

UNIT - II

SPUR GEARS AND PARALLEL AXIS HELICAL GEARS

GEAR DRIVES:

Introduction:

A Gear drive is a mechanical drive which transmits power through toothed wheels called as gears.

In the gear drive arrangement the smaller wheel ~~is~~ is named as pinion and the higher wheel is known as wheel or gear regardless of the nature of working. Such as the smaller wheel may be operated as driving member or driven member as soon.

Classification of Gears:

1) Based on the position of the axes of gear shafts such as

(a) Parallel :- Ex :- Spur, helical, herring-bone gears.

(b) Intersecting - Bevel gear

(c) Non Intersecting and non Parallel - Crossed helical, Worm gear - hypoid, Spheroid, Planoid, beveloid, helican, face gear etc.

2) Based on the type of engagement

(a) External gearing

(b) Internal gearing

(c) Rack and pinion gear

(d) Sector gearing.

3, Based on the position of teeth on the wheel rim.

4, Based on the peripheral speed of gears

(a) Low speed gears $v < 3 \text{ m/s}$

(b) Medium speed gears $3 < v < 15 \text{ m/s}$

(c) High speed gears $v > 15 \text{ m/s}$

5, Based on profile

(a) Involute profile gear

(b) Cycloidal profile gear

(6) Based on pressure angle

(a) Gears with 20° pressure angle.

(7) Based on tooth height (or) working depth.

Design Parameters:

1, Tooth Profile

2, Base Circle

3, Pitch Circle

4, Gear Centre

5, Line of Centre

6, Addendum Circle

7, Dedendum Circle

8, Point of Contact

9, Patch of Contact

10, Line of action

11, Pressure angle

12, Module

13, Circular Pitch

14, Base Pitch.

- 15, Diametral Pitch
- 16, Addendum
- 17, Dedendum
- 18, Clearance
- 19, Tooth height (or) total depth
- 20, Working depth
- 21, Face width
- 22, Face
- 23, Flank
- 24, Top Land.
- 25, Bottom Land
- 26, Centre distances
- 27, Height factor
- 28, Tooth thickness
- 29, width of Spaces
- 30, Backlash
- 31, Gear ratio.

Contact Ratio:

In a spur gear drive, when are pinion drives the wheel the contact b/w teeth of pinion and wheel will be gradual from up to good and end of teeth.

The ratio of its length of action to the circular pitch is termed as contact ratio.

Law of Gearing:

The law of gearing states the for obtaining a constant velocity ratio at any instant of teeth, the common normal at each point of contact should always pass through the pitch point situated on the line joining the centres of rotation of the pair of mating gear.

Design Analysis of Spur Gear Drive:

1, Force acting on gear tooth

$$M_t = \frac{60P}{2\pi N}$$

$$T_t = \frac{\text{Torque}}{\text{Pinion radius}} = \frac{M_t}{\frac{d_1}{2}}$$

$$\text{normal force } F_n = \frac{F_t}{\cos \alpha} = \frac{2M_t}{d_1 \cos \alpha}$$

$$\text{Centre distance } a = \frac{d_2 + d_1}{2}$$

2, Load Concentrations:

$$k = \frac{\sigma_{\max}}{q}$$

3, Dynamic Load

$$k_d = \left(1 + \frac{F_i}{F_a} \right)$$

$$M_d = \text{Design torque} = M_t k k_d$$

4, Induced Stresses on gear tooth:

5, Dynamic effects:

↳ For ordinary and Commercial cut gears made with $v < 10$ m/s

$$C_v = \frac{3}{3+v}$$

↳ For accurately hobbed and generated gears with m/s

$$1, C_v = \frac{6}{6+v}$$

3. For Precision gear with grinding and Lapping operation

$$C_v = \frac{0.75}{1+v} + 0.25$$

$$F_d = \frac{F_i}{C_w}$$

6) Factor of Safety:

Factor of Safety the bending

$$F_{\text{bending}} = \frac{\text{Beam Strength of gear tooth}}{\text{Dynamic load at gear tooth}}$$

The recommended factor of safety 1.5 to 2

Fos for Pitch

$$S_{\text{pitch}} = \frac{\text{Gear Strength of gear tooth}}{\text{Dynamics load gear tooth}}$$

① Problem:

Design a part of Spur Gears to transmit 20 kw at a pinion speed of 1400 rpm, The transmission ratio is 4, which assume suitable materials and stresses.

