given gears ... manner as solved in Example 5.17.

Example 5.19 ***Design a spur gear drive to transmit 22.5 kW at 900 r.p.m. Speed reduction is 2.5. Materials for pinion and wheel are C 15 steel and cast iron grade 30 respectively. Take pressure angle of 20° and working life of the gears as 10000 hrs.***

A

Sem 16Mark

Given Data : $P = 22.5$ kW ; $N_1 = 900$ r.p.m. ; $i = 2.5$; $\phi = 20°$; $N = 10000$ hrs.

To find : Design a spur gear.

* Pinion → C 15 steel
* Wheel → Cast iron grade 30

Solution : Since the ***materials for pinion and wheel are different, therefore we have to design the pinion first and check both pinion and wheel.***

1. ***Gear ratio :*** Given $i = 2.5$ ... (Given)

$$i = \frac{N_1}{N_2} = \frac{Z_2}{Z_1}$$

2. ***Material selection :***

Pinion : C15 steel, case hardened to 55 RC and core hardness < 350, and

Wheel : C.I. grade 30. ... (Given)

3. ***Gear life :*** Given $N = 10000$ hrs ... (Given)

Gear life in terms of number of cycles is given by

$$N = 10000 \times 60 \times 900 = 54 \times 10^7 \text{ cycles}$$

4. ***Design torque [M_t] :***

PSG 8.15

$$[M_t] = M_t \cdot \underbrace{K \cdot K_d}$$

$P = \frac{2\pi N_1 \cdot M_t}{60}$

where $M_t = \frac{60 \times P}{2\pi N_1} = \frac{60 \times 22.5 \times 10^3}{2\pi \times 900} = 238.73$ N-m, and

Given PSG 8.15 $K \cdot K_d = 1.3$ PSG 8.15 $M_t = [M_t \times K \times K_d]$ → 8.15 ... (assum

∴ Design torque, $[M_t] = 238.73 \times 1.3 = 310.35$ N-m → M_t

5. *Calculation of E_{eq}, $[\sigma_b]$ and $[\sigma_c]$:* PSG 8.14

(i) *To find E_{eq}:* From Table 5.20, for pinion steel and cast iron (> 280 N/mm ... equivalent Young's modulus, $E_{eq} = 1.7 \times 10^5$ N/mm². 8.14

(ii) *To find $[\sigma_b]$:* The design bending stress $[\sigma_b]$ is given by

PSG 8.18 $$[\sigma_b] = \frac{1.4 \times K_{bl}}{n \cdot K_\sigma} \times \sigma_{-1}$$ assuming rotation in one direction only.

PSG 8.20 ✓ From Table 5.14, for steel (HB ≤ 350) and N ≥ 10^7, $K_{bl} = 1$. (surface hardness; No of cycles; Life Factor for bending)

8.19 ✓ From Table 5.17, for steel case hardened, factor of safety $n = 2$.

8.19 ✓ From Table 5.15, for steel case hardened, stress concentration factor $K_\sigma = 1.2$.

8.19 ✓ From Table 5.16, for forged steel, $\sigma_{-1} = 0.25\,(\sigma_u + \sigma_y) + 50$. Endurance limit Stress

1.40 But from Table 5.3, for C 15, $\sigma_u = 490$ N/mm² and $\sigma_y = 240$ N/mm². Page 5.11 (Tensile strength; Yield stress)

∴ $$\sigma_{-1} = 0.25\,(490 + 240) + 50 = 232.5 \text{ N/mm}^2$$

Then, $$[\sigma_b] = \frac{1.4 \times 1}{2 \times 1.2} \times 232.5 = 135.625 \text{ N/mm}^2$$

(iii) *To find $[\sigma_c]$:* The design contact stress $[\sigma_c]$ is given by (compressive)

8.16 $$[\sigma_c] = C_R \cdot HRC \cdot K_{cl}$$ Pinion

where
8.16 C_R = 22, for C 15 case hardened steel, from Table 5.18,
8.16 HRC = 55 to 63, for C 15 steel, from Table 5.18, and
8.17 K_{cl} = 0.585, for HB > 350, N ≥ 25 × 10^7, from Table 5.19.

