



**Topic: 3. 6 – EVOLUTES**

6. Find the evolute of the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Soln:

The parametric equation of the hyperbola,

$$x = a \sec \theta; \quad y = b \tan \theta.$$

$$\frac{dx}{d\theta} = a \sec \theta \tan \theta; \quad \frac{dy}{d\theta} = b \sec^2 \theta.$$

$$y_1 = \frac{b \sec^2 \theta}{a \sec \theta \tan \theta} = \frac{b}{a} \cdot \frac{1}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta}$$

$$y_1 = \frac{b}{a} \operatorname{cosec} \theta$$

$$y_2 = \frac{d}{d\theta} \left( \frac{b}{a} \operatorname{cosec} \theta \right) \cdot \frac{d\theta}{dx} \quad \left| \frac{d}{d\theta} \operatorname{cosec} \theta = -\frac{\operatorname{cosec} \theta}{\cot \theta} \right.$$

$$= -\frac{b}{a} \operatorname{cosec} \theta \cdot \cot \theta \cdot \frac{1}{a \sec \theta \tan \theta}$$

$$y_2 = -\frac{b}{a^2} \frac{\cos^3 \theta}{\sin^3 \theta} = -$$



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$$\bar{x} = x - \frac{y_1}{y_2} (1 + y_1^2)$$

$$= a \sec \theta - \frac{b}{a} \operatorname{cosec} \theta \left( \frac{-a^2}{b} \frac{\sin^3 \theta}{\cos^3 \theta} \right) (1 + \frac{b^2}{a^2} \operatorname{cosec}^2 \theta)$$

$$= a \sec \theta + a \frac{1}{\sin \theta} \frac{\sin^3 \theta}{\cos^3 \theta} \left[ a^2 + \frac{b^2}{a^2} \frac{1}{\sin^2 \theta} \right]$$

$$= a \sec \theta + a \frac{\sin^2 \theta}{\cos^3 \theta} \left[ \frac{a^2 \sin^2 \theta + b^2}{a^2 \sin^2 \theta} \right]$$

$$= a \sec \theta + \frac{1}{a \cos^3 \theta} \left[ a^2 (1 - \cos^2 \theta) + b^2 \right]$$

$$= a \sec \theta + \frac{1}{a \cos^3 \theta} \left[ a^2 - a^2 \cos^2 \theta + b^2 \right]$$

$$\bar{x} = a \sec \theta + \frac{1}{a} \sec^3 \theta \left[ a^2 - a^2 \cos^2 \theta + b^2 \right]$$

$$a \bar{x} = a^2 \sec \theta + \sec^3 \theta \left[ a^2 - a^2 \cos^2 \theta + b^2 \right]$$

$$= a^2 \sec \theta + a^2 \sec^3 \theta - a^2 \cos^2 \theta \sec^3 \theta + b^2 \sec^3 \theta$$

$$= a^2 \sec \theta + a^2 \sec^3 \theta - a^2 \sec \theta + b^2 \sec^3 \theta$$

$$(a \bar{x})^{\frac{2}{3}} = (a^2 + b^2)^{\frac{2}{3}} \sec^2 \theta$$



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$$\begin{aligned}\bar{y} &= y + \frac{(1+y_1^2)}{y_2} \\ &= b \tan \theta + \left( \frac{-a^2 \sin^3 \theta}{b \cos^3 \theta} \right) \left( 1 + \frac{b^2}{a^2} \operatorname{cosec}^2 \theta \right) \\ &= b \tan \theta - \frac{a^2}{b} \tan^3 \theta \left[ \frac{a^2 + b^2 \operatorname{cosec}^2 \theta}{a^2} \right] \\ \bar{y} &= b \tan \theta - \frac{\tan^3 \theta}{b} [a^2 + b^2 \operatorname{cosec}^2 \theta] \\ b\bar{y} &= b^2 \tan \theta - a^2 \tan^3 \theta - b^2 \tan^3 \theta \operatorname{cosec}^2 \theta \\ &= b^2 \tan \theta - a^2 \tan^3 \theta - b^2 \tan \theta \sec^2 \theta \\ &= b^2 \tan \theta - a^2 \tan^3 \theta - b^2 \tan \theta (1 + \tan^2 \theta) \\ b\bar{y} &= -(a^2 + b^2) \tan^3 \theta \\ (b\bar{y})^{\frac{2}{3}} &= (a^2 + b^2)^{\frac{2}{3}} \tan^2 \theta \\ (a\bar{x})^{\frac{2}{3}} - (b\bar{y})^{\frac{2}{3}} &= (a^2 + b^2)^{\frac{2}{3}} (\sec^2 \theta - \tan^2 \theta) \\ (a\bar{x})^{\frac{2}{3}} - (b\bar{y})^{\frac{2}{3}} &= (a^2 + b^2)^{\frac{2}{3}} \\ \text{changing } \bar{x} \text{ and } \bar{y} \text{ by } x \text{ and } y \\ (ax)^{\frac{2}{3}} - (by)^{\frac{2}{3}} &= (a^2 + b^2)^{\frac{2}{3}}\end{aligned}$$