Example 5.19 ***Design a spur gear drive to transmit 22.5 kW at 900 r.p.m. Speed reduction is 2.5. Materials for pinion and wheel are C 15 steel and cast iron grade 30 respectively. Take pressure angle of 20° and working life of the gears as 10000 hrs.***

Sem 16 Mark

Given Data : $P = 22.5$ kW ; $N_1 = 900$ r.p.m. ; $i = 2.5$; $\phi = 20°$; $N = 10000$ hrs.

* Pinion → C 15 steel
* Wheel → Cast iron grade 30

To find : Design a spur gear.

Solution : Since the *materials for pinion and wheel are different, therefore we have to design the pinion first and check both pinion and wheel.*

1. ***Gear ratio :*** $i = 2.5$ ... (Given)

$$i = \frac{N_1}{N_2} = \frac{Z_2}{Z_1}$$

2. ***Material selection :***

Pinion : C15 steel, case hardened to 55 RC and core hardness < 350, and

Wheel : C.I. grade 30. ... (Given)

3. ***Gear life :*** $N = 10000$ hrs ... (Given)

Gear life in terms of number of cycles is given by

$$N = 10000 \times 60 \times 900 = 54 \times 10^7 \text{ cycles}$$

4. ***Design torque $[M_t]$:*** PSG 8.15

$$[M_t] = M_t \cdot K \cdot K_d$$

where $$M_t = \frac{60 \times P}{2\pi N_1} = \frac{60 \times 22.5 \times 10^3}{2\pi \times 900} = 238.73 \text{ N-m, and}$$

$$K \cdot K_d = 1.3 \quad \text{... (assume)}$$

$\therefore$ Design torque, $[M_t] = 238.73 \times 1.3 = 310.35$ N-m

5. Calculation of E_{eq}, $[\sigma_b]$ and $[\sigma_c]$:

(i) To find E_{eq}: From Table 5.20, for pinion steel and cast iron (> 280 N/mm²), equivalent Young's modulus, $E_{eq} = 1.7 \times 10^5$ N/mm².

(ii) To find $[\sigma_b]$: The design bending stress $[\sigma_b]$ is given by

$$[\sigma_b] = \frac{1.4 \times K_{bl}}{n \cdot K_\sigma} \times \sigma_{-1}$$ assuming rotation in one direction only.

From Table 5.14, for steel (HB ≤ 350) and N ≥ 10^7, $K_{bl} = 1$.

From Table 5.17, for steel case hardened, factor of safety $n = 2$.

From Table 5.15, for steel case hardened, stress concentration factor $K_\sigma = 1.2$.

From Table 5.16, for forged steel, $\sigma_{-1} = 0.25\,(\sigma_u + \sigma_y) + 50$.

But from Table 5.3, for C 15, $\sigma_u = 490$ N/mm² and $\sigma_y = 240$ N/mm².

$$\therefore \quad \sigma_{-1} = 0.25\,(490 + 240) + 50 = 232.5 \text{ N/mm}^2$$

Then, $$[\sigma_b] = \frac{1.4 \times 1}{2 \times 1.2} \times 232.5 = 135.625 \text{ N/mm}^2$$

(iii) To find $[\sigma_c]$: The design contact stress $[\sigma_c]$ is given by

$$[\sigma_c] = C_R \cdot HRC \cdot K_{cl}$$

where $C_R = 22$, for C 15 case hardened steel, from Table 5.18,

$HRC = 55$ to 63, for C 15 steel, from Table 5.18, and

$K_{cl} = 0.585$, for HB > 350, N ≥ 25×10^7, from Table 5.19.

$$\therefore \quad [\sigma_c] = 22 \times 63 \times 0.585 = 810.81 \text{ N/mm}^2$$

6. Calculation of centre distance (a) :

We know that, $$a \geq (i+1)\sqrt[3]{\left(\frac{0.74}{[\sigma_c]}\right)^2 \times \frac{E_{eq}\,[M_t]}{i\,\psi}}$$

where $$\psi = \frac{b}{a} = 0.3 \quad \text{... (assumed initially)}$$

$$\therefore \quad a \geq (2.5+1)\sqrt[3]{\left(\frac{0.74}{810.81}\right)^2 \times \frac{1.7 \times 10^5 \times 310.35 \times 10^3}{2.5 \times 0.3}}$$

$$\geq 135.94 \text{ mm or } a = 136 \text{ mm.}$$

7. To find z_1 and z_2 :

(i) For 20° full depth system, select $z_1 = 18$.

