Example 4.1 *A truck equipped with a 9.5 kW engine uses a roller chain as the final drive to the rear axle. The driving sprocket runs at 900 r.p.m. and the driven sprocket at 400 r.p.m. with a centre distance of approximately 600 mm. Select the roller chain.*

Given Data : N = 9.5 kW ; N_1 = 900 r.p.m. ; N_2 = 400 r.p.m. ; a_0 = 600 mm.

To find : Select (*i.e.*, design) the roller chain.

Chain drive

☺ ***Solution :***

1. Determination of the transmission ratio (i) :

$$\text{Transmission ratio, } i = \frac{N_1}{N_2} = \frac{900}{400} = 2.25$$

(Since the transmission ratio can be calculated from the given data, therefore we need not to consult Table 4.2)

2. Selection of number of teeth on the driver sprocket (z_1) :

From Table 4.3, $z_1 = 27$ (for $i = 2$ to 3) is selected.

3. Determination of number of teeth on the driven sprocket (z_2) :

$$z_2 = i \times z_1 = 2.25 \times 27 = 60.75 \approx 61$$

Recommended value, $z_{2max} = 100$ to 120

$\therefore$ $z_2 =$ **61** is satisfactory.

4. Selection of standard pitch (p) :

We know that Centre distance, $a = (30 - 50)\,p$

$$\therefore \quad \text{Maximum pitch, } p_{max} = \frac{a}{30} = \frac{600}{30} = 20 \text{ mm}$$

$$\text{and} \quad \text{Minimum pitch, } p_{min} = \frac{a}{50} = \frac{600}{50} = 12 \text{ mm}$$

Any standard pitch between 12 mm and 20 mm can be chosen. But to get a quicker solution, it is always preferred to take the standard pitch closer to p_{max}. Refer Table 4.4.

$\therefore$ Standard pitch, $p =$ **15.875 mm** is chosen.

5. Selection of the chain :

Assume the chain to be duplex. Consulting Table 4.5, the selected ***chain number*** is ***10A-2 / DR50.***

6. Calculation of total load on the driving side of the chain (P_T) :

(i) Tangential force (P_t) :

$$P_t = \frac{1020\,N}{v}$$

where N = Transmitted power in kW = 9.5 kW

v = Chain velocity in m/s

$$= \frac{z_1 \times p \times N_1}{60 \times 1000} = \frac{27 \times 15.875 \times 900}{60 \times 1000} = 6.43 \text{ m/s}$$

$$\therefore \quad P_t = \frac{1020\,N}{v} = \frac{1020 \times 9.5}{6.43} = 1507 \text{ N}$$

(ii) Centrifugal tension (P_c) :

$$P_c = mv^2$$

From Table 4.5, $m = 1.78$ kg/m

$$\therefore \quad P_c = 1.78\,(6.43)^2 = 73.59 \text{ N}$$

Chain drive

(iii) Tension due to sagging (P_s) :

$$P_s = k \cdot w \cdot a$$

From Table 4.6, $k = 6$ (for horizontal)

$w = mg = 1.78 \times 9.81 = 17.46$ N

a = Initial centre distance = 0.6 m

$\therefore \quad P_s = 6 \times 17.46 \times 0.6 = 62.82$ N

(iv) Total load (P_T) : $P_T = P_t + P_c + P_s$

$= 1507 + 73.59 + 62.82 = 1643.4$ N

7. Calculation of service factor (k_s) :

We know that the service factor,

$$k_s = k_1 \cdot k_2 \cdot k_3 \cdot k_4 \cdot k_5 \cdot k_6$$

From Table 4.7, $k_1 = 1.25$ (for load with mild shocks)

From Table 4.8, $k_2 = 1$ (for adjustable supports)

From Table 4.9, $k_3 = 1$ ($\because$ we have used $a_p = (30 \text{ to } 50)\, p$)

From Table 4.10, $k_4 = 1$ (for horizontal drive)

From Table 4.11, $k_5 = 1$ (for drop lubrication)

From Table 4.12, $k_6 = 1.25$ (for 16 hours / day running)

