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Topic: 3.8 – ENVELOPES

Erwelope ; A wince which touches each member of a family of worke is called the envelope of that family of curves, The envelope of a family of aerves is the locues of the ultimate points of intersection of the consecutive members of the family. Method 1; for finding envelope: 1) If the family of workes is expressed as a quadratic equation of the parameter, say, AX2+13A+c=0 where A, B, c are functions of x and y and I is the parameter then the envelope of this family is given by B-4AC=0. Analytic method to find the Envelope of the family of across. 1. Differentiate f(x,y,c)=0 partially w.r. t the Parameters c. 2. Eliminate 'c' from f(x, y, c) = 0 th of (x, y, c)=0 " We get the envelope of the family.



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problems: 1. Find the envelope of the family of straight lines Y=mx+am2; m being the parameter, Rota: Given Y=mx+am2, am+mx-y=0, This is quadratic in m, so the envelope is B-HAC=0, here A=a x2-4a(-y)=0 C=-y > x+ + + ay = 0 2. Find the envelope of the family of y=mx+a' where 'a' is a constan Univer y=mx+a y = m x +a my= my+a. m'n - my +a =0. this is a quadratic in 'm'. so the envelope is B2- HAC = 0 (-y2)-4xa=0 (i.e) y= Hax.





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3. Find the envelope of the family of straight Lines. X coso + ysino = a, where O being parameter Adni Given X coso + y sin 0 = a -> 1) () different = -x sine + ycose = 0 -> @ squaring and adding () +1 (2) the envelope $\chi^{2}(\cos^{2}\theta + \sin^{2}\theta) + \chi^{2}(\sin^{2}\theta + \cos^{2}\theta) = a^{2}$ (ie) x2+y2=a2, which is a circle. 4. Find the envelope of the family of straight Lines y=mx+ Vaint+be where 'm' is the parameter. xoln: y-mx = Va2m2+12 (y-mx) = a2m2+b2=) y2-2mxy+mx=am+ $m^{2}(x^{2}-a^{2})-2mxy+y^{2}-b^{2}=0$ Which is quadratic is 'm', here $A = \chi^2 - a^2$; $B = -2\chi y$; $C = y^2 - b^2$ B²-4AC= 4x²y²-4(x²-a²)(y²-b²)=0 4x2y2- 4(x2y2- x2b2 -a2y2+a2b2)=0 4x2y2 - 4x2y2 + 4x2 b2 + 4a2y2 - 4a2b2=0 $i H^{2}b^{2} = 4x^{2}b^{2} + 4a^{2}y^{2} = 4a^{2}b^{2}$ $i H^{2}b^{2} = 7^{2} + y^{2}b^{2} = 1$





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5. Find the envelope of the family of lines % wso + y sin 0 = 1; 0 being the parameter. Notini Given & LOSO + 4 Gind=1 -Ditt. p.w.r. to (1) w.r. t'O' we get. - * sind + 4 650 = 0 -> 0 Forwaring and adding (the (2 coso + 4 sino)2 + (-2 sino+4 coso)=12+02 7/2 cos20 + 42 Lin20 + 2xy coso sin0. + $\frac{x^2}{a^2}$ sin²0 + $\frac{y^2}{b^2}$ cos²0 - $\frac{2xy}{ab}$ cos 0 sin0 = 1 $\frac{\chi^{2}}{2} \left[\cos^{2}\theta + \sin^{2}\theta \right] + \frac{y^{2}}{b^{2}} \left[\cos^{2}\theta + \sin^{2}\theta \right] = 1$ 2+4/2=1





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b. Find the envelope of x sec 0 - ytan 0 = a where 'O' being the parameter. Given X Sec 0 - y tan 0 = a -> 0 x seco = a +y +an Q. Aquaring both sides. x sec = a + day tand +y tand x2(1+tan 0) = a2 + 2ay tano + y2 +an20 x2 + x2 +año = y2 +año + 2ay +ano + a2 (y2-x2)+an0 + 2ay tano + (a2-x2)=0 $(1.8)(y^2 - x^2)m^2 + 2aym + (a^2 + x^2) = 0$. Where 'm = tand which is a quadratic form in m Here $A = y^2 - x^2$, B = aay; $C = a^2 - x^2$ The envelope is B-4AC=0. $4a^{2}y^{2} - 4(y^{2} - x^{2})(a^{2} - x^{2}) = 0$ hay - 4[a2y2-x2y2-x2+x4]=0 4ªy2 - 4ay +x2y2+ +4x2a - 4x4 =0 > by x => y2+a2-x2=0 $\chi^2 - \eta^2 = \alpha^2$.