(ii) $z_2 = i \times z_1 = 2.5 \times 18 = 45$

8. Calculation of module (m) :

We know that, $m = \dfrac{2a}{z_1 + z_2} = \dfrac{2 \times 136}{18 + 45} = 4.32$ mm

From Table 5.8, the nearest higher standard module, $m = 5$ mm.

9. Revision of centre distance :

New centre distance, $a = \dfrac{m(z_1 + z_2)}{2} = \dfrac{5(18 + 45)}{2} = 157.5$ mm

10. Calculation of b, d_1, v and ψ_p :

- ✓ Face width (b): $b = \psi \cdot a = 0.3 \times 157.5 = 47.25$ mm.
- ✓ Pitch diameter of pinion (d_1): $d_1 = m \cdot z_1 = 5 \times 18 = 90$ mm.
- ✓ Pitch line velocity (v): $v = \dfrac{\pi d_1 N_1}{60} = \dfrac{\pi \times 90 \times 10^{-3} \times 900}{60} = 4.24$ m/s.
- ✓ $\psi_p = \dfrac{b}{d_1} = \dfrac{47.25}{90} = 0.525$.

11. Selection of quality of gear :

From Table 5.22, for $v = 4.24$ m/s, IS quality 8 gears are selected.

12. Revision of design torque [M_t] :

- ✓ ***Revise K :*** From Table 5.11, for $\psi_p = 0.525$, $K \approx 1.03$.
- ✓ ***Revise K_d :*** From Table 5.12, for IS quality 8 and $v = 4.24$ m/s, $K_d = 1.4$.
- ✓ ***Revise [M_t] :*** $[M_t] = M_t \cdot K \cdot K_d$

$$= 238.73 \times 1.03 \times 1.4 = 344.24 \text{ N-m}$$

13. Check for bending :

✓ ***Calculation of induced bending stress, σ_b :***

$$\sigma_b = \frac{(i+1)}{a \cdot m \cdot b \cdot y} [M_t]$$

where y = Form factor = 0.377, for $z_1 = 18$, from Table 5.13.

$$\therefore \quad \sigma_b = \frac{(2.5 + 1) \times 344.24 \times 10^3}{157.5 \times 5 \times 47.25 \times 0.377} = 85.89 \text{ N/mm}^2$$

✓ We find $\sigma_b < [\sigma_B]$. Therefore *the design is satisfactory.*

14. Check for wear strength :

✓ ***Calculation of induced contact stress, σ_c :***

$$\sigma_c = 0.74 \frac{i+1}{a} \sqrt{\frac{i+1}{i\,b} \times E_{eq}\,[M_t]}$$

$$= 0.74 \left(\frac{2.5+1}{157.5}\right) \sqrt{\left(\frac{2.5+1}{2.5 \times 47.25}\right) \times 1.7 \times 10^5 \times 344.24 \times 10^3}$$

$$= 684.76 \text{ N/mm}^2$$

✓ We find $\sigma_c < [\sigma_c]$. Therefore ***the design is safe and satisfactory.***

15. Check for wheel :

(i) Calculation of $[\sigma_b]_{wheel}$ and $[\sigma_c]_{wheel}$:

Wheel material : CI grade 30.

Wheel speed : $N_2 = \frac{N_1}{i} = \frac{900}{2.5} = 360$ r.p.m.

Life of wheel = 10000 hrs = $10{,}000 \times 60 \times 360 = 21.6 \times 10^7$ cycles

To find $[\sigma_b]_{wheel}$: The design bending stress for wheel is given by

$$[\sigma_b]_{wheel} = \frac{1.4 \times K_{bl}}{n \cdot K_\sigma} \times \sigma_{-1},$$ assuming rotation in one direction only.

✓ From Table 5.14, for cast iron wheel, $K_{bl} = \sqrt[9]{\frac{10^7}{N}} = \sqrt[9]{\frac{10^7}{21.6 \times 10^7}} = 0.918.$

✓ From Table 5.17, for cast iron, $n = 2$.

