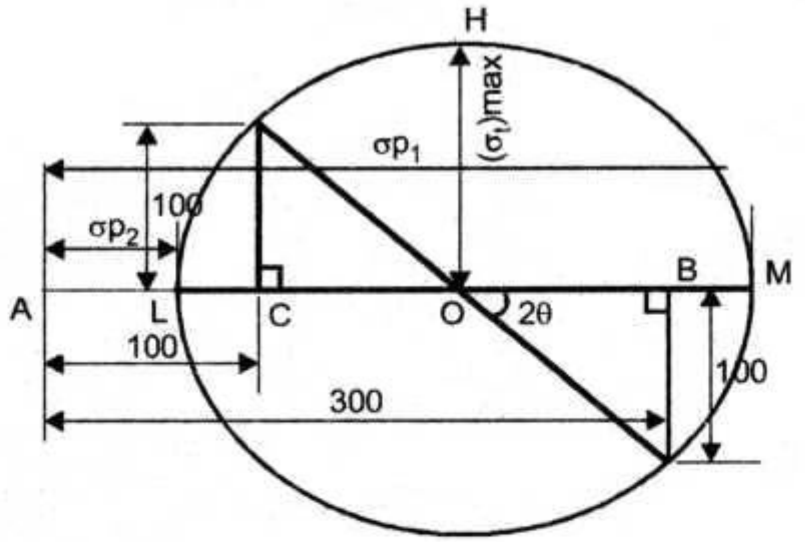
$\sigma_2 = 10$ N/mm^2

$$= \frac{10}{2} = 5 \text{ cm}$$

q = 10 N/mm^2

$$= \frac{10}{2} = 5 \text{ cm}$$





1. Locate a point A and draw a horizontal line through A.

2. Take AB = σ_1 = 15 cm and AC = σ_2 = 5 cm towards right side of A (since both are tensile).

3. Draw perpendiculars through B and C and mark BF = CG = q = 5 cm.

4. Bisect BC at O.

5. Taking O as center and radius equal to OG (or OF) draw a circle cutting the horizontal line drawn through A at L and M.

6. AM = Major principal stress

AL = Minor principal stress

OH = Radius of circle = Maximum shear stress

From the diagram

AM = 17.1 cm = 17.1 × 2 = 34.2 N/mm^2

AL = 2.93 cm = 2.93 × 2 = 5.86 N/mm^2

OH = radius= 7.05 cm= 7.05 × 2 = 14.1 N/mm^2

$2\theta = 45°$(or) $\theta = \frac{45}{2}$ = 22.5°

Answer

Maximum principal stress, σ_{P1} = 34.2 N/mm^2

Minimum principal stress, σ_{P1} = 5.86 N/mm^2

Maximum shear stress, $(\sigma_t)_{max}$ = 14.1 N/mm^2

Direction of principal planes, θ_1, θ_2 = 22.5° and 112.5°.