



**Topic: 3. 4 – CIRCLE OF CURVATURE**

Equation of the circle of curvature.

Let  $(\bar{x}, \bar{y})$  be the centre of curvature and  $\rho$  be the radius of curvature corresponding to a point  $(x, y)$  of the given curve. The equation of the circle of curvature is

$$(x - \bar{x})^2 + (y - \bar{y})^2 = \rho^2.$$

1. find the circle of curvature at the point  $(\frac{a}{4}, \frac{a}{4})$  of the curve  $\sqrt{x} + \sqrt{y} = \sqrt{a}$

Soln:

circle of curvature formula is

$$(x - \bar{x})^2 + (y - \bar{y})^2 = \rho^2$$

$$\text{where } \bar{x} = x - \frac{y_1}{y_2} (1 + y_1^2)$$

$$\bar{y} = y + \frac{(1 + y_1^2)}{y_2}$$



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Given  $\sqrt{x} + \sqrt{y} = \sqrt{a}$ .

Diff. w.r.t  $x$ .

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = 0.$$
$$\Rightarrow y_1 = -\frac{\sqrt{y}}{\sqrt{x}} = \frac{-\sqrt{y}}{\sqrt{x}}.$$
$$y_1\left(\frac{a}{4}, \frac{a}{4}\right) = -\frac{\sqrt{\frac{a}{4}}}{\sqrt{\frac{a}{4}}} = -1.$$
$$y_2 = -\left[ \frac{\sqrt{x} \cdot \frac{1}{2\sqrt{y}} \left(\frac{dy}{dx}\right) - \sqrt{y} \left(\frac{1}{2\sqrt{x}}\right)}{x} \right]$$
$$y_2\left(\frac{a}{4}, \frac{a}{4}\right) = -\left[ \frac{\sqrt{\frac{a}{4}} \cdot \frac{1}{2\sqrt{\frac{a}{4}}} (-1) - \sqrt{\frac{a}{4}} \cdot \frac{1}{2\sqrt{\frac{a}{4}}}}{\frac{a}{4}} \right]$$
$$= -\left[ \frac{-\frac{1}{2} - \frac{1}{2}}{\frac{a}{4}} \right] = \frac{4}{a}.$$
$$r = \frac{(1 + y_1^2)^{3/2}}{y_2} = \frac{(1 + 1)^{3/2}}{4/a} = 2^{3/2} \cdot \frac{a}{4}$$
$$r = \frac{a}{\sqrt{2}}$$
$$\bar{x} = x - \frac{y_1 (1 + y_1^2)}{y_2}$$



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$$\begin{aligned}\bar{x}(a/4, a/4) &= \frac{a}{4} - \frac{(-1)}{4/a} (1 + (-1)^2) \\ &= \frac{a}{4} + \frac{a}{4}(2) = \frac{2a+a}{4} = \frac{3a}{4}\end{aligned}$$

$$\bar{y} = y + \frac{(1+y_1^2)}{y_2}$$

$$\bar{y}(a/4, a/4) = \frac{a}{4} + \frac{(1+1)}{4/a} = \frac{a}{4} + \frac{a}{4}(2) = \frac{3a}{4}$$

The centre of curvature is  $(\frac{3a}{4}, \frac{3a}{4})$

∴ The equation of centre of curvature is

$$(x - \bar{x})^2 + (y - \bar{y})^2 = r^2 \dots$$

$$(x - \frac{3a}{4})^2 + (y - \frac{3a}{4})^2 = \frac{a^2}{2}$$

2. Find the equation of the circle of curvature at  $(c, c)$  on  $xy = c^2$ .

Soln: circle of curvature formula is

$$(x - \bar{x})^2 + (y - \bar{y})^2 = r^2$$

Given  $xy = c^2$ .

$$y = \frac{c^2}{x} \Rightarrow y_1 = \frac{dy}{dx} = -\frac{c^2}{x^2}$$

$$y_2 = \frac{2c^2}{x^3}$$

$$y_1(c, c) = -\frac{c^2}{c^2} = -1 ; y_2(c, c) = \frac{2c^2}{c^3} = \frac{2}{c}$$



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$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2} = \left[1+(-1)^2\right]^{3/2} \cdot \frac{c}{2} = 2^{3/2} \cdot \frac{c}{2}$$
$$\rho = c\sqrt{2}$$
$$\bar{x} = x - \frac{y_1}{y_2} (1+y_1^2)$$
$$\bar{x}_{(c,c)} = c - (-1) \frac{c}{2} [1+(-1)^2] = c + \frac{2c}{2} = 2c.$$
$$\bar{y} = y + \frac{(1+y_1^2)}{y_2} = c + \frac{(1+(-1)^2)}{2} \cdot c$$
$$= c + \frac{2c}{2} = 2c.$$

$\therefore$  the equation of circle is  $(x-2c)^2 + (y-2c)^2 = 2c^2$