

Example 5.20 It is desired to determine the proportions of a spur gear drive to transmit 8 kW from a shaft rotating at 1200 r.p.m. to a low speed shaft, with a reduction of 3:1. Assume that the teeth are 20° full depth involute, with 24 teeth on the pinion. The pinion is to be of 40 C 8 normalized steel and gear of 30 C 8 normalized steel. Assume that the starting torque is 130% of the rated torque.

Given Data : $P = 8 \text{ kW}$; $N_1 = 1200 \text{ r.p.m.}$; $i = 3$; $\phi = 20^\circ$; $z_1 = 24$;

Starting torque = $1.3 \times$ rated torque.

To find : Design a spur gear.

Solution :

1. Gear ratio : $i = 3$... (Given)

2. Material selection : Pinion = 40 C 8 normalized steel ; and
Gear = 30 C 8 normalized steel. ... (Given)

3. Gear life : Assume 20,000 hours.

$\therefore N = 20000 \times 60 \times 1200 = 144 \times 10^7$ cycles

4. Design torque $[M_t]$: $[M_t] = M_t \times K \times K_d$ ^{8.15}

where $M_t = \frac{60 \times P}{2\pi N_1} = \frac{60 \times 8 \times 10^3}{2\pi \times 1200} = 63.66 \text{ N-m, and}$

$K \cdot K_d = 1.3$ ^{8.15} ... (Assume)

$\therefore [M_t] = 63.66 \times 1.3 = 82.76 \text{ N-m}$

5. Calculation of E_{eq} , $[\sigma_b]$ and $[\sigma_c]$:

(i) To find E_{eq} : From Table 5.20, $E_{eq} = 2.15 \times 10^5 \text{ N/mm}^2$ for steel.

(ii) To find $[\sigma_b]$: Design bending stress, $[M_t] = \frac{1.4 \times K_{bl}}{n \cdot K_\sigma} \times \sigma_{-1}$ ^{8.18}

- ✓ $K_{bl} = 1$, for steel HB ≤ 350 and $N \geq 10^7$, from Table 5.14,
- ✓ $n = 2$, for steel normalized, from Table 5.17,
- ✓ $K_{\sigma} = 1.5$, for steel normalized, from Table 5.15,
- ✓ $\sigma_{-1} = 0.35 \sigma_u + 120$, for alloy steel, from Table 5.16
 $= 0.35 \times 720 + 120 = 372 \text{ N/mm}^2$.

... [$\because \sigma_u = 720 \text{ N/mm}^2$]

$$\therefore [\sigma_b] = \frac{1.4 \times 1}{2 \times 1.5} \times 372 = 173.6 \text{ N/mm}^2$$

(iii) To find $[\sigma_c]$: Design contact stress, $[\sigma_c] = C_B \times \text{HB} \times K_{cl}$

where $C_B = 2.5$, for alloy steel normalized, from Table 5.18,

HB ≤ 350 , from Table 5.18, and

$K_{cl} = 1$, for steel, HB ≤ 350 and $N \geq 10^7$, from Table 5.19.

$$\therefore [\sigma_c] = 2.5 \times 300 \times 1 = 750 \text{ N/mm}^2$$

6. Centre distance (a): Assume $\psi = 0.3$.

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$$a \geq (i+1) \sqrt[3]{\left(\frac{0.74}{[\sigma_c]}\right)^2 \times \frac{E_{eq} [M_t]}{i \psi}}$$

$$\geq (3+1) \sqrt[3]{\left(\frac{0.74}{750}\right)^2 \times \frac{2.15 \times 10^5 \times 82.76 \times 10^3}{3 \times 0.3}}$$

$$\geq 107.2 \text{ or } a = 110 \text{ mm}$$

7. Given that $z_1 = 24$. $\therefore z_2 = i z_1 = 3 \times 24 = 72$.

8. Module (m): $m = \frac{2a}{z_1 + z_2} = \frac{2 \times 110}{24 + 72} = 2.29 \text{ mm}$

From Table 5.8, the nearest higher standard module, $m = 2.5 \text{ mm}$.

9. Revised centre distance: $a = \frac{m(z_1 + z_2)}{2} = \frac{2.5(24 + 72)}{2} = 120 \text{ mm}$.

10. Calculation of b , d , v and ψ_p :

✓ Face width (b): $b = \psi \times a = 0.3 \times 120 = 36 \text{ mm}$

✓ Pitch diameter of pinion (d_1): $d_1 = m \cdot z_1 = 2.5 \times 24 = 60 \text{ mm}$.

✓ Pitch line velocity (v): $v = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 60 \times 10^{-3} \times 1200}{60} = 3.77 \text{ m/s}$

✓ $\psi_p = \frac{b}{d_1} = \frac{36}{60} = 0.6$

11. Quality of gear: From Table 5.22, for $v = 3.77 \text{ m/s}$, IS quality 8 gears are selected.

12. Revised design torque $[M_t]$:

From Table 5.11, for $\psi = 0.6$, $K = 1.03$.