∴ $$[\sigma_c] = 22 \times 63 \times 0.585 = 810.81 \text{ N/mm}^2$$

6. *Calculation of centre distance (a) :* PSG 8.13

Design ① We know that, (Center Distance) 8.13
$$a \geq (i+1)\sqrt[3]{\left(\frac{0.74}{[\sigma_c]}\right)^2 \times \frac{E_{eq}\,[M_t]}{i\,\psi}}$$
(Eq you had 8.14; Design Torque 8.15; Design Contact Stress 8.16; Gear ratio; 8.14)

where PSG 8.14 $\psi = \frac{b}{a} = 0.3$ ... (assumed initially)

∴ $$a \geq (2.5+1)\sqrt[3]{\left(\frac{0.74}{810.81}\right)^2 \times \frac{1.7 \times 10^5 \times 310.35 \times 10^3}{2.5 \times 0.3}}$$

$a \geq 135.94$ mm or $a = 136$ mm.

a = 136

SPUR

Assume 5.61 5.65 Page no of both

ψ psi

8.13

D → 1
C → 2
C → 1

7. To find z_1 and z_2 :

(i) For 20° full depth system, select $z_1 = 18$.

8.22

(ii) $z_2 = i \times z_1 = 2.5 \times 18 = 45 \rightarrow z_2$

PSG 8.22 Find Previous value

8. Calculation of module (m) :

We know that, 8.22 $m = \dfrac{2a}{z_1 + z_2} = \dfrac{2 \times 136}{18 + 45} = 4.32$ mm

8.2

From Table 5.8, the nearest higher standard module, $m = 5$ mm. ← m

9. Revision of centre distance :

PSG 8.22

New centre distance, 8.22 $a = \dfrac{m\,(z_1 + z_2)}{2} = \dfrac{5\,(18 + 45)}{2} = 157.5$ mm

10. Calculation of b, d_1, v and ψ_p :

8.14

- ✓ Face width (b) : $b = \psi \cdot a = 0.3 \times 157.5 = 47.25$ mm. ← b

8.22

- ✓ Pitch diameter of pinion (d_1) : $d_1 = m \cdot z_1 = 5 \times 18 = 90$ mm.

8.15

- ✓ Pitch line velocity (v) : $v = \dfrac{\pi\, d_1\, N_1}{60} = \dfrac{\pi \times 90 \times 10^{-3} \times 900}{60} = 4.24$ m/s. → v

8.15

- ✓ $\psi_p = \dfrac{b}{d_1} = \dfrac{47.25}{90} = 0.525.$

11. Selection of quality of gear :

8.3

From Table 5.22, for $v = 4.24$ m/s, IS quality 8 gears are selected. 5.62

12. Revision of design torque [M_t] :

8.15

- ✓ ***Revise K :*** From Table 5.11, for $\psi_p = 0.525$, $K \approx 1.03$.

8.16

- ✓ ***Revise K_d :*** From Table 5.12, for IS quality 8 and $v = 4.24$ m/s, $K_d = 1.4$.
- ✓ ***Revise [M_t] :*** $[M_t] = M_t \cdot K \cdot K_d$ 8.15

$$= 238.73 \times 1.03 \times 1.4 = 344.24 \text{ N-m}$$

13. Check for bending :

Checking ②

✓ ***Calculation of induced bending stress, σ_b :*** PSG 8.13 A

8.13

$$\sigma_b = \frac{(i+1)}{a \cdot m \cdot b \cdot y}\,[M_t]$$

where 8.18 y = Form factor = 0.377, for $z_1 = 18$, from Table 5.13.

$$\therefore \quad \sigma_b = \frac{(2.5+1) \times 344.24 \times 10^3}{157.5 \times 5 \times 47.25 \times 0.377} = 85.89 \text{ N/mm}^2$$

✓ We find $\sigma_b < [\sigma_B]$. Therefore ***the design is satisfactory.***

SPUR

Contact compressive

PSG 8.13

checking (1)