$\therefore \quad k_s = 1.25 \times 1 \times 1 \times 1 \times 1 \times 1.25 = 1.5625$

8. Calculation of design load :

Design load $= P_T \times k_s = 1643.4 \times 1.5625 = 2567.8$ N

9. Calculation of working factor of safety (FS_w) :

$$FS_w = \frac{\text{Breaking load Q from Table 4.5}}{\text{Design load}} = \frac{44400}{2567.8} = 17.29$$

10. Check for factor of safety :

Consulting Table 4.13, for smaller sprocket speed of 900 r.p.m. and pitch 15.875 mm, the required minimum factor of safety is 11. Therefore the working factor of safety is greater than the recommended minimum factor of safety. Thus ***the design is safe and satisfactory.***

11. Check for the bearing stress in the roller :

We know that $\sigma_{roller} = \dfrac{P_t \times k_s}{A}$; where $A = 140\ mm^2$ from Table 4.5.

$$= \frac{1507 \times 1.5625}{140} = 16.8\ N/mm^2$$

Consulting Table 4.14, for smaller sprocket speed of 900 r.p.m. and pitch 15.875 mm, the allowable bearing stress is 22.4 N/mm². Therefore the induced stress is less than the allowable bearing stress. Thus ***the design is safe and satisfactory.***

12. Calculation of length of chain (L) :

Number of links, $$l_p = 2\,a_p + \left(\frac{z_1 + z_2}{2}\right) + \frac{[\,(z_2 - z_1)/2\pi\,]^2}{a_p}$$

where $$a_p = \frac{a_0}{p} = \frac{\text{Centre distance}}{\text{pitch}} = \frac{600}{15.875} = 37.795$$

$$\therefore \quad l_p = 2\,(37.795) + \left(\frac{27+61}{2}\right) + \frac{[\,(61-27)/2\pi\,]^2}{37.795} = 120.36$$

$$\approx 122 \text{ links (rounded off to an even number)}$$

$\therefore$ Actual length of chain, $L = l_p \times p = 122 \times 15.875 =$ **1936.75 mm**

13. Calculation of exact centre distance (a) :

We know that $$a = \frac{e + \sqrt{e^2 - 8\,M}}{4} \times p$$

where $$e = l_p - \left(\frac{z_1 + z_2}{2}\right) = 122 - \left(\frac{27+61}{2}\right) = 78$$

and $$M = \left(\frac{z_2 - z_1}{2\pi}\right)^2 = \left(\frac{61-27}{2\pi}\right)^2 = 29.28$$

$$\therefore \quad a = \frac{78 + \sqrt{78^2 - 8 \times 29.28}}{4} \times 15.875 = 613.11 \text{ mm}$$

Decrement in centre distance for an initial sag $= 0.01\,a = 0.01\,(613.11) = 6.1311$ mm

$\therefore$ Exact centre distance $= 613.11 - 6.1311 =$ **606.978 mm**

14. Calculation of sprocket diameters :

Smaller sprocket :

Pcd of smaller sprocket, $$d_1 = \frac{p}{\sin\,(180/z_1)}$$

$$= \frac{15.875}{\sin\,(180/27)} = \mathbf{136.74 \text{ mm}}$$

and Sprocket outside diameter, $d_{01} = d_1 + 0.8\,d_r$

where d_r = Diameter of roller, from Table 4.5 = 10.16 mm

$$\therefore \quad d_{01} = 136.74 + 0.8 \times 10.16 = \mathbf{144.868 \text{ mm}}$$

Chain

Larger sprocket :

$$\text{Pcd of larger sprocket, } d_2 = \frac{p}{\sin(180/z_2)} = \frac{15.875}{\sin(180/61)}$$

$$= \mathbf{308.38\ mm}$$

and Sprocket outside diameter, $d_{02} = d_2 + 0.8\, d_r$

$$= 308.38 + 0.8 \times 10.16 = \mathbf{316.51\ mm}$$

Example 4.2 ***The transporter of a heat treatment furnace is driven by a 4.5 kW, 1440 r.p.m. induction motor through a chain drive with a speed reduction ratio of 2.4. The transmission is horizontal with bath type of lubrication. Rating is continuous with 3 shifts per day. Design the complete chain drive.***

Chain drive

Given Data : N = 4.5 kW ; N_1 = 1440 r.p.m. ; i = 2.4.

