



Topic: 1.1 – INTRODUCTION TO MATRICES

Matrix: UNIT - I

A system of 'mn' numbers (elements) arranged in a rectangular arrangement along 'm' rows and 'n' columns bounded by the brackets [] (or) () is called an m by n matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1j} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2j} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{i1} & a_{i2} & \dots & a_{ij} & \dots & a_{in} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mj} & \dots & a_{mn} \end{bmatrix}$$

In short $A = [a_{ij}]$, $i = 1, 2, \dots, m$
 $j = 1, 2, \dots, n$

Here each a_{ij} is called an element of the matrix in i^{th} row and j^{th} column.

Order of a matrix:

The order of a matrix is denoted by the number of its rows and columns.

Row matrix:

A matrix having a single row is called a row matrix. Eg. $[1, -1, 3, 5]_{1 \times 4}$



Column matrix:

A matrix having a single column is called a column matrix. Eg: $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}_{3 \times 1}$.

Note: Row and column matrices are sometimes called row vectors and column vectors.

Square matrix:

A matrix having n rows and n columns is called a square matrix of order n .

Eg: $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}_{2 \times 2}$.

Note: In the square matrix $A = (a_{ij})$ the elements $a_{11}, a_{22}, a_{33}, \dots, a_{nn}$ are called the diagonal elements of A . The sum of the diagonal elements of a square matrix A is called the trace of A .

Null (or) zero matrix:

In a matrix if all the elements are zero, then the matrix is called a null (or) zero matrix is denoted by O .

Eg: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2 \times 2}$.



Diagonal matrix:

In a square matrix all the elements except in the main diagonal are zeros, then the matrix is called a diagonal matrix.

$$\text{Eq: } A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}_{3 \times 3}$$

Scalar matrix:

A square matrix in which all the elements of its leading diagonal are equal and the other elements are zeros is called a scalar matrix.

$$\text{Eq: } A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}_{3 \times 3}$$

Symmetric matrix:

A square matrix $A = [a_{ij}]$ is said to be symmetric when $a_{ij} = a_{ji}$ for all i, j
(ie) $(i, j)^{\text{th}}$ element = $(j, i)^{\text{th}}$ element.

[condition: A matrix A is symmetric if $A = A^T$]

$$\text{Eq: } A = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} ; A^T = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$$

Here $A = A^T$ Hence A is a symmetric matrix]



Skew Symmetric matrix:

A square matrix $A = [a_{ij}]$ is said to be Skew Symmetric when $a_{ij} = -a_{ji} \forall i, j$

[condition: A matrix A is said to be Skew Symmetric if $A = -A^T$]

Eg: $A = \begin{bmatrix} 0 & 3 & 2 \\ -3 & 0 & 5 \\ -2 & -5 & 0 \end{bmatrix}$ $-A^T = \begin{bmatrix} 0 & 3 & 2 \\ -3 & 0 & 5 \\ -2 & -5 & 0 \end{bmatrix}$

Here $A = -A^T$. Hence A is a Skew Symmetric matrix

Inverse of a matrix (or) Reciprocal matrix:

If A is non-singular matrix $\frac{1}{|A|} \text{adj} A$ is defined to be the reciprocal of the matrix A (or) the inverse of the matrix A. It is denoted by A^{-1} .

$$A^{-1} = \frac{1}{|A|} \text{adj} A$$

It can be shown that $AA^{-1} = A^{-1}A = I$.