14. *Check for wear strength :*

✓ ***Calculation of induced contact stress, σ_c :***

8.13
$$\sigma_c = 0.74 \frac{i+1}{a} \sqrt{\frac{i+1}{i\,b} \times E_{eq} [M_t]}$$

$$= 0.74 \left(\frac{2.5+1}{157.5}\right) \sqrt{\left(\frac{2.5+1}{2.5 \times 47.25}\right) \times 1.7 \times 10^5 \times 344.24 \times 10^3}$$

$$= 684.76 \text{ N/mm}^2$$

✓ We find $\sigma_c < [\sigma_c]$. Therefore ***the design is safe and satisfactory.***

15. *Check for wheel :*

(i) ***Calculation of $[\sigma_b]_{wheel}$ and $[\sigma_c]_{wheel}$:***

Wheel material : CI grade 30.

Wheel speed : $N_2 = \dfrac{N_1}{i} = \dfrac{900}{2.5} = 360$ r.p.m.

$\therefore$ Life of wheel = 10000 hrs = $10{,}000 \times 60 \times 360 = 21.6 \times 10^7$ cycles

To find $[\sigma_b]_{wheel}$: The design bending stress for wheel is given by

8.18
$$[\sigma_b]_{\text{wheel}} = \frac{1.4 \times K_{bl}}{n \cdot K_\sigma} \times \sigma_{-1},$$ assuming rotation in one direction only.

8.20 ✓ From Table 5.14, for cast iron wheel, $K_{bl} = \sqrt[9]{\dfrac{10^7}{N}} = \sqrt[9]{\dfrac{10^7}{21.6 \times 10^7}} = 0.918.$

8.19 ✓ From Table 5.17, for cast iron, $n = 2$.

✓ From Table 5.15, for cast iron, $K_\sigma = 1.2$.

✓ From Table 5.16, for cast iron, $\sigma_{-1} = 0.45\,\sigma_u$.

But from Table 5.3, for cast iron, $\sigma_u = 290$ N/mm².

8.19. $CI = \sigma_{-1} = 0.45 \times \sigma_u$

$$\sigma_{-1} = 0.45 \times 290 = 130.5 \text{ N/mm}^2$$

Then, $$[\sigma_b]_{\text{wheel}} = \frac{1.4 \times 0.918}{2 \times 1.2} \times 130.5 = 69.88 \text{ N/mm}^2$$

To find $[\sigma_c]_{wheel}$: The design contact stress for wheel is given by

8.16
$$[\sigma_c]_{\text{wheel}} = C_B \cdot HB \cdot K_{cl}$$

where 8.16 C_B = 2.3, for cast iron grade 30, from Table 5.18,

8.16 HB = 200 to 260, for cast iron, from Table 5.18, and

8.17 $K_{cl} = \sqrt[6]{\dfrac{10^7}{N}} = \sqrt[6]{\dfrac{10^7}{21.6 \times 10^7}} = 0.879$, for cast iron, from Table 5.19.

$\therefore [\sigma_c]_{\text{wheel}} = 2.3 \times 260 \times 0.879 = 525.64$ N/mm²

(ii) *Check for bending :*

✓ Calculation of induced bending stress for wheel σ_{b2} :

$$\sigma_{b1} \times y_1 = \sigma_{b2} \times y_2$$

where σ_{b1} and σ_{b2} = Induced bending stresses in the pinion and wheel respectively, and

y_1 and y_2 = Form factors for pinion and wheel respectively.

From Table 5.13, $y_2 = 0.471$, for $z_2 = 45$.

$\sigma_{b1} = 85.89$ N/mm² and $y_1 = 0.377$ ... (already calculated)

$\therefore \quad 85.89 \times 0.377 = \sigma_{b2} \times 0.471$ $\quad \sigma_{b2} = \frac{\sigma_{b1} y_1}{y_2}$

or $\quad \sigma_{b2} = 68.75$ N/mm²

✓ We find $\sigma_{b2} < [\sigma_b]_{wheel}$. Therefore ***the design is satisfactory.***

(iii) *Check for wear strength :* Since contact area is same, therefore $\sigma_{c\ wheel} = \sigma_{c\ pinion} = 684.76$ N/mm². Here $\sigma_{c\ wheel} > [\sigma_c]_{wheel}$. It means, wheel does not have the required wear resistance. So, in order to decrease the induced contact stress, increase the face width (b) value or in order to increase the design contact stress, increase the surface hardness, say to 340 HB. Increasing the surface hardness will give $[\sigma_c] = 2.3 \times 340 \times 0.879 = 687.34$ N/mm². Now we find $\sigma_c < [\sigma_c]$. So the ***design is safe and satisfactory.***

16. *Calculation of basic dimensions of pinion and wheel :* Refer Table 5.10.

