TOPIC: 2.2 –SERIES: TYPES AND CONVERGENCE

Series

Infinite Series

If $a_1, a_2, \ldots a_n, \ldots$ be an infinite sequence of real numbers, then

$a_1 + a_2 + a_3 + \cdots + a_n + \cdots \infty$ is called an infinite series.

An infinite series is denoted by $\sum a_n$ and the sum of its first 'n' terms is denoted by S_n.

Convergent, Divergent & Oscillatory of a series

Consider the infinite series.

$$\sum a_n = a_1 + a_2 + \cdots + a_n + \cdots \infty.$$

Let the sum of first 'n' terms be

$$S_n = a_1 + a_2 + \cdots + a_n.$$

The convergence or divergence of the series $\sum a_n$ is defined in terms of the convergence or divergence of the sequence $\{S_n\}$.

(i) $\sum a_n$ is said to converge if the sequence $\{S_n\}$ converges.

② $\sum a_n$ is said to be diverge if the sequence $\{S_n\}$ diverges.

③. $\sum a_n$ is said to oscillatory if the sequence $\{S_n\}$ does not tend to a unique limit as $n \to \infty$.

①. Examine the convergence of the series.

$1+2+3+\cdots+n+\cdots\infty$.

Soln. Given. $\sum a_n = 1+2+3+\cdots+n+\cdots\infty$.

Let $S_n = 1+2+3+\cdots+n$.

$$S_n = \frac{n(n+1)}{2}$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{n(n+1)}{2}$$

$$= \frac{1}{2} \lim_{n\to\infty} n(n+1)$$

$$\lim_{n\to\infty} S_n = \infty.$$

$\therefore$ The S_n divergent

$\therefore$ $\sum a_n$ is also divergent.

(2). $1+3+5+7+\cdots\infty$.

Soln:

Gn. $1+3+5+7+\cdots\infty$ are in A.P

$$t_n = a+(n-1)d.$$

$a=1, d=2,$

$$t_n = 1+(n-1)2$$
$$= 2n-1.$$

$$S_n = 1+3+5+\cdots+(2n-1)$$
$$= \frac{n}{2}\left[2a+(n-1)d\right]$$
$$= \frac{n}{2}\left[2+(n-1)2\right]$$
$$= n^2.$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} n^2 = \infty.$$

$\therefore$ S_n is divergent. So $\sum a_n$ is diverge

(3). $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots$

Soln:

The given series $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{n(n+1)}$

$$t_n = \frac{1}{n(n+1)}$$

$$S_n = \sum \frac{1}{n(n+1)}$$

$$= \sum \frac{1}{n} - \frac{1}{n+1}$$

Here $u_1 = 1 - \frac{1}{2}$

$$u_2 = \frac{1}{2} - \frac{1}{3}$$

$$u_3 = \frac{1}{3} - \frac{1}{4}$$

$$\cdots\cdots\cdots$$

$$u_n = \frac{1}{n} - \frac{1}{n+1}$$

$$\cdots\cdots\cdots$$

$$S_n = u_1 + u_2 + u_3 + \cdots + u_n$$

$$= 1 - \frac{1}{n+1}$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(1 - \frac{1}{n+1}\right)$$

$$= 1 - 0$$

$$= 1 \text{ finite}$$

$\therefore$ The series is convergent.