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Multiplication of large integers and strassen’s matrix

Multiplication of large integers

- Over 100 Decimal digits long required manipulation of Integers
- Such Integers are too long to fit in single word of modern computers, they required special treatment
- So, we are using classic method Pen and Pencil algorithm for multiplying to n-digit integers
- n-digit -1st number * n-digit-2nd number = n^2 digit multiplication

Formula

- Pair of 2 digit integers

  a = a1 a0

  b = b1 b0

  Their product is c.

  $c = a * b = C2\ 10^{2} + c1\ 10^{1} + c0$, where

  c2 = a1 * b1 -> Product of 1st digit

  c0 = a0 * b0 -> Product of 2nd digit

  c1 = (a1 + a0) * (b1 + b0) – (c2 + c0) -> product of sum of a's digit and sum of b's digit minus sum of c2 and c0

Formula

- Apply Divide and Conquer technique
- First half of a's digit is a1 and second half by a0. Same as this for b, b1 and b0
- Using $c = a * b = C2\ 10^{2} + c1\ 10^{1} + c0$ this formula,

$$c = a*b = (a1\ 10^{n/2}+a0) * (b1\ 10^{n/2}+b0)$$

$$\Rightarrow (a1*b1)10^{n} + (a1*b0 + a1*b1)\ 10^{n/2} + (a0*b0)$$

$$\Rightarrow C2\ 10^{2}+c1\ 10^{1} +c0$$

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Analysis of Multiplication of large Integers

- T(n) = 3 T(n/2)

Therefore, time complexity for all the cases,

$3\log_2 n$

trassen's algorithm for two 2x2 matrices :

$$\begin{matrix} c_{00} & c_{01} \\ c_{10} & c_{11} \end{matrix} = \begin{matrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{matrix} * \begin{matrix} b_{00} & b_{01} \\ b_{10} & b_{11} \end{matrix}$$

$$= \begin{matrix} m_1 + m_4 - m_5 + m_7 & m_3 + m_5 \\ m_2 + m_4 & m_1 + m_3 - m_2 + m_6 \end{matrix}$$

$m_1 = (a_{00} + a_{11}) * (b_{00} + b_{11})$
$m_2 = (a_{10} + a_{11}) * b_{00}$
$m_3 = a_{00} * (b_{01} - b_{11})$
$m_4 = a_{11} * (b_{10} - b_{00})$
$m_5 = (a_{00} + a_{01}) * b_{11}$
$m_6 = (a_{10} - a_{00}) * (b_{00} + b_{01})$
$m_7 = (a_{01} - a_{11}) * (b_{10} + b_{11})$