8.22

✓ Module : m = **5 mm**

5.39 Page

✓ Face width : b = **47.25 mm**

✓ Height factor : f_0 = **1** for full depth teeth. 8.22

8.22

✓ Bottom clearance : $c = 0.25\,m = 0.25 \times 5 = \mathbf{1.25\ mm.}$

8.22 PSG

8.22

✓ Tooth depth : $h = 2.25\,m = 2.25 \times 5 = \mathbf{11.25\ mm.}$

✓ Pitch circle diameter : d

$d_1 = m \cdot z_1 = 5 \times 18 = \mathbf{90\ mm}$; and

8.22

$d_2 = m \cdot z_2 = 5 \times 45 = \mathbf{225\ mm.}$

✓ Tip diameter : d_a

$d_{a1} = (z_1 + 2 f_0)\, m = (18 + 2 \times 1)\, 5 = \mathbf{100\ mm}$; and

8.22

$d_{a2} = (z_2 + 2 f_0)\, m = (45 + 2 \times 1)\, 5 = \mathbf{235\ mm}$

✓ Root diameter : d_f

$d_{f1} = (z_1 - 2 f_0)\, m - 2c$ 8.22

8.22

$= (18 - 2 \times 1)\, 5 - 2 \times 1.25 = \mathbf{77.5\ mm}$; and

$d_{f2} = (z_2 - 2 f_0)\, m - 2c$ 8.22

$= (45 - 2 \times 1)\, 5 - 2 \times 1.25 = \mathbf{212.5\ mm}$

SPUR

Example 5.20 *It is desired to determine the proportions of a spur gear drive to transmit 8 kW from a shaft rotating at 1200 r.p.m. to a low speed shaft, with a reduction of 3 : 1. Assume that the teeth are 20° full depth involute, with 24 teeth on the pinion. The pinion is to be of 40 C 8 normalized steel and gear of 30 C 8 normalized steel. Assume that the starting torque is 130% of the rated torque.*

Given Data : $P = 8$ kW; $N_1 = 1200$ r.p.m. ; $i = 3$; $\phi = 20°$; $z_1 = 24$; SPUR

Starting torque = 1.3 × rated torque.

To find : Design a spur gear.

☺ Solution :

1. Gear ratio : $i = 3$ ... (Given)

2. Material selection : Pinion = 40 C 8 normalized steel ; and

Gear = 30 C 8 normalized steel. ... (Given)

3. Gear life : Assume 20,000 hours.

$$\therefore N = 20000 \times 60 \times 1200 = 144 \times 10^7 \text{ cycles}$$

4. Design torque $[M_t]$:

$$[M_t] = M_t \times K \times K_d$$ 8.15

where

$$M_t = \frac{60 \times P}{2\pi N_1} = \frac{60 \times 8 \times 10^3}{2\pi \times 1200} = 63.66 \text{ N-m, and}$$

$$K \cdot K_d = 1.3$$ 8.15 ... (Assume)

$$\therefore \quad [M_t] = 63.66 \times 1.3 = 82.76 \text{ N-m}$$

5. Calculation of E_{eq}, $[\sigma_b]$ and $[\sigma_c]$:

(i) To find E_{eq} : From Table 5.20, $E_{eq} = 2.15 \times 10^5$ N/mm² for steel. 8.14

(ii) To find $[\sigma_b]$: Design bending stress, $[M_t] = \dfrac{1.4 \times K_{bl}}{n \cdot K_\sigma} \times \sigma_{-1}$ 8.18

8.20 ✓ $K_{bl} = 1$, for steel HB ≤ 350 and $N \geq 10^7$, from Table 5.14,

8.19 ✓ $n = 2$, for steel normalized, from Table 5.17.

