



Topic: 1.5 – PROPERTIES OF EIGEN VALUES & EIGEN VECTORS

Properties of Eigen values and Eigen vectors

- * The sum of the Eigen values of a matrix is the sum of the elements of the principal (or) diagonal.
- * The product of the Eigen values of a matrix is the determinant value of the given matrix say $|A|$.
- * A square matrix A and its transpose A^T have the same Eigen values.
- * The characteristic roots of a triangular matrix are just the diagonal elements of the matrix.
- * If λ is an Eigen value of a matrix A , then $1/\lambda$ is the Eigen value of the matrix A^{-1} .
- * If λ is an Eigen value of an orthogonal matrix, then $1/\lambda$ is also one of its Eigen values.
- * The Eigen values of a real symmetric matrix are real numbers.
- * The similar matrices have the same Eigen values.
- * Two Eigen vectors X_1 and X_2 are called orthogonal if $X_1^T X_2 = 0$
- * If A and B are two matrices and B is non-singular, then A and $B^{-1}AB$ have the same Eigen values.

Problems:

1. Find the sum and product of Eigen values of $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & 2 & 0 \end{bmatrix}$
- $\text{Givn } A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & 2 & 0 \end{bmatrix}$ \Rightarrow Sum of Eigen values = Sum of diagonal elements
- $$\lambda_1 + \lambda_2 + \lambda_3 = -2 + 1 + 0$$
- $$\lambda_1 + \lambda_2 + \lambda_3 = -1$$



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$$\begin{aligned} \text{ii) Product of Eigen values} &= |A| \\ \lambda_1 \lambda_2 \lambda_3 &= -2(12) - 2(-6) - 3(5) \\ \lambda_1 \lambda_2 \lambda_3 &= -27 \end{aligned}$$

2. The product of two Eigen values of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16. Find the third Eigen value.

Gn. the product of two Eigen values is 16
To find the third Eigen value λ_3 by using property

$$\begin{aligned} \text{Product of Eigen values} &= |A| \\ \lambda_1 \lambda_2 \lambda_3 &= 6(9-1) + 2(-6+2) + 2(2-6) \\ 16 \cdot \lambda_3 &= 6(8) + 2(-4) + 2(-4) \\ 16 \cdot \lambda_3 &= 48 - 8 - 8 \\ \lambda_3 &= 32/16 \\ \lambda_3 &= 2 \end{aligned}$$

3. If 3 and 15 are two Eigen values of $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. Find $|A|$ without expanding the determinant.

Gn. $\lambda_1 = 3$; $\lambda_2 = 15$
To find $|A|$ without expanding the determinant by using property

$$\begin{aligned} \lambda_1 + \lambda_2 + \lambda_3 &= d_1 + d_2 + d_3 \\ 3 + 15 + \lambda_3 &= 8 + 7 + 3 \\ \lambda_3 &= 0 \end{aligned}$$

W.k.t. Product of Eigen values = $|A|$

$$\begin{aligned} \lambda_1 \lambda_2 \lambda_3 &= |A| \\ 3 \cdot 15 \cdot 0 &= |A| \\ |A| &= 0 \end{aligned}$$



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4. If 2, 2, 3 are the Eigen values of $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$, find the Eigen values of A^T .

By property, A square matrix A and its transpose A^T have the same Eigen values.

∴ Eigen values of $A = 2, 2, 3$
Eigen values of $A^T = 2, 2, 3$.

3. Find the Eigen values of adjoint of A if $A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 4 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$$A^{-1} = \frac{\text{adj.}A}{|A|}$$

$$A^{-1}|A| = \text{adj.}A$$

Eigen values of $\text{adj.}A = |A|$. Eigen values of A^{-1} ——— ①

$$|A| = 3(4-0) - 2(0-0) + 1(0)$$

$$|A| = 12$$

Eigen values of A is 1, 3, 4 [because Eigen values of triangular matrix is the coefficients on the diagonals]

Eigen values of A^{-1} is $1, \frac{1}{3}, \frac{1}{4}$.

$$\text{Sub all in ① we get, Eigen values of adj.}A \left. \begin{array}{l} \\ \\ \end{array} \right\} = 12 \left(1, \frac{1}{3}, \frac{1}{4} \right) \\ = 12, 4, 3$$

Orthogonal Matrices :

A square matrix A is said to be orthogonal if

$$AA^T = A^T A = I \quad (\because AA^{-1} = A^{-1}A = I)$$

A matrix A is orthogonal if $A^T = A^{-1}$



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1. Check whether the matrix B is orthogonal $B = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Given $B = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$B^T = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$BB^T = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} \cos^2 \theta + \sin^2 \theta + 0 & -\sin \theta \cos \theta + \cos \theta \sin \theta + 0 & 0 + 0 + 0 \\ -\sin \theta \cos \theta + \sin \theta \cos \theta + 0 & \sin^2 \theta + \cos^2 \theta + 0 & 0 + 0 + 0 \\ 0 + 0 + 0 & 0 + 0 + 0 & 0 + 0 + 0 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

Similarly $B^T B = I$

Hence the given matrix is orthogonal.