TOPIC: 2.2 –SERIES: TYPES AND CONVERGENCE

Series.

Infinite Series

If $a_1, a_2, \dots a_n, \dots$ be an infinite sequence of real numbers, then

$a_1 + a_2 + a_3 + \dots + a_n + \dots \infty$ is called an infinite series.

An infinite series is denoted by $\sum a_n$ and the sum of its first 'n' terms is denoted by S_n.

Convergent, Divergent & Oscillatory of a series

Consider the infinite series.

$$\sum a_n = a_1 + a_2 + \dots + a_n + \dots \infty.$$

Let the sum of first 'n' terms be

$$S_n = a_1 + a_2 + \dots + a_n.$$

The convergence or divergence of the series $\sum a_n$ is defined in terms of the convergence or divergence of the sequence $\{S_n\}$.

i.e., (1) $\sum a_n$ is said to converge if the sequence $\{S_n\}$ converges.

② Σa_n is said to be diverge if the sequence $\{S_n\}$ diverges.

③. Σa_n is said to oscillatory if the sequence $\{S_n\}$ does not tend to a unique limit as $n \to \infty$.

①. Examine the convergence of the series.
$1+2+3+\cdots+n+\cdots\infty$.

Soln. Given. $\Sigma a_n = 1+2+3+\cdots+n+\cdots\infty$.

Let $S_n = 1+2+3+\cdots+n$.

$$S_n = \frac{n(n+1)}{2}.$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{n(n+1)}{2}$$

$$= \frac{1}{2} \lim_{n\to\infty} n(n+1)$$

$$\lim_{n\to\infty} S_n = \infty.$$

$\therefore$ The S_n divergent

$\therefore$ Σa_n is also divergent.

(2). $1+3+5+7+\cdots\infty$.

Soln

Gn. $1+3+5+7+\cdots\infty$ are in A.P

$$t_n = a+(n-1)d.$$

$$a=1,\ d=2,$$

$$t_n = 1+(n-1)2$$

$$= 2n-1.$$

$$S_n = 1+3+5+\cdots+(2n-1)$$

$$= \frac{n}{2}\left[2a+(n-1)d\right]$$

$$= \frac{n}{2}\left[2+(n-1)2\right]$$

$$= n^2.$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} n^2 = \infty.$$

$\therefore$ S_n is divergent. So $\sum a_n$ is diverge

(3). $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots$

Soln:

The given series $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{n(n+1)}$

$$t_n = \frac{1}{n(n+1)}$$

$$S_n = \sum \frac{1}{n(n+1)}$$

$$= \sum \frac{1}{n} - \frac{1}{n+1}$$

Here $u_1 = 1 - \frac{1}{2}$

$$u_2 = \frac{1}{2} - \frac{1}{3}$$

$$u_3 = \frac{1}{3} - \frac{1}{4}$$

$$\cdots\cdots\cdots$$

$$u_n = \frac{1}{n} - \frac{1}{n+1}$$

$$\cdots\cdots\cdots$$

$$S_n = u_1 + u_2 + u_3 + \cdots + u_n$$

$$= 1 - \frac{1}{n+1}$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(1 - \frac{1}{n+1}\right)$$

$$= 1 - 0$$

$$= 1 \text{ finite}$$

$\therefore$ The series is convergent.

Series of positive terms:

General properties:

1. Convergence of a series remains unchanged by the replacement, inclusion (or) omission of a finite number of terms.

2. A series remains convergent, divergent (or) oscillatory when each term of it is multiplied by a fixed number other than zero.

3. A series of positive terms either converges (or) diverges to $+\infty$., omitting the negative terms, the sum of first n terms tends to either a finite limit (or) $+\infty$.

4. Every finite series is a convergent series.

Problem:

Series of positive terms.

1. If all the terms after few negative terms in an infinite series are positive, such a series is a positive term series.

Problem:

Series of positive terms.

1. If all the terms after few negative terms in an infinite series are positive, such a series is a positive term series.

Eg: $-10-6-1+5+12+20+ \cdots$

2. A series of positive terms either converges (or) diverges to ∞, for the sum of first n terms, omitting the negative terms, tends to either a finite limit (or) ∞.

3. Necessary condition for convergence. If a +ve term series Σu_n is convergent, then $\lim_{n\to\infty} u_n = 0$. but the converse not true.

4. Test for divergence.

If $\lim_{n\to\infty} u_n \neq 0$, the series Σu_n must be divergent.