✓ From Table 5.15, for cast iron, $K_\sigma = 1.2$.

✓ From Table 5.16, for cast iron, $\sigma_{-1} = 0.45\,\sigma_u$.

But from Table 5.3, for cast iron, $\sigma_u = 290$ N/mm².

$$\sigma_{-1} = 0.45 \times 290 = 130.5 \text{ N/mm}^2$$

Then, $$[\sigma_b]_{wheel} = \frac{1.4 \times 0.918}{2 \times 1.2} \times 130.5 = 69.88 \text{ N/mm}^2$$

To find $[\sigma_c]_{wheel}$: The design contact stress for wheel is given by

$$[\sigma_c]_{wheel} = C_B \cdot HB \cdot K_{cl}$$

where C_B = 2.3, for cast iron grade 30, from Table 5.18,

HB = 200 to 260, for cast iron, from Table 5.18, and

$K_{cl} = \sqrt[6]{\frac{10^7}{N}} = \sqrt[6]{\frac{10^7}{21.6 \times 10^7}} = 0.879$, for cast iron, from Table 5.19.

$$\therefore [\sigma_c]_{wheel} = 2.3 \times 260 \times 0.879 = 525.64 \text{ N/mm}^2$$

*(ii) **Check for bending :***

✓ Calculation of induced bending stress for wheel σ_{b2} :

$$\sigma_{b1} \times y_1 = \sigma_{b2} \times y_2$$

where σ_{b1} and σ_{b2} = Induced bending stresses in the pinion and wheel respectively, and

y_1 and y_2 = Form factors for pinion and wheel respectively.

From Table 5.13, $y_2 = 0.471$, for $z_2 = 45$.

$\sigma_{b1} = 85.89$ N/mm² and $y_1 = 0.377$ ... (already calculated)

$\therefore$ $85.89 \times 0.377 = \sigma_{b2} \times 0.471$ $\qquad \sigma_{b2} = \frac{\sigma_{b1} y_1}{y_2}$

or $\sigma_{b2} = 68.75$ N/mm²

✓ We find $\sigma_{b2} < [\sigma_b]_{wheel}$. Therefore ***the design is satisfactory.***

*(iii) **Check for wear strength :*** Since contact area is same, therefore $\sigma_{c\ wheel} = \sigma_{c\ pinion} = 684.76$ N/mm². Here $\sigma_{c\ wheel} > [\sigma_c]_{wheel}$. It means, wheel does not have the required wear resistance. So, in order to decrease the induced contact stress, increase the face width (b) value or in order to increase the design contact stress, increase the surface hardness, say to 340 HB. Increasing the surface hardness will give $[\sigma_c] = 2.3 \times 340 \times 0.879 = 687.34$ N/mm². Now we find $\sigma_c < [\sigma_c]$. So the ***design is safe and satisfactory.***

16. *Calculation of basic dimensions of pinion and wheel :* Refer Table 5.10.

✓ Module : m = **5 mm**

✓ Face width : b = **47.25 mm**

✓ Height factor : f_0 = **1** for full depth teeth.

✓ Bottom clearance : $c = 0.25\,m = 0.25 \times 5 =$ **1.25 mm.**

✓ Tooth depth : $h = 2.25\,m = 2.25 \times 5 =$ **11.25 mm.**

✓ Pitch circle diameter : $d_1 = m \cdot z_1 = 5 \times 18 =$ **90 mm** ; and

$d_2 = m \cdot z_2 = 5 \times 45 =$ **225 mm.**

✓ Tip diameter : $d_{a1} = (z_1 + 2 f_0)\, m = (18 + 2 \times 1)\, 5 =$ **100 mm ;** and

$d_{a2} = (z_2 + 2 f_0)\, m = (45 + 2 \times 1)\, 5 =$ **235 mm**

✓ Root diameter : $d_{f1} = (z_1 - 2 f_0)\, m - 2c$

$= (18 - 2 \times 1)\, 5 - 2 \times 1.25 =$ **77.5 mm;** and

$d_{f2} = (z_2 - 2 f_0)\, m - 2c$

$= (45 - 2 \times 1)\, 5 - 2 \times 1.25 =$ **212.5 mm**