Topic: 3.3 – CENTRE OF CURVATURE

(3)

Centre and Radius of curvature.

Circle of curvature:

The curvature at any point 'p' of a curve is equal to the curvature of the circle which passes through p and two close points on the curve on either side of p such a circle exists for each point of the curve. It is called the circle of curvature of the curve at the point.

radius of curvature:

The radius of this circle is called the radius of curvature of the curve at that point.

centre of the circle:

The centre of the circle is called the centre of curvature of the curve at that point.

2) write the formula for centre of curvature.

Soln: If $(\bar{x}, \bar{y})$ is the centre of curvature at a point (x, y) on a curve

$$\bar{x} = x - \frac{y_1}{y_2}(1+y_1^2) \; ; \; \bar{y} = y + \frac{(1+y_1^2)}{y_2}.$$

1. Find the centre of curvature at the point $(am^2, 2am)$ on the parabola $y^2 = 4ax$.

Soln: Given $x = am^2$, $y = 2am$.

$$\frac{dx}{dm} = 2am \;;\; \frac{dy}{dm} = 2a$$

$$y_1 = \frac{dy}{dm} \cdot \frac{dm}{dx} = \frac{2a}{2am} = \frac{1}{m}.$$

$$y_2 = \frac{d}{dm}\left(\frac{dy}{dx}\right)\frac{dm}{dx} = \frac{d}{dm}\left(\frac{1}{m}\right)\frac{dm}{dx}$$

$$= -\frac{1}{m^2} \cdot \frac{1}{2am} = -\frac{1}{2am^3}.$$

$$\bar{x} = x - \frac{y_1}{y_2}(1 + y_1^2).$$

$$= am^2 - \frac{1}{m}\left(\frac{-2am^3}{1}\right)^2\left(1 + \frac{1}{m^2}\right)$$

$$= am^2 + \frac{2am^2(m^2+1)}{m^2} = am^2 + am^2 + 2a$$

$$= 3am^2 + 2a$$

$$\bar{Y} = y + \frac{(1+y_1^2)}{y_2} = 2am + \frac{\left(1 + \frac{1}{m^2}\right)}{\frac{1}{2am^3}}.$$

$$= 2am + \frac{(m^2+1)}{m^2} \cdot \frac{2am^3}{(-1)} = 2am - 2am^3 - 2am$$

$$\bar{Y} = -2am^3$$

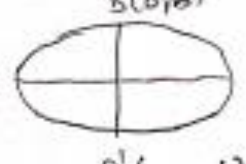
$\therefore$ The centre of curvature is $(3am^2 + 2a, -2am^3)$.

2) prove that if the centre of the curvature of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at one end of the minor axis lies at the other end, then the eccentricity of the ellipse is $\frac{1}{\sqrt{2}}$.

soln:

The ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \rightarrow (1)$

B(0,b)



B'(0,−b)

BB' is the minor axis. B is (0,b)

B' is (0,−b)

Diff. (1) w.r.t x, we get

$$\frac{2x}{a^2}+\frac{2y}{b^2}\frac{dy}{dx}=0.$$

$$y_1=\frac{dy}{dx}=-\frac{2x}{a^2}\cdot\frac{b^2}{2y}=-\frac{b^2x}{a^2y}.$$

$$y_2=\frac{d^2y}{dx^2}=-\frac{b^2}{a^2}\left[\frac{y(1)-x\cdot\frac{dy}{dx}}{y^2}\right]$$

$$y_1(0,b)=0$$

$$y_2(0,b)=-\frac{b^2}{a^2}\left[\frac{b}{b^2}\right]=-\frac{b}{a^2}.$$

Let $(\bar{x},\bar{y})$ be the centre of curvature at (0,b)

$$\bar{x}=x-\frac{y_1}{y_2}(1+y_1^2) \quad \& \quad \bar{y}=y+\frac{(1+y_1^2)}{y_2}$$

$$\bar{x}(0,b)=0-0=0$$

$$\bar{y}_{(0,b)} = b + \frac{a^2}{-b^2}(1+0) = b - \frac{a^2}{b}.$$

The centre of curvature is $\left(0, b - \frac{a^2}{b}\right)$ this is given to be the point $(0, -b)$ the other end B' of the minor axis.

$$b - \frac{a^2}{b} = -b \Rightarrow b^2 - a^2 = -b^2$$

$$2b^2 = a^2 \rightarrow (2)$$

But $b^2 = a^2(1-e^2)$ where e is being eccentricity using in (2)

$$a^2 = 2a^2(1-e^2)$$

$$1 - e^2 = \frac{1}{2} \Rightarrow e^2 = \frac{1}{2}$$

$$e = \frac{1}{\sqrt{2}}$$

centre of curvature:

The centre of curvature $(\bar{x}, \bar{y})$ at any point $P(\bar{x}, \bar{y})$ on the curve $y = f(x)$ are

$$\bar{x} = x - \frac{y_1}{y_2}(1 + y_1^2)$$

$$\bar{y} = y + \frac{1}{y_2}(1 + y_1^2).$$