✓ $K_\sigma = 1.5$, for steel normalized, from Table 5.15,

8.19 ✓ $\sigma_{-1} = 0.35\,\sigma_u + 120$, for alloy steel, from Table 5.16

$= 0.35 \times 720 + 120 = 372$ N/mm². 1.40 ... [$\because \sigma_u = 720$ N/mm²]

$\sigma_b = \dfrac{1.4 \times K_{bl}}{n\, K_\sigma}$ 8.18 $\quad \therefore [\sigma_b] = \dfrac{1.4 \times 1}{2 \times 1.5} \times 372 = 173.6$ N/mm²

(iii) To find $[\sigma_c]$: Design contact stress, $[\sigma_c] = C_B \times HB \times K_{cl}$

where $C_B = 2.5$, for alloy steel normalized, from Table 5.18,

$HB \leq 350$, from Table 5.18, and

$K_{cl} = 1$, for steel, HB ≤ 350 and $N \geq 10^7$, from Table 5.19.

$\therefore \quad [\sigma_c] = 2.5 \times 300 \times 1 = 750$ N/mm²

6. *Centre distance (a) :* Assume $\psi = 0.3$.

Design ① 8.13

$$a \geq (i+1)\sqrt[3]{\left(\frac{0.74}{[\sigma_c]}\right)^2 \times \frac{E_{eq}\,[M_t]}{i\,\psi}}$$

$$\geq (3+1)\sqrt[3]{\left(\frac{0.74}{750}\right)^2 \times \frac{2.15 \times 10^5 \times 82.76 \times 10^3}{3 \times 0.3}}$$

≥ 107.2 or $a = 110$ mm

7. Given that $z_1 = 24$. $\therefore z_2 = i\, z_1 = 3 \times 24 = 72$.

8. *Module (m) :* 8.83 $m = \dfrac{2a}{z_1 + z_2} = \dfrac{2 \times 110}{24 + 72} = 2.29$ mm

From Table 5.8, the nearest higher standard module, $m = 2.5$ mm.

9. *Revised centre distance :* 8.82 $a = \dfrac{m\,(z_1 + z_2)}{2} = \dfrac{2.5\,(24 + 72)}{2} = 120$ mm.

10. *Calculation of b, d, v and ψ_p*

✓ Face width (b) : 8.14 $b = \psi \times a = 0.3 \times 120 = 36$ mm

✓ Pitch diameter of pinion (d_1) : $d_1 = m \cdot z_1$ 8.22 $= 2.5 \times 24 = 60$ mm.

✓ Pitch line velocity (v) : $v = \dfrac{\pi\, d_1\, N_1}{60}$ 8.11 $= \dfrac{\pi \times 60 \times 10^{-3} \times 1200}{60} = 3.77$ m/s

✓ 8.15 $\psi_p = \dfrac{b}{d_1} = \dfrac{36}{60} = 0.6$

11. *Quality of gear :* From Table 5.22, for $v = 3.77$ m/s, IS quality 8 gears are selected

8.13

D → 1
C → 2
C → 1

12. *Revised design torque* [M_t] :

From Table 5.11, for $\psi = 0.6$, $K = 1.03$.

From Table 5.12, for IS quality 8, HB ≤ 350 and $v = 3.77$ m/s, $K_d = 1.55$.

$$\therefore \quad [M_t] = M_t \cdot K \cdot K_d \quad \text{8.15}$$

$$= 63.66 \times 1.03 \times 1.55 = 101.63 \text{ N-m}$$

13. *Check for bending* :

8.13 A

check (2)

Induced bending stress, $\sigma_b = \dfrac{i+1}{a \cdot m \cdot b \cdot y} [M_t]$

where $y = 0.414$, for $z_1 = 24$, from Table 5.13.

$$\therefore \quad \sigma_b = \frac{(3+1)}{120 \times 2.5 \times 36 \times 0.414} \times 101.63 \times 10^3 = 90.9 \text{ N/mm}^2$$