Solution

Power to be transmitted $P = 20 \text{ kw}$
Pinion Speed, $n = 1400 \text{ rpm}$

Gear ratio $i = 4$

$$\text{Gear Speed} = \frac{P}{i} = \frac{1400}{4} = 350 \text{ rpm.}$$

Let the arrangement may be external gearing.

Material Selection:

Now the materials for Pinion and Gear are selected as follow

Name	Material	σ_c in N/mm^2	σ_t N/mm^2
Pinion	15Ni2Cr1Mo15	950	320
Gear	C45	500	340

Minimum Centre distance

$$a \geq i + 1 \left[\frac{74}{\sigma_c} \right]^2 \left[\frac{1 + m_1}{i_4} \right]^{1/2}$$

$$E = 215 \times 10^5 \text{ N/mm}^2 \text{ table 7.14}$$

$$\sigma_c = 500 \text{ N/mm}^2$$

$$i = 4$$

$$\psi = \frac{b}{a} = 0.3$$

$$[M_E] = M_E k k_b$$

$$M_E = \frac{60P}{2\pi N}$$

$$= \frac{60 \times 20000}{2\pi \times 1400}$$

$$= 136.4 \text{ N}$$

$$= 136.4 \times 10^3 \text{ N}\cdot\text{mm}$$

$$k k_b = 1.3 \text{ initially}$$

$$M_E = 136.4 \times 10^3 \times 1.3$$

$$= 177.3 \times 10^3 \text{ N}\cdot\text{mm}$$

Substitute the above value in the equation

$$a \geq (i+1) \left[\frac{74}{500} \right]^2 \approx \frac{2.15 \times 10^5 \times 117.3 \times 10^3}{4 \times 3} \Bigg]^{1/3}$$

3, Minimum Module

$$m \geq 1.26 \left[\frac{M}{\sigma_b 4m^2} \right]^{1/3}$$

$$M = 177.3 \times 10^3 \text{ N-mm}$$

$$\sigma_b = 140 \text{ N/mm}^2$$

$$V_{az} = \frac{b}{m} = 10 \text{ (Assumed)}$$

$$Z_1 = 20 \text{ Assumed.}$$

$$y = 0.389, Z_1 = 20 \text{ Table 7.18}$$

$$m \geq 1.28 \left[\frac{177.3 \times 10^3}{140 \times 10 \times 20 \times 0.389} \right]^{1/3} \approx 3.2$$

$$m = 4 \text{ mm from table}$$

$$Z_1 = \frac{2a}{m(i+1)}$$

$$Z_1 = 2ad \quad Z_2 = i Z_1$$

$$= 4 \times 22 = 88 \text{ teeth.}$$

4, Pitch Circle diameter:

$$\text{Pitch Circle dia for Pinion } d_1 = m Z_1 \\ = 4 \times 22 = 88 \text{ mm}$$

$$\text{Gear } d_2 = m Z_2 = 4 \times 88 = 352$$

5, Corrected Centre distance:

$$a = \frac{d_1 + d_2}{2} = \frac{88 + 352}{2} = 220 \text{ mm}$$

6, Face width

$$b = \psi a = 0.3 \times 220 = 66 \text{ mm}$$

$$b = 4m = 10 \times 4 = 40 \text{ mm}$$

$$b = 66 \text{ (Higher Value)}$$

$$b/d = \frac{66}{88} = 0.75$$

$$k = 1.5$$

$$V = \frac{\pi d_1 n_1}{60 \times 1000}$$

$$= \frac{\pi \times 88 \times 1400}{60 \times 1000} = 6.45 \text{ m/s}$$

$$m_t = m \pm k \cdot k_c$$

$$= 136.4 \times 10^3 \times 1.05 \times 1.4$$

$$= 200.5 \times 10^3 \text{ N}\cdot\text{m}$$

7, Checking of induced Compressive Stress and bending stress

$$\begin{aligned} \sigma_c &= 0.74 \left(\frac{i+1}{n} \right) \left(\frac{i+1}{16} \times E_m \right)^{1/2} \text{ Table 7.3} \\ &= 0.74 \left(\frac{4+1}{200} \right) \left(\frac{4+1}{4 \times 66} \times 2.15 \times 10^5 \times 200.5 \times 10^3 \right) \\ &= 486 \text{ N/mm}^2 \end{aligned}$$

Wdy bending stress

$$\begin{aligned} \sigma_v &= \frac{(i+1) m_t}{a \cdot m \cdot b} \\ &= \frac{4+1 \times 200.5 \times 10^3}{220 \times 4 \times 66 \times 0.402} \\ &= 482 \text{ N/mm}^2, \quad \sigma_b = 140 \text{ N/mm}^2 \end{aligned}$$

8) Other Parameters:

$$\text{Addend } h_i = f_o m = 1 \times 4 = 4 \text{ mm}$$

$$h_f = (f_f + c) m = (1 + 0.25) 4 \\ = 5 \text{ mm}$$

Tip Centre dia

$$d_b = d_1 + 2h_o$$

$$= 88 + 2 \times 4$$

$$= 96 \text{ mm.}$$

T, P Circular dia of Gear

$$d_2 = d_2 + 2h$$

$$= 352 + 2 \times 4 = 360 \text{ mm}$$

Root Circle dia of D = PCD of Dia + 2 ded

$$d_1 = d_1 - (2 \times b_1)$$

$$= 88 - 2 \times 5 = 78 \text{ mm}$$

Root Circle dia of Pitch = PCD - 2 \times d

$$= d_1 - 2 \times b_1$$

$$= 88 - 2 \times 5 = 78 \text{ mm}$$

Root Circle dia $d_2 = \text{PCD of Gear} - 2 \times \text{ded}$

$$= d_2 - 2 \times b_2$$

$$= 352 - (2 \times 5) = 342 \text{ mm}$$

$$h = h_a + h_f = 4 + 5 = 9 \text{ mm}$$

Result

Description	Pinion	Gear
1, Material	15 Ni 2 Cr Mo Steel	C45 Steel
2, No of teeth	22 22	88
3, Pitch Circle dia	86 mm	352 mm

Result:

4, Tip Circle dia	96 mm	360 mm
5, Root Circle dia	78	342 mm
6, face width	66	66
7, Module	4 mm	4 mm
8, Tooth height	9 mm	9 mm
9, Centre distance between	= 220 mm	

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Helical Gears

Helical gears are the modified form of spur gears in which all the teeth are cut at a constant angle known as helix angle to the axis of the gear.

Types of helical Gears

1) Parallel helical gear.

* They operate on two parallel shafts.

2) Crossed helical gears.

Advantages:

1) Noise

2) Load Carrying Capacity

3) Manufacture

Design Procedure:

The steps given below may be observed for the design of helical and herring bone gear.

- 1) Note down the amount of Power to be transmitted. Pinion Speed, Gear ratio Life of Gear drive and otherwise Condition.
- 2) Calculate the minimum Centre distance
- 3) Based on bending Stress evaluate the minimum normal module.
- 4) Select the nearest standard module
- 5) Correct the number of teeth of Pinion.
- 6) Determine the Pitch Circle diameter.
- 7) Decide the corrected Centre distance
- 8) Determine the face width of gear teeth
- 9) Check the induced Compressive Stress and bending Stress with their design value as given in table 7.10
- 10) Find the essential Parameters of gear drives
- 11) Draw the schematic sketch of helical gear drive health

Problem:

A pair of helical gear is to be designed to transmit 30kw at a pinion speed of 1500rpm. The velocity ratio is 3, selecting suitable materials determine the dimension of the gears

Given Data

$$\text{Power } P = 30 \text{ kW}$$

$$\text{Pinion Speed } n_1 = 1500 \text{ rpm}$$

$$\text{Velocity ratio } i = 3$$

material for Pinion Gear as

15Ni2Cr1Mo15 Steel.

$$[\sigma_c] = 950 \text{ N/mm}^2$$

$$[\sigma_b] = 320 \text{ N/mm}^2$$

① Minimum Centre distance

$$a \geq (i+1) \sqrt[3]{\left(\frac{7}{\sigma_b}\right)^2 \frac{E(m)}{14}} \quad \text{Table 7.8}$$

$$M_t = M_t k k_d$$

$$\begin{aligned} M_t &= \frac{60P \times 10^6}{2\pi n} \\ &= \frac{60 \times 30 \times 10^6}{2\pi \times 1500} \end{aligned}$$

$$k k_d = 1.3$$

$$M_t = M_E k k_d$$

$$= 191 \times 10^3 \times 1.3$$

$$= 248 \times 10^3 \text{ N}\cdot\text{mm}$$

$$\psi = \frac{b}{a} = 0.5 \text{ and } E = 2.15 \times 10^3 \text{ N/mm}^2$$

$$a \geq 3 + 1 \sqrt{\left(\frac{7}{950}\right)^2 \times \frac{2.15 \times 10^3 \times 248 \times 10^3}{3 \times 0.5}}$$

$$\geq 108 \text{ mm}$$

To find module

$$m_n \geq 1.15 \cos^3 \beta^3 \sqrt{\frac{M_E}{\gamma (\sigma_b) \psi n Z}}$$

$$Z_1 = 20$$

$$\psi_m = \frac{b}{m_f} = 10 \text{ Assumed.}$$

$$\beta = \text{helix angle} = 20^\circ$$

$$Z = \frac{Z_1}{\cos^3 \beta} = \frac{20}{\cos^3 20} = 24.1$$

$$\psi_v \text{ for } Z_n = 24 = 0.414$$

$$m_n \geq 1.15 \cos 20 \times \sqrt{\frac{248 \times 10^3}{414 \times 320 \times 10 \times 20}}$$

$$\geq 2.3 \text{ mm}$$

To find out the number of teeth of
Pinion and gear

$$Z_1 = \frac{2a \cos \beta}{m_c (C+1)}$$

$$= \frac{2 \times 108 \times \cos 20^\circ}{2.5(3+1)}$$

$$= 20.3 = 22$$

$$Z_2 = i Z_1 = 3 \times 22 = 66$$

To find Corrected Centre distance

$$a = \frac{m_n}{\cos \beta} \left(\frac{z_1 + z_2}{2} \right) = \frac{2.5}{\cos 20^\circ} \left(\frac{22 + 66}{2} \right)$$

$$= 1107 \text{ mm}$$

Since this corrected centre distance is more than its minimum value and design is Centre

To Check Induced Stress

$$\sigma_c = 7 \left(\frac{i+1}{a} \right) \sqrt{\frac{1+C}{i_b}} \times E (m_t)$$

$$b = \psi a = 0.5 \times 1117 = 58.5 \text{ mm}$$

$$b = \psi m_n \quad m_n = 10 \times 2.5 = 25 \text{ mm}$$

$$\sigma_c = 7 \left(\frac{3+1}{1117} \right) \sqrt{\frac{3+1}{3 \times 60}} \times 2.15 \times 10^6 \times 248 \times 10^{-3}$$

$$= 824 \text{ N/mm}^2 < \sigma_c = 950 \text{ N/mm}^2$$

$$\sigma_c = \frac{7(i+1) m_t}{a b m_n \psi_n}$$

$$z_1 = 28.5 \text{ is } 0.427$$

$$\sigma_b = \frac{7 \times (3+1) \times 248 \times 10^3}{117 \times 60 \times 2.5 \times 0.427}$$

$$= 92.7 \text{ N/mm}^2 \leq \sigma_b 320 \text{ N/mm}^2$$

Pitch Circle dia of Pinion

$$d_1 = \frac{m_n}{\cos \beta} \times Z_1 = \frac{2.5}{\cos 20} \times 22 = 58.5 \text{ mm}$$

$$d_2 = \frac{m_n}{\cos \beta} \times Z_2 = \frac{2.5}{\cos 20} \times 66 = 175.5 \text{ mm}$$

Addendum $h_a = m_n = 2.5 \text{ mm}$

Dedendum $h_f = 1.25 m_n = 1.25 \times 2.5$

$$= 3.125 \text{ mm}$$

T₁ P Circle dia of

$$d_{a1} = d_1 + 2h_a$$

$$= 58.5 + (2 \times 2.5)$$

$$= 63.5 \text{ mm}$$

$$d_{a2} = d_2 + 2h_a$$

$$d_{a2} = 180.5 \text{ mm}$$

Root Circle dia of Pinion $d_1 = d_1 - 2h_f$

$$= 58.5 - (2 \times 3.125)$$

$$= 52.25$$

$$d_1 = d_1 - 2h_f$$

$$= 58.5 - 2 \times 3.125$$

$$= 52.25$$

$$d_2 = d_2 - 2h_f$$

$$= 175.5 - (2 \times 3.125)$$

$$= 169.25 \text{ mm}$$

Result:

Sl no	Description	Pinion	Gear
1,	Material	13 Ni 2 Cr 1 mol 5	15 Ni 2 Cr 1 mol 5
2,	No. of teeth	22	66
3,	Pitch Circle dia	58.5 mm	175.5 mm
4,	Tip Circle dia	63.5 mm	180.5 mm
5,	Root Circle dia	52.25 mm	169.25 mm
6,	Face width	60 mm	60 mm
7,	Normal module	2.5	2.5
8,	Centre distance	= 117 mm	
9,	Helix angle	= 20°	