From Table 5.12, for IS quality 8, HB ≤ 350 and $v = 3.77$ m/s, $K_d = 1.55$.

D \rightarrow 1

C \rightarrow 2

C \rightarrow 1

$$\begin{aligned} \therefore [M_t] &= M_t \cdot K \cdot K_d \quad \text{B.15} \\ &= 63.66 \times 1.03 \times 1.55 = 101.63 \text{ N-m} \end{aligned}$$

13. Check for bending:

Induced bending stress, $\sigma_b = \frac{i+1}{a \cdot m \cdot b \cdot y} [M_t]$ B.13 A check (2)

where $y = 0.414$, for $z_1 = 24$, from Table 5.13.

$$\therefore \sigma_b = \frac{(3+1)}{120 \times 2.5 \times 36 \times 0.414} \times 101.63 \times 10^3 = 90.9 \text{ N/mm}^2$$

We find $\sigma_b < [\sigma_b]$. Thus *the design is satisfactory*.

14. Check for wear strength: Induced contact stress is given by

$$\begin{aligned} \sigma_c &= 0.74 \frac{i+1}{a} \sqrt{\frac{i+1}{i b} \times E_{eq} [M_t]} \quad \text{B.13} \quad \text{check (1)} \\ &= 0.74 \left(\frac{3+1}{120} \right) \sqrt{\left(\frac{3+1}{3 \times 36} \right) \times 2.15 \times 10^5 \times 101.63 \times 10^3} \\ &= 701.71 \text{ N/mm}^2 \end{aligned}$$

We find $\sigma_c < [\sigma_c]$, thus *the design is safe and satisfactory*.

15. Check for plastic deformation:

$$M_t = \text{Rated torque} = 63.66 \text{ N-m} \quad \dots \text{ (already calculated)}$$

Given that starting torque is 130% of rated torque.

$$\begin{aligned} \therefore [M_t]_{\max} &= \text{Maximum instantaneous torque} = 1.3 \times M_t \\ &= 1.3 \times 63.66 = 82.758 \text{ N-m} \end{aligned}$$

(i) Check for bending: Induced bending stress due to maximum instantaneous torque is given by

$$\sigma_{b \max} = \sigma_b \frac{[M_t]_{\max}}{M_t} \quad \text{B.21} = 90.9 \times \frac{82.758}{63.66} = 118.17 \text{ N/mm}^2 \quad [\because \sigma_b = 90.9 \text{ N/mm}^2]$$

From Table 5.23, for steel HB ≤ 350 , permissible bending stress is given by

$$[\sigma_b]_{\max} = 0.8 \sigma_y = 0.8 \times 540 = 432 \text{ N/mm}^2$$

Since $\sigma_{b \max} < [\sigma_b]_{\max}$, *the design is satisfactory*. $[\because \sigma_y = 540 \text{ N/mm}^2]$

(ii) **Check for wear strength** : Induced contact stress due to maximum instantaneous torque is given by

$$\sigma_{c \max} = \sigma_c \times \frac{[M_t]_{\max}}{M_t} \quad \text{B.21}$$

$$= 701.71 \times \frac{82.758}{63.66} = 912.22 \text{ N/mm}^2 \quad [\because \sigma_c = 701.71 \text{ N/mm}^2]$$

From Table 5.24, for steel HB ≤ 350 , permissible contact stress is given by

$$[\sigma_c]_{\max} = 3.1 \sigma_y = 3.1 \times 540 = 1674 \text{ N/mm}^2$$

Since $\sigma_{c \max} < [\sigma_c]_{\max}$, *the design is safe and satisfactory against plastic deformation also.*

16. Basic dimensions of pinion and gear : Refer Table 5.10. B.22

- ✓ Module : $m = 2.5 \text{ mm}$
- ✓ Face width : $b = 36 \text{ mm}$
- ✓ Height factor : $f_0 = 1$
- ✓ Bottom clearance : $c = 0.25 m = 0.25 \times 2.5 = 0.625 \text{ mm}$
- ✓ Tooth depth : $h = 2.25 m = 2.25 \times 2.5 = 5.625 \text{ mm}$
- ✓ Pitch circle diameter : $d_1 = m \cdot z_1 = 2.5 \times 24 = 60 \text{ mm}$; and $d_2 = m \cdot z_2 = 2.5 \times 72 = 180 \text{ mm}$.
- ✓ Tip diameter : $d_{a1} = (z_1 + 2 f_0) m = (24 + 2 \times 1) 2.5 = 65 \text{ mm}$; and $d_{a2} = (z_2 + 2 f_0) m = (72 + 2 \times 1) 2.5 = 185 \text{ mm}$.
- ✓ Root diameter : $d_{f1} = (z_1 - 2 f_0) m - 2 c = (24 - 2 \times 1) 2.5 - 2 \times 0.625 = 53.75 \text{ mm}$; and $d_{f2} = (z_2 - 2 f_0) m - 2 c = (72 - 2 \times 1) 2.5 - 2 \times 0.625 = 173.75 \text{ mm}$.