We find $\sigma_b < [\sigma_b]$. Thus ***the design is satisfactory.***

14. *Check for wear strength* : Induced contact stress is given by

check (1)

$$\sigma_c = 0.74 \frac{i+1}{a} \sqrt{\frac{i+1}{i\,b} \times E_{eq} [M_t]} \quad \text{8.13}$$

$$= 0.74 \left(\frac{3+1}{120}\right) \sqrt{\left(\frac{3+1}{3 \times 36}\right) \times 2.15 \times 10^5 \times 101.63 \times 10^3}$$

$$= 701.71 \text{ N/mm}^2$$

We find $\sigma_c < [\sigma_c]$, thus ***the design is safe and satisfactory.***

15. *Check for plastic deformation* :

M_t = Rated torque = 63.66 N-m ... (already calculated)

Given that starting torque is 130% of rated torque.

$$\therefore \quad [M_t]_{max} = \text{Maximum instantaneous torque} = 1.3 \times M_t$$

$$= 1.3 \times 63.66 = 82.758 \text{ N-m}$$

*(i) **Check for bending :*** Induced bending stress due to maximum instantaneous torque is given by

8.21

$$\sigma_{b\,max} = \sigma_b \frac{[M_t]_{max}}{M_t} = 90.9 \times \frac{82.758}{63.66} = 118.17 \text{ N/mm}^2 \qquad [\because \sigma_b = 90.9 \text{ N/mm}^2]$$

From Table 5.23, for steel HB ≤ 350, permissible bending stress is given by

$$[\sigma_b]_{max} = 0.8\,\sigma_y = 0.8 \times 540 = 432 \text{ N/mm}^2$$

Since $\sigma_{b\,max} < [\sigma_b]_{max}$, ***the design is satisfactory.*** $[\because \sigma_y = 540 \text{ N/mm}^2]$

(ii) Check for wear strength : Induced contact stress due to maximum instantaneous torque is given by

$$\sigma_{c\,max} = \sigma_c \times \frac{[M_t]_{max}}{M_t}$$ 8.21

$$= 701.71 \times \frac{82.758}{63.66} = 912.22 \text{ N/mm}^2 \qquad [\because \sigma_c = 701.71 \text{ N/mm}^2]$$

From Table 5.24, for steel HB ≤ 350, permissible contact stress is given by

$$[\sigma_c]_{max} = 3.1\,\sigma_y = 3.1 \times 540 = 1674 \text{ N/mm}^2$$

Since $\sigma_{c\,max} < [\sigma_c]_{max}$, ***the design is safe and satisfactory against plastic deformation also.***

8.22

8.22

16. ***Basic dimensions of pinion and gear :*** Refer Table 5.10.

- ✓ Module : m = **2.5 mm**
- ✓ Face width : b = **36 mm**
- ✓ Height factor : f_0 = **1**
- ✓ Bottom clearance : $c = 0.25\,m = 0.25 \times 2.5 =$ **0.625 mm**
- ✓ Tooth depth : $h = 2.25\text{ m} = 2.25 \times 2.5 =$ **5.625 mm**
- ✓ Pitch circle diameter : $d_1 = m \cdot z_1 = 2.5 \times 24 =$ **60 mm** ; and

  $d_2 = m \cdot z_2 = 2.5 \times 72 =$ **180 mm.**

- ✓ Tip diameter : $d_{a1} = (z_1 + 2\,f_0)\,m = (24 + 2 \times 1)\,2.5 =$ **65 mm** ; and

  8.22

  $d_{a2} = (z_2 + 2\,f_0)\,m = (72 + 2 \times 1)\,2.5 =$ **185 mm.**

22

PSG

- ✓ Root diameter : $d_{f1} = (z_1 - 2\,f_0)\,m - 2\,c$

  $= (24 - 2 \times 1)\,2.5 - 2 \times 0.625 =$ **53.75 mm** ; and

  $d_{f2} = (z_2 - 2\,f_0)\,m - 2\,c = (72 - 2 \times 1)\,2.5 - 2 \times 0.625$

  $=$ **173.75 mm.**

SPUR