To find : Design the chain drive.

☺ ***Solution :***

1. ***Transmission ratio,*** i = 2.4 (Given) $\therefore\ N_2 = \frac{N_1}{i} = \frac{1440}{2.4} = 600$ r.p.m.

2. ***To find*** z_1 **:** From Table 4.3, $z_1 = \mathbf{27}$ (for $i = 2$ to 3) is chosen.

3. ***To find*** z_2 **:** $z_2 = i \times z_1 = 2.4 \times 27 = 64.8 \approx 65$

Recommended $z_{2\,max}$ = 100 to 120. $\therefore\ z_2 = \mathbf{65}$ is satisfactory.

4. ***Standard pitch (p) :*** Since the centre distance is not given, we have to assume the initial centre distance, say $a = 500$ mm.

We know that $a = (30 - 50)\,p$

$$\therefore \quad p_{max} = \frac{a}{30} = \frac{500}{30} = 16.6 \text{ mm}$$

$$\text{and} \quad p_{min} = \frac{a}{50} = \frac{500}{50} = 10 \text{ mm}$$

From Table 4.4, in between 10 and 16.6 mm, a standard pitch, $p = \mathbf{15.875}$ mm is chosen.

5. ***Selection of chain :*** Assume the chain to be ***simplex.***

From Table 4.5, the ***10A-1 / R50 chain number*** is chosen.

6. ***Calculation of total load on the driving side*** (P_T) **:**

$$P_T = P_t + P_c + P_s$$

(i) $$P_t = \frac{1020\ N}{v}$$

where N = Transmitted power in kW = 4.5 kW (Given)

$$v = \text{Velocity of chain in m/s}$$

$$= \frac{z_1 \times p \times N_1}{60 \times 1000} = \frac{27 \times 15.875 \times 1440}{60 \times 1000} = 10.287 \text{ m/s}$$

$$\therefore \quad P_t = \frac{1020 \times 4.5}{10.287} = 446.19 \text{ N}$$

(ii) $P_c = mv^2$

From Table 4.5, $m = 1.01$ kg/m

$$\therefore \quad P_c = 1.01\,(10.287)^2 = 106.88 \text{ N}$$

(iii) $P_s = k \cdot w \cdot a$

From Table 4.6, $k = 6$ (for horizontal)

$w = mg = 1.01 \times 9.81 = 9.908$ N/m and $a = 0.5$ m

$$\therefore \quad P_s = 6 \times 9.908 \times 0.5 = 29.72 \text{ N}$$

(iv) $\therefore$ Total load, $P_T = 466.19 + 106.88 + 29.72 = 582.79$ N

7. Service factor : $k_s = k_1 \cdot k_2 \cdot k_3 \cdot k_4 \cdot k_5 \cdot k_6$

From Table 4.7, $k_1 = 1.25$ (for load with mild shocks)

From Table 4.8, $k_2 = 1$ (for adjustable supports)

From Table 4.9, $k_3 = 1$ (since we have used $a = (30 \text{ to } 50)\,p$)

From Table 4.10, $k_4 = 1$ (for horizontal drive)

From Table 4.11, $k_5 = 0.8$ (for bath type lubrication)

From Table 4.12, $k_6 = 1.5$ (for continuous running *i.e.*, 3 shifts / day)

$$\therefore \quad k_s = 1.25 \times 1 \times 1 \times 1 \times 0.8 \times 1.5 = 1.5$$

8. Design load $= P_T \times k_s = 582.79 \times 1.5 = 874.19$ N

9. Working factor of safety $= \dfrac{\text{Breaking load Q from Table 4.5}}{\text{Design load}}$

$$FS_w = \frac{22200}{874.19} = 25.39$$

From Table 4.13, for smaller sprocket speed 1440 r.p.m. and pitch 15.875 mm, the recommended minimum value of factor of safety (n') is 13.2. Since the working factor of safety is greater than the recommended minimum value of factor of safety, therefore ***the design is safe and satisfactory.***

10. Bearing stress in the roller : From Table 4.5, $A = 70$ mm²

$$\sigma = \frac{P_t \times k_s}{A} = \frac{446.19 \times 1.5}{70} = 9.56 \text{ N/mm}^2$$

chain

From Table 4.14, for smaller sprocket speed 1440 r.p.m. and pitch 15.875 mm, the allowable bearing stress is 18.5 N/mm². Since the induced stress is less than the allowable bearing stress, ***the design is safe and satisfactory.***

11. Actual length of chain (L) :

$$\text{Number of links, } l_p = 2\,a_p + \left(\frac{z_1 + z_2}{2}\right) + \frac{[\,(z_2 - z_1)/2\pi\,]^2}{a_p}$$

where $$a_p = \frac{a_0}{p} = \frac{500}{15.875} = 31.496$$

$$\therefore \quad l_p = 2\,(31.496) + \left(\frac{27+65}{2}\right) + \frac{[\,(65-27)/2\pi\,]^2}{31.496}$$

$$= 110.153 \approx 112 \quad \text{(rounded off to an even number)}$$

$$\therefore \quad \text{Actual length of chain, L} = l_p \times p = 112 \times 15.875 = \mathbf{1778\ mm}$$

12. Exact centre distance :

$$a = \frac{e + \sqrt{e^2 - 8\,\mathrm{M}}}{4} \times p$$

where $$e = l_p - \left(\frac{z_1 + z_2}{2}\right) = 112 - \left(\frac{27+65}{2}\right) = 66$$

and $$\mathrm{M} = \left[\frac{(z_2 - z_1)}{2\pi}\right]^2 = \left(\frac{65-27}{2\pi}\right)^2 = 36.57$$

$$\therefore \quad a = \frac{66 + \sqrt{66^2 - 8 \times 36.57}}{4} \times 15.875 = 514.92\text{ mm}$$

Decrement in centre distance for an initial sag, $\Delta a = 0.01\,a = 5.149$ mm

$$\therefore \quad \text{Exact centre distance} = 514.92 - 5.149 = \mathbf{509.77\ mm}$$

13. Sprocket diameters :

For smaller sprocket :

$$\text{Pcd} = \frac{p}{\sin(180/z_1)} = \frac{15.875}{\sin(180/27)} = \mathbf{136.74\ mm}$$

and Sprocket outside diameter, $d_{01} = d_1 + 0.8\,d_r$

From Table 4.5, d_r = Diameter of roller = 10.16 mm

$$\therefore \quad d_{01} = 136.74 + 0.8 \times 10.16 = \mathbf{144.87\ mm}$$

For larger sprocket :

$$\text{Pcd} = \frac{p}{\sin(180/z_2)} = \frac{15.875}{\sin(180/65)} = \mathbf{328.58\ mm}$$

and Sprocket outside diameter, $d_{02} = d_2 + 0.8\,d_r = 328.58 + 0.8 \times 10.16$

$$= \mathbf{336.71\ mm}$$

—x—x—x—

Chain drive

Sem
16 Mark

Example 3.1 ***Design a wire rope for an elevator in a building 60 metres high and for a total load of 20 kN. The speed of the elevator is 4 m/sec and the full speed is reached in 10 seconds.***

Wire rope

Given Data : Height = 60 m ; $W = 20$ kN $= 20 \times 10^3$ N ; $v = 4$ m/sec = 240 m/min; $t = 10$ sec.

To find : Design a wire rope.

☺ ***Solution :***

1. Selection of suitable wire rope : Given that the wire rope is used for an elevator, *i.e.*, for hoisting purpose. So lets use **6 × 19 rope** (refer Table 3.1).

Wire rope.

2. *Calculation of design load :* Assuming a larger factor of safety of 15, the design load is calculated.

$$\text{Design load} = \text{Load to be lifted} \times \text{Assumed factor of safety}$$

$$= 20 \times 15 = \mathbf{300\ kN}$$

3. *Selection of wire rope diameter (d) :* From Table 3.4, taking the design load as the breaking strength, the wire rope diameter is selected as 25 mm.

$\therefore$ d = **25 mm** for σ_u = 1600 to 1750 N/mm² and breaking strength = 340 kN.

4. *Calculation of sheave diameter (D) :* From Table 3.5, for 6 × 19 rope and class 4,

$$\frac{D_{min}}{d} = 27 \quad \text{(for velocity upto 50 m/min)}$$

Since the given lifting speed is 240 m/min (= 4 m/s), therefore D_{min}/d ratio should be modified. Thus for every additional speed of 50 m/min, D_{min}/d ratio has to be increased by 8%.

$$\therefore \quad \text{Modified } \frac{D_{min}}{d} = 27 \times (1.08)^{5-1} = 36.73 \text{ say } 40. \qquad \left[\because \frac{240}{50} \approx 5\right]$$

$$\text{The sheave diameter,} \quad D = 40 \times d = 40 \times 25 = \mathbf{1000\ mm}$$

5. *Selection of the area of useful cross-section of the rope (A) :* From Table 3.6, for 6 × 19 rope,

$$A = 0.4\, d^2 = 0.4\,(25)^2 = \mathbf{250\ mm^2}$$

6. *Calculation of wire diameter (d_w) :*

$$\text{Wire diameter, } d_w = \frac{d}{1.5\sqrt{i}}$$

where i = Number of strands × Number of wires in each strand

$= 6 \times 19 = 114$

$$\therefore \quad d_w = \frac{25}{1.5\sqrt{114}} = 1.56 \text{ mm}$$

7. *Selection of weight of rope (W_r) :*

From Table 3.4,

$$\text{Approximate mass} = 2.41 \text{ kg / m}$$

$$\therefore \quad \text{Weight of rope / m} = 2.41 \times 9.81 = 23.6 \text{ N/m}$$

$$\text{and} \quad \text{Weight of rope, } W_r = 23.6 \times 60 = \mathbf{1416\ N}$$

8. *Calculation of various loads :*

(i) Direct load, $W_d = W + W_r = 20000 + 1416 = 21416$ N

(ii) Bending load, $W_b = \sigma_b \times A = \dfrac{E_r \cdot d_w}{D} \times A$

$$= \frac{0.84 \times 10^5 \times 1.56}{1000} \times 250 = 32760 \text{ N}$$

[Take $E_r = 0.84 \times 10^5$ N / mm²]

(iii) Acceleration load, $W_a = \left(\dfrac{W + W_r}{g}\right) a$

where a = Acceleration of the load $= \dfrac{v_2 - v_1}{t_1} = \dfrac{4-0}{10}$

$= 0.4$ m/s²

$\therefore$ $W_a = \left(\dfrac{20000 + 1416}{9.81}\right) 0.4 = 873.23$ N

(iv) Starting load (W_{st}) :

When there is no slack in the rope, starting load is given by

$$W_{st} = 2 \cdot W_d = 2\,(W + W_r) = 2\,(20000 + 1416) = 42832 \text{ N}$$

9. *Calculation of effective loads on the rope :*

(i) Effective load during normal working, $W_{en} = W_d + W_b$

$= 21416 + 32760 = 54176$ N

(ii) Effective load during acceleration of the load, $W_{ea} = W_d + W_b + W_a$

$= 21416 + 32760 + 873.23$

$= 55049.23$ N

(iii) Effective load during starting, $W_{est} = W_b + W_{st}$

$= 32760 + 42832 = 75592$ N

10. *Calculation of working factor of safety (FS_w) :*

Working factor of safety $= \dfrac{\text{Breaking load from Table 3.4 for the selected rope}}{\text{Effective load during acceleration } (W_{ea})}$

$$= \frac{340000}{55049.23} = 6.176$$

11. *Check for safe design :* From Table 3.2, for hoists and class 4, the recommended factor of safety = 6.

Since the working factor of safety is greater than the recommended factor of safety therefore the ***design is safe